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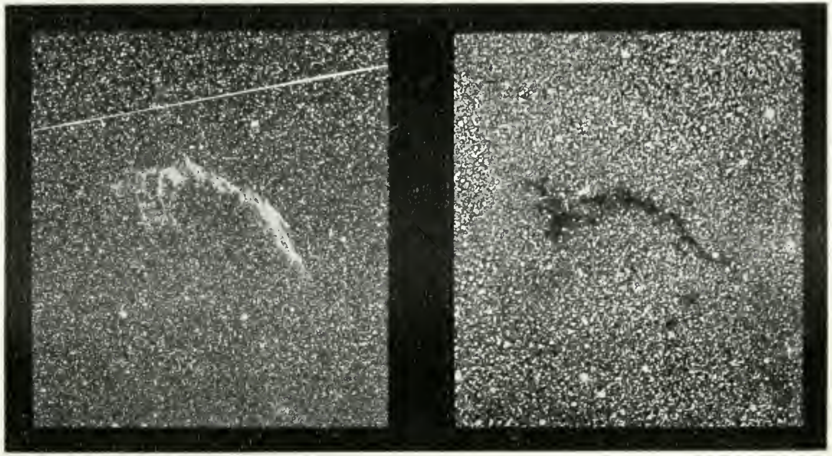
PLATE I



North

Gaseous Nebula N.G.C. 6995
1909 July 15. Exposure 5^h43^m

Dark Object in Cepheus
Shown on Plate II

Scale: 1 cm = 27′

THE
ASTROPHYSICAL JOURNAL
AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XLIII JANUARY 1916 NUMBER 1

SOME OF THE DARK MARKINGS ON THE SKY AND WHAT THEY SUGGEST

By E. E. BARNARD

Our knowledge of the universe is confined to the stars and their attendant phenomena and to the nebulae. It is admitted that there are a great many dark stars, and there is even a claim that there are more dark than bright stars. This is entirely plausible, because we must assume that the stars ultimately die out or cease to give light. That they are everlasting, so far as their light is concerned, is more than improbable. If this is reasonable with respect to the stars, what can be said of the nebulae? If we can free ourselves from the belief that the function of a nebula is to become a star, or stars, we can imagine some similar condition to that of the stars to prevail among these great bodies, that is, that they may become dark in the course of time. Should this be true we shall have space peopled with bright and dark nebulae. Indeed, we shall have all the grades between a bright nebula and one entirely devoid of light.

Photographs of the sky often show us large dark markings comparable in size with the nebulae, scattered here and there over the heavens. They are found especially in the region of the Milky Way. The first impression these objects give is that they are openings in the rich regions of stars through which we look out into the blackness of space beyond. The more familiar we become with them, however, the less this explanation appeals to us, and we soon begin to suspect that most of them are really dark or feebly luminous bodies shown in relief against a brighter background. Some of them are undoubtedly real vacancies, or places where the stars are actually thinned out, but these particular ones at once proclaim their nature and leave one in little doubt as to their explanation.

At present we are interested only in those dark markings that give the impression of being real objects and not simply vacancies. What their nature is we do not know, and the spectroscope cannot help us because the objects are devoid of light, or nearly so. But there is strong evidence that they are of the nature of the nebulae —that is, that they are dark nebulae.

Some of the dark stars make their presence known by their perturbing effect on the motions of bright stars, and by the eclipsing of their light. Their existence would otherwise be unknown, for their angular dimensions are too small for them to be seen in relief against any luminous background. Non-luminous nebulae, though their presence may never be known by any disturbance in neighboring bodies, would, if more or less opaque, be large enough to be seen if projected against a brighter background.

I think the general belief among astronomers is that a nebula remains luminous, and finally develops into a star or system of stars—that is, its ultimate destiny is a stellar condition. This opposes any supposition that a nebula may become dark by the loss of its light. For the presence of a dark nebula, however, it is not necessary that it should have lost its light. It may never have been luminous. It is possible that the original condition of a nebula is dark. It may be that some nebulae never become luminous, or it may still be possible that there are great non-luminous, opaque bodies in space that are not necessarily related to the nebulae, though this is perhaps far-fetched. From the fact that some of the nebulae appear to be partly dark and partly transparent, we might infer that they may, in the course of time, become either wholly bright or wholly dark.

In the case of Hind's variable nebula in Taurus, we have undeniable proof that a nebula may lose its light. Some fifty or sixty years ago this nebula lost all or most of its light, and from a conspicuous object in a small telescope, it ceased to be visible even in the most powerful instruments. Though it is again feebly visible in very powerful telescopes, it is mainly by photography that we can see it at all. What has occurred to this may happen to any other nebula, but the changes may take a much longer period of time.

I have shown that in certain cases a nebula may be opaque, or partly so. This is strikingly evident in the case of the great nebula about ν Scorpii and the one about ρ Ophiuchi.[1] In the case of the first of these nebulae it was shown that parts of it seemed to transmit the light of the stars behind it, and other parts—especially the darker or more obscure portions—seemed to cut out the light of the stars entirely. This possible opacity of a nebula leads us to a probable explanation of some of the smaller starless regions. That they are due to the interposition of masses of dark nebulosity seems more than probable, especially as long-exposure photographs frequently show feeble traces of nebulosity in these starless spots. A striking illustration of this is shown in the dark spot in Taurus (1855. 0, $\alpha=4^h10^m$, $\delta=+27°55'$) where what appears to be a dark hole in the sky is exactly filled with excessively feeble nebulosity with dark details.[2] There are other equally striking examples of this connection of feeble nebulosity with vacancies. Perhaps a more remarkable example of this condition is that of the great nebula of ρ Ophiuchi and its immediate surroundings. All that is needed to make these dark bodies visible is a luminous region behind them. This is supplied in one way by the rich stellar regions of the Milky Way. An excellent example of how such a thing may be possible is shown by a phenomenon that presented itself to me one beautiful transparent moonless night in the summer of 1913, while I was photographing the southern Milky Way with the Bruce telescope. I was struck with the presence of a group of tiny cumulous clouds scattered over the rich star-clouds of Sagittarius. They were remarkable for their smallness and definite outlines—

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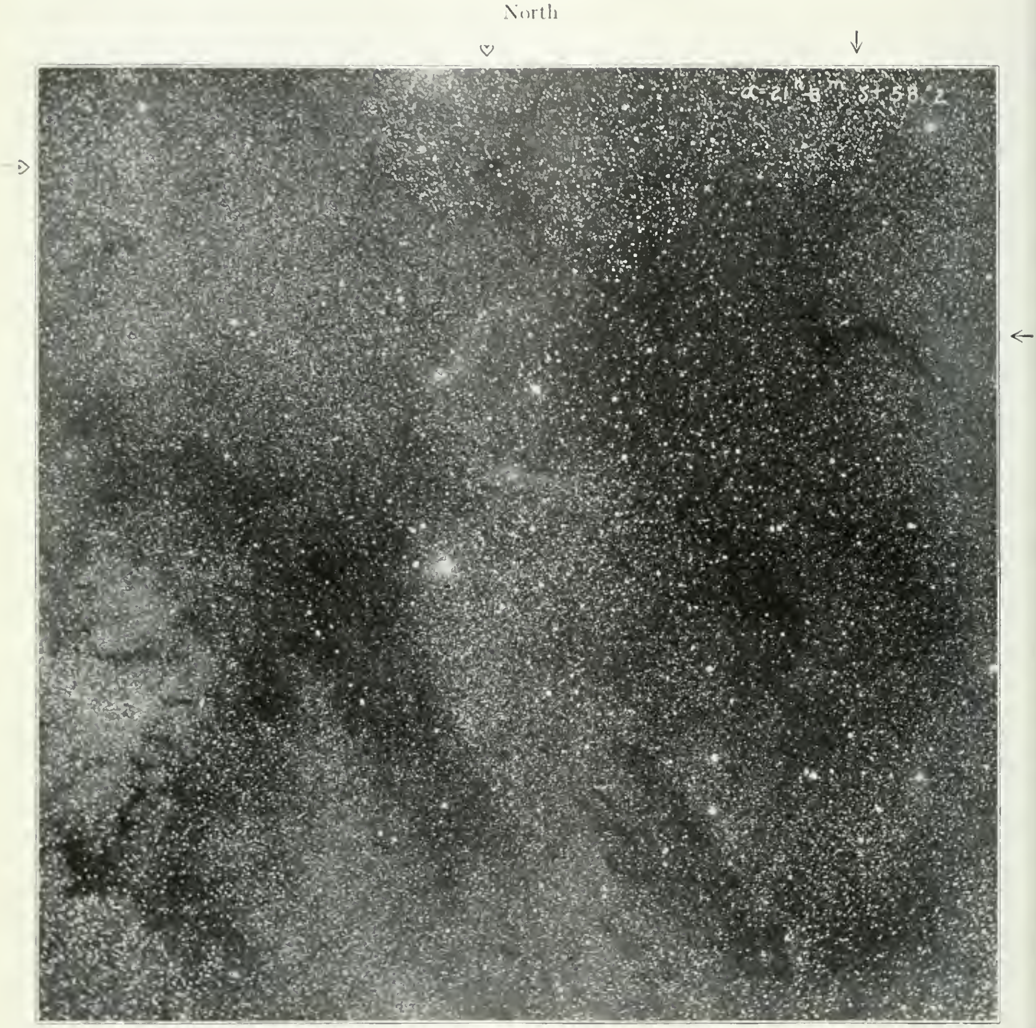
[1] *Astrophysical Journal*, 31, 8, 1910.

[2] *Ibid.*, 25, 218, 1907.

some not being larger than the moon. Against the bright background they appeared as conspicuous and black as drops of ink. They were in every way like the black spots shown on photographs of the Milky Way, some of which I was at that moment photographing. The phenomenon was impressive and full of suggestion. One could not resist the impression that many of the black spots in the Milky Way are due to a cause similar to that of the small black clouds mentioned above—that is, to more or less opaque masses between us and the Milky Way. I have never before seen this peculiarity so strongly marked from clouds at night, because the clouds have always been too large to produce the effect.

Enough has been said, perhaps, to show that, whatever their condition may have been or may become, there are dark opaque objects scattered here and there over the heavens, which resemble, in all but light, the regular nebulae of the sky. Perhaps if we show a close resemblance in form and size of one of these to one of the well-known nebulae, it may aid us in connecting the two kinds of objects. For this purpose I have prepared Plate I from my photographs, which shows two different objects in different parts of the sky, one representing the bright nebulae and the other the "dark" nebulae. The photographs are on exactly the same scale and show that the two objects are of almost exactly the same angular size. The first is the nebula N.G.C. 6995 in Cygnus (1860.0, $\alpha = 20^{h}51^{m}$, $\delta = +30^{\circ}41'$), the other a dark marking recently found by me in Cepheus, in the position: 1860.0, $\alpha = 20^{h}48^{m}$, $\delta = +59^{\circ}\!.6$. There is a striking resemblance in the forms of the two objects; but one is a luminous nebula and the other a dark—what? One can readily see that if the nebula were to lose its light, it would, if dense enough, still be shown against the sky and would strongly resemble the dark object. For this and many other reasons I am constrained to believe that the dark object is really a non-luminous nebula seen against a luminous background. There is no visible evidence of the ordinary nebulosity in the immediate region of this dark object; that is, the background is not the ordinary diffused nebulosity so frequently shown on photographs, nor is the stellar stratum dense enough to

PLATE II



Region of Milky Way South of Alpha Cephei, Showing Two Dark Objects at Points Indicated

10-inch Bruce Telescope. 1910 Oct. 1. Exposure $6^h 2^m$
Center at $\alpha = 21^h 8^m$, $\delta = +58^\circ.2$ (1855.0). Scale: 1 cm = 34′.8

serve as a luminous background. There is every evidence in the original photograph of a continuous, uniform luminosity over this entire region that is real and not due to the ordinary fogging of the plate by atmospheric luminosity. The idea has therefore suggested itself to me, and the suggestion has been strengthened by the appearance of other regions, that there is possibly a feeble luminosity in space, sufficiently strong to impress itself on the sensitive plate by prolonged exposures. I am, of course, familiar with the fact that our atmosphere is more or less luminous and that it fogs the plate with prolonged exposures, but palpably this could never serve as a background for the distant nebulae.

That the "dark" nebula of Plate I is between us and the stars in this region is clearly proved by the fact that it blots out the few stars that must be behind it. From an inspection of the original negatives it is clear that its visibility is not due to that fact, however, for the stars are too few at that point to serve as a luminous background.

That this luminous condition of the sky is widely extended in this region is shown by the presence of another similar, but much smaller, dark spot on the same plate, one degree south of α Cephei, which has a small star in it. This small object is close to and south of the 8.9 magnitude star B.D.$+61^\circ 2903$. It is in the position 1855.0, $\alpha = 21^h 11^m.2$, $\delta = +61^\circ.8$, and is well shown on Plate II.

This last plate shows near its center a large fragmentary ring of nebulosity, part of which seems to be connected with a nebulous star, B.D.$+57^\circ 2309 (6^M.5)$. It lies in the position, roughly, $\alpha = 21^h 10^m$, $\delta = +58\frac{1}{2}^\circ$.

On the eastern edge of the plate is part of a remarkable nebula which is ramified with singular "dark lanes," which somewhat resemble the dark lanes in Taurus.[1] The nebulous star ($+57^\circ 2309$) and nebulosities mentioned above were first shown on my plates with the Willard lens taken at the Lick Observatory.[2]

Two photographs were made with the 6-inch and 10-inch lenses of the Bruce telescope on September 30 and October 1, 1910, with exposures of 5^h5^m and 6^h2^m respectively. Plates I and II were made from the second of these negatives. I had hopes of making an entire night's exposure on this region, but the conditions have not been favorable. Such a long exposure was begun on October 9, 1915, with the dark object central on the plate, but after nearly six hours the sky suddenly clouded. The resulting negative verifies the other two, and shows that the marking is apparently on the sky itself, that is, it is not due to the presence of any ordinary nebulous background. Though this marking is purely a silhouette, and has no detail on its surface, there is much detail in the outline of its eastern end, which is more or less convoluted and has sharply defined projections from it like the horns of an insect. The body of it is much broken up.

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[1] *Astrophysical Journal*, 25, 218, 1907.

[2] See *Publications of the Lick Observatory*, Vol. 11, Plate 82.

If I have proved that there are dark objects in the heavens that are shown on photographs through being projected on a luminous ground I have opened the way to prove something else. This very fact of there being a luminous background may prove of the greatest value to us in our solution of the problems of space, because one form of this background suggests a feeble luminosity through interstellar regions and perhaps beyond.

If we assume that dark and opaque bodies exist in space, we could expect their presence to be shown by several means, all of which must be capable of producing a luminous background for the object. The kind of background will also give us some idea of the relative distances of these dark bodies. Such a body, if between us and the Milky Way, would appear black and conspicuous. We have many cases where such objects apparently exist in the Milky Way. One of the finest examples is that of the small black spot in a bright star cloud in Sagittarius (1855.0, $\alpha = 18^h7^m$, $\delta = -18°17'$), which seems to be an opaque body in relief against the mass of bright stars. If one of these bodies were projected against a nebulous ground it would also be visible. We have examples of this, one being the dark spot in the nebulous strip running south from ζ Orionis,[1] which is probably a darker mass of the nebula itself, seen in relief against the more luminous portion of the nebula. Another beautiful example of this kind is shown in photographs of the very elongated nebula 24 Comae[1] (N.G.C. 4565), which seems to be an object similar to the great nebula of Andromeda, with its edge toward us, where the darker outer periphery of the nebula is seen cutting across the brighter central region as a black irregular streak.

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[1] *Astrophysical Journal*, 38, 496, 1913.

These examples are exceedingly interesting and important, because in the case of those objects seen against the Milky Way, it shows that these dark nebulae in many cases are nearer than the small stars that form the main body of the Milky Way. This in itself is not surprising to me, for we have many examples of bright nebulae that must be nearer than the more distant stars, such as the great nebula of ρ Ophiuchi, parts of which are also opaque nebulosity, and the nebula about σ Scorpii. Both of these are intimately connected with bright stars and must be relatively near to us. But there are others of these dark bodies that do not have a bright stellar background to show them, for they are seen in regions poor in stars, nor does their visibility depend upon the ordinary nebulous background, for there are no evidences of such nebulosity in the regions in which they appear. Without the stellar background or the presence of ordinary nebulosity these dark objects should not be visible. That they are distinctly visible against the sky under these circumstances would suggest, as their visibility would absolutely depend upon a more or less luminous background, that the sky beyond, for some reason, is itself luminous. If this uniform luminosity covered a very large region of the sky without any definite form or structure, there might arise a suspicion that distant space itself is more or less feebly luminous. There are a number of these dark spots whose presence is due to projection against a luminous background other than that produced by stars or nebulosity. They look like dark markings on the sky itself.

What is the cause of this luminous background where there is no ordinary nebulosity and where the stars are too few to produce it? There seems to be one possible explanation. Let us suppose space to be filled with the feeblest luminosity, so feeble, indeed, that at the distance of the fixed stars it is not sensible. Then, as space is supposed to be of very great extent, this luminosity would increase in apparent density until it would finally become dense enough to affect the sensitive plate. This then would serve as a background to show in relief any large non-luminous object that might exist in space.

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[1] See W. S. Franks, *Monthly Notices*, 65, 160, 1904.

It is not necessary to attempt here any explanation as to the cause of this supposed luminosity of space. Indeed there is no direct claim that it actually exists. It is only suggested as a possible cause of certain phenomena that seem best interpreted by its presence—and from the further fact that the photographic plate seems to show its existence over large regions of the sky.

In respect to the illustrations: on the original negative the dark object is but feebly contrasted with the sky. To show it more clearly it was necessary, in order to gain contrast, to copy it repeatedly. This has produced the coarse grain on Plate I and gives the impression of a dense stellar background which only partly exists—the west portion of the object being projected on a much richer region of stars. The original plate is slightly fogged to the right of the dark nebula, which produces the luminous appearance close to the edge of the plate. The small plate is intended mainly to show the form of the object. The large one gives a truer representation of this region of the sky, with the exception of the slight fogging at the right side of the plate. The bright line on the photograph of N.G.C. 6995 is a large meteor trail.

An important fact that may come from our knowledge of the existence of dark nebulae is that their masses must be much greater than would be assumed for the ordinary nebulae, because they are perfectly opaque and must be relatively dense, and hence comparatively massive. If this is so, then, we must take into account these great masses in a study of the motions of the stars as a whole.

YERKES OBSERVATORY
WILLIAMS BAY, WIS.
October 26, 1915

FACTORS AFFECTING THE RELATION BETWEEN PHOTO-ELECTRIC CURRENT AND ILLUMINATION

By HERBERT E. IVES, SAUL DUSHMAN, AND E. KARRER

I. INTRODUCTION

Experiments described in an earlier paper[1] showed that the relationship between illumination and current in potassium photo-electric cells as ordinarily constructed for photometric purposes is not rectilinear. No explanation was discovered for the extremely varied relationships found from cell to cell, but in general it was established that the relationship between illumination and current is a complicated function of applied voltage, electrode distance, and gas pressure. The investigations now reported have resulted in discovering the reason for the effects previously found, and in establishing the conditions necessary for a straight-line relation between illumination and photo-electric current. The various steps in the investigation will be described in the order in which they fall chronologically, for while some of them followed false scents, they furnished information which may answer questions arising in other connections.

II. EFFECT OF VARIATION OF GAS PRESSURE. CHARACTERISTICS OF HIGH-VACUA CELLS

The work previously reported included experiments on the effect of variation of the pressure of the gas within the cell. These showed gas pressure to be one of the significant variables. The new experiments fall under two heads: first, experiments on altering the pressure without opening or otherwise mechanically disturbing the tube; and, secondly, experiments with tubes with much better vacua than those previously used.

The experiments under the first head, in so far as they bear on effects of both illumination and voltage, were confined to a single cell. This, lettered *d* in the previous paper, lent itself, because of its manner of construction, to a trial of the effect on the vacuum of a long-continued glow discharge. This discharge could be passed, as is evident from inspection of its construction (figured in the earlier paper), from the long platinum electrode to the guard-ring, without presumably affecting the potassium surface.

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[1] Ives, "The Illumination-Current Relationship in Potassium Photo-electric Cells," *Astrophysical Journal*, 39, 428, 1914.

The cell was measured first before the passage of the discharge, both for the voltage-current and for the illumination-current relationships, the latter at two voltages, 261 and 174. It was then connected to a 5000-volt transformer, and a discharge passed until the vacuum reached the point where the current would no longer flow, at which point similar measurements were made. It was then connected to a small induction coil and run for a long period until no further discharge would pass. It was measured at this point, and then again after standing undisturbed overnight.

The four sets of measurements are shown in Fig. 1. Considering first the voltage-current curves, perhaps the most striking fact brought out is that the effect on sensibility of a gaseous atmosphere is altogether different at different voltages. Thus at voltages below 200 (with this particular cell) the effect of the gas is to inhibit the photo-electric current, notwithstanding the fact that the current is being assisted by ionization by collision, as is evident from the type of the voltage-current curves. It is evidently not permissible to describe a cell as being filled with gas "to the pressure which gives greatest sensibility," because this is one pressure for one voltage and another for a second. A further point of interest in connection with the voltage-current curves is that even at the best vacuum attained, saturation is not approached. This is presumably due, not alone to the imperfect vacuum, but to the inadequate anode area.

Before taking up the illumination-current relation in this cell, an additional and more complete set of data on the effect of gas pressure on sensibility may be reported.

The cell used for this study was similar in construction to those used by Elster and Geitel. It consisted of a 2-inch bulb with a platinum wire as an anode. The cathode was prepared by melting the ordinary potassium in a side tube which was connected to the

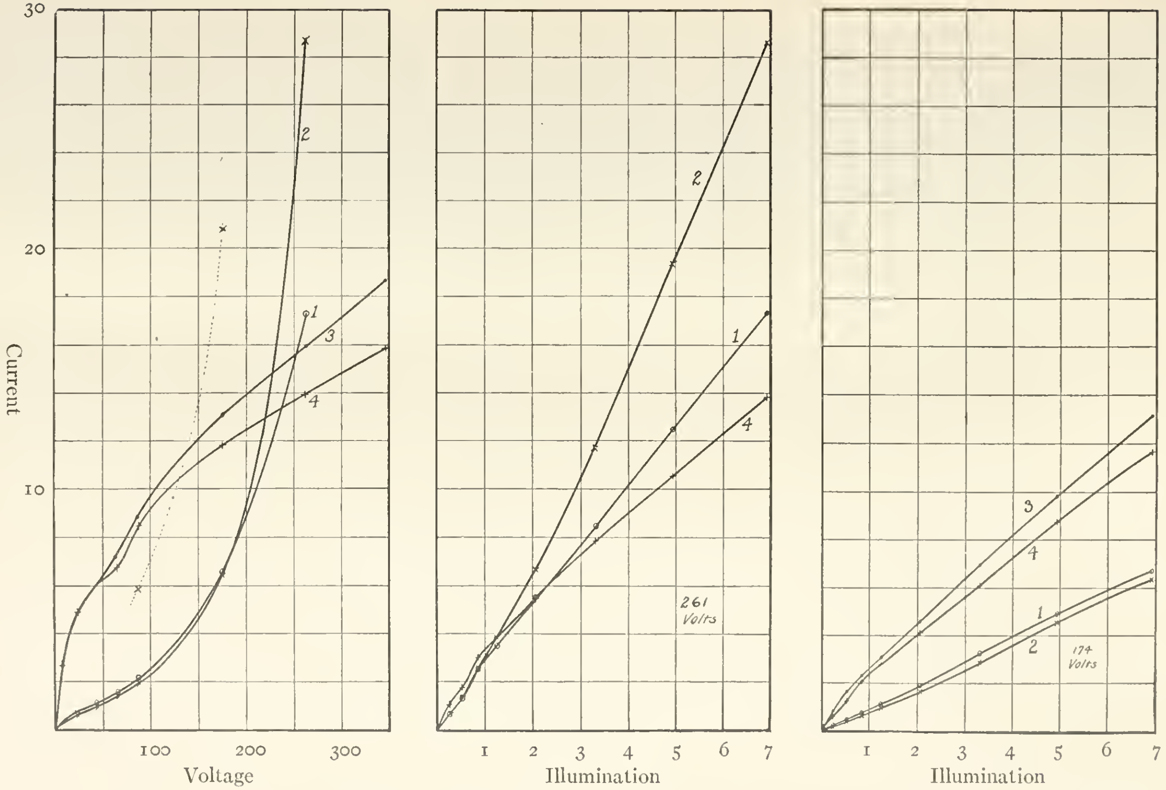

FIG. 1.—Characteristics of cell *d*, as pressure and other conditions were changed by the passage of a glow discharge between anode and guard-ring.

cell by means of a U-tube. The molten metal was distilled from adherent oxide into the cell under vacuum. The latter was connected by means of a mercury-seal stopcock to a McLeod gauge and reservoir containing purified hydrogen gas. A liquid-air trap was inserted between the cell and the rest of the system, thus preventing water vapor and carbon dioxide gas from entering the cell.

Observations were taken of the photo-electric current with constant illumination but different voltages. The source of the illumination was a 172 c.p. tungsten lamp placed about 4 inches above the potassium surface. The relative positions of the lamp and cell, as well as the voltage over the lamp, were maintained constant during the different sets of measurements. Fig. 2 illustrates graphically the results of these observations at different pressures of hydrogen gas. The numbers in parentheses indicate the order in which the observations were carried out.

The freshly prepared cell had a residual gas pressure of about 4 bar. This may have been either nitrogen or hydrogen gas—more likely the latter. The current-voltage curve obtained under these conditions is shown by (1), Fig. 2. It will be noted that the current varied only slightly with the voltage, but as the variation was linear, there was apparently no tendency toward a limiting current-value.

Hydrogen was allowed to enter the cell until a pressure of 1900 bar was attained, and the observations shown in curve (2) were obtained. The pressure was then reduced to 1380 bar and 600 bar in succession, as shown in curves (3) and (4). During the latter set of measurements an interesting effect was observed. The voltage applied to the cell was 160 at first and was then gradually increased to 220. The galvanometer deflection was observed to increase correspondingly from about 400 to 1450 divisions. On the changing of the voltage, however, from 220 to 240 volts, the photoelectric current increased at least tenfold and a glow appeared in the cell, indicating that a Geissler discharge had occurred. On the decreasing of the voltage now to 180, the galvanometer deflection was found to be 1700, that is, the sensitiveness of the cell had been increased considerably by the discharge. That this increase was not temporary is shown by the character of the subsequent curves.

Fig. 2, curve (5), gives the results of the observation obtained at 600 bar after the discharge. The general shape of the curves has remained unaltered, but the sensitiveness at the same voltage has increased in a ratio of approximately 2 to 1.

As the pressure of hydrogen is lowered from 260 bar to 1.9 bar, the current-voltage curves become flatter, so that (12) is quite similar to (1), but owing to the discharge previously mentioned, the photo-electric currents are over twice as great for the same voltage.

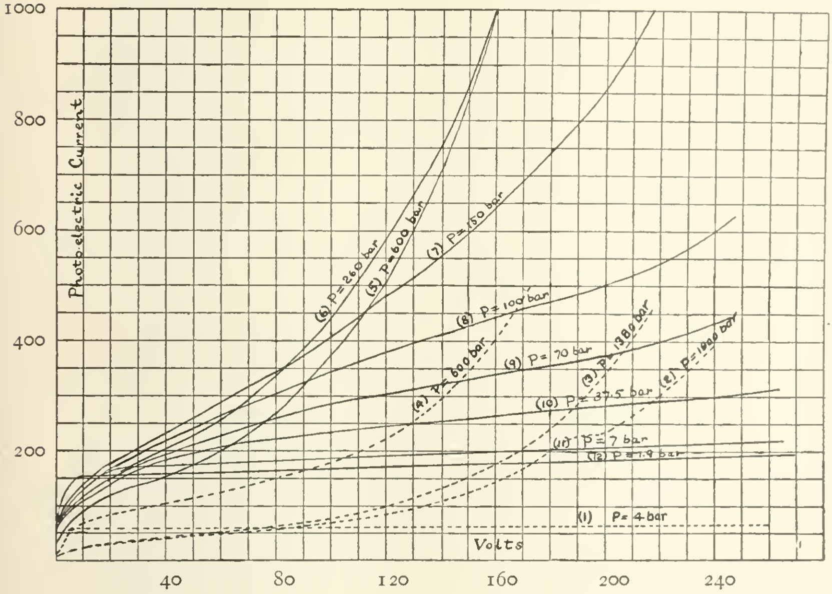

FIG. 2.—Voltage-current curves of cell, measured on pump, at various gas pressures. Dotted curves taken before passage of glow discharge, full curves after.

Any discharges, whether from a source of direct current at 240 volts (at higher pressures), or from an induction coil (at pressures of 1 to 2 bar), increased the sensitiveness of the cell, and usually there was an evolution of gas, as if the effect of the discharge consisted in decomposing some photo-electrically inactive compound on the surface. This hypothesis would also explain the subsequent "fatigue" which usually followed the increased sensibility. For, assuming a static equilibrium between the residual gas in the cell and the gas present on the surface (either chemically combined or as an absorbed layer), the tendency would be for this equilibrium to re-establish itself after the discharge had ceased. That thin gas films inhibit the electronic emission from the surface of incandescent metals has already been shown pretty conclusively by the work of Langmuir,[1] and it appears quite probable that in the case of photo-electronic emission these effects would be even more pronounced.

Turning now to the illumination-current curves for cell *d*, a complicated state of affairs is found. At the lower voltage we have a complete transformation from curves that are convex to the illumination axis to curves that are concave. At the higher voltages similar changes occur, on which are superposed other smaller irregularities, and in addition, in agreement with the voltage-current curves, the relative sizes of the currents are different at the two voltages. Thus at 174 volts the order of increasing size of current is 2, 1, 4, 3; at 261 volts it is (neglecting the confusion near the origin) 4, 3, 1, 2.

These curves show in most convincing form how far from true is the unqualified statement that the photo-electric current is directly proportional to illumination. They show further, apparently, that the gas pressure, under the conditions present in certain cells, has an enormous influence on the actual relationship. We say *apparently* because, while the gas pressure was greatly changed, this change was also probably accompanied by changes in the amount of metal, grease, etc., in the cell walls, owing to the passage of the glow discharge. These latter changes, by altering the capacity of the cell walls to become charged, were, in view of our later work, probably the more important factors.

The experiments falling under the second head were undertaken in order to learn the character of the relationship of illumination to current when all gas is removed, as nearly as that condition can be approximated with available means of evacuation. Eight cells were constructed, differing from the previous ones in that the evacuation was performed with the Gaede molecular pump, in conjunction with a liquid-air trap to remove all grease and other vapors.

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[1] Langmuir, "Thermionic Currents in High Vacua," *Physikalische Zeitschrift*, **15**, 524, 1914.

The glass of the cells was baked at a temperature of 350° C. for some time previous to the introduction of the potassium, and the latter was distilled into place, usually in a number of stages, in one case (to be described later) as many as seven. In one cell, *p*, the vacuum was further improved by the process of burning out a tungsten filament after the cell was sealed off from the pump. The pressure of the residual gas in these cells is believed to be not more than 10^{-3} bar.

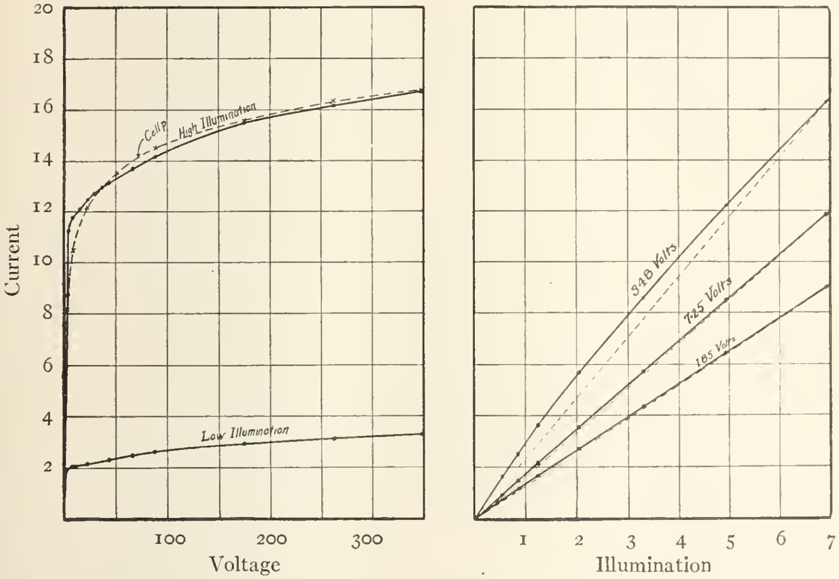

FIG. 3.—Characteristics of the typical high-vacuum cell (*m*)

These high-vacua cells differed from the earlier cells in several respects. They were all of considerably larger dimensions. While, in the earlier ones the bulbs were only 3 or 4 cm in diameter, these were from 5 to 8 or 9, and in one case still larger. The anodes in these cells were either large loops of platinum wire, or, in the later ones, grids of tungsten wire. Guard-rings were usually dispensed with, as arrangements were made to work with comparatively large currents and low sensibility of the electrometer system.

Fig. 3 exhibits the characteristics of one of these cells, *m*. The voltage-current relation is characteristic of them all. The current rises very rapidly up to about 10 volts, after which it mounts very slowly. Complete saturation is not reached at the highest voltages employed (350) in the electrometer measurements. Even at 5000 volts these cells have not shown complete saturation. The photo-electric current increases linearly with the voltage and apparently shows no tendency to approach a constant value. This failure to saturate may be due to several causes. The anode may not be sufficiently large. The grid electrodes used in several of the cells were intended to assist toward saturation, but, as is shown by the curve for cell *p* (Fig. 3), the improvement is little if, indeed, there is any. Again, there may be sufficient gas, or perhaps potassium or mercury vapor, present to prevent saturation. (The possibility of metallic vapor being present is, however, practically ruled out by the fact that surrounding the cell with carbon dioxide snow does not noticeably affect the voltage-current curve.) Probably, however, the most active cause is the reflection of electrons from the walls of the cell, which, as is well established, interferes with saturation.

Coming now to the illumination-current data, the curves of Fig. 3 are representative of all the cells only in certain features. It is characteristic of these cells that there are present none of the double curvatures and other local irregularities of the small cells. It is also characteristic of these cells that the greatest curvature is shown by the high-voltage curves.

But in the most significant of all characteristics, namely, the direction of curvature, the data on cell *m* are not representative for the reason that no two of these cells are alike. About half show illumination-current curves bending upward; the rest show curves bending downward. The certain elimination of nearly all the gaseous atmosphere and the reduction of all the cells to the same condition of vacuum, as shown by their characteristic curves, have not reduced them to uniformity in their behavior to light.

The extraordinary diversity of behavior of these cells is shown by the group of curves in Fig. 4. These, obtained (with the exception of cell *e*) from the new high-vacua cells, were measured upon a sensitive galvanometer, the currents being about one hundred times those worked with in the previous investigation. This shift of instruments was made partly to make doubly sure that no instrumental peculiarities were figuring in the results, and partly because it was becoming evident that all the illumination-current peculiarities were accentuated with high voltages and currents.

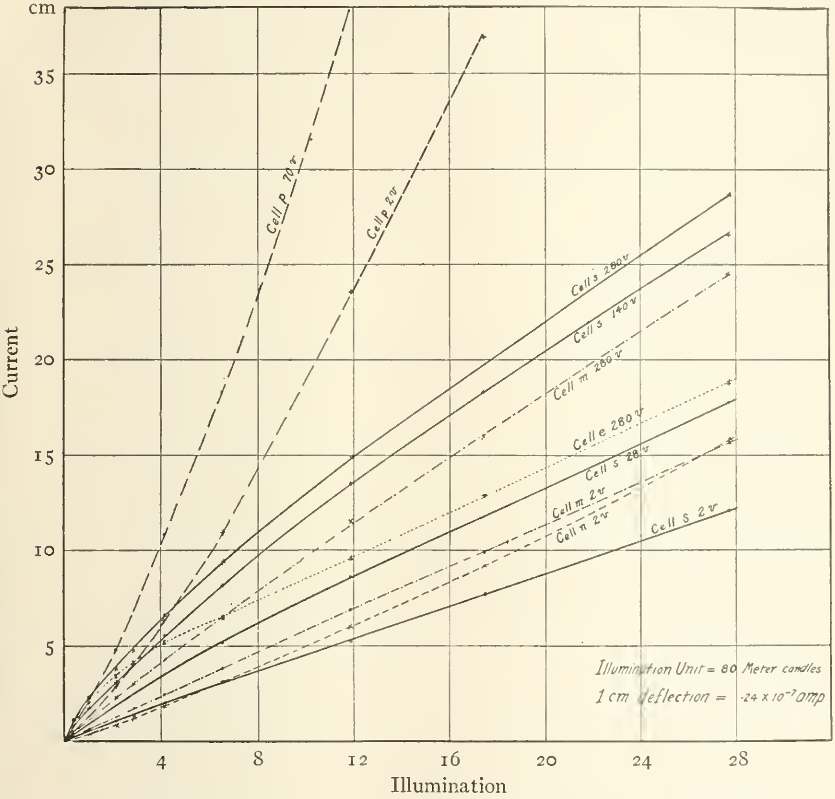

FIG. 4.—Illumination-current characteristics of a group of high-vacua cells

It is evident from these curves that the real cause of the non-rectilinear light-current relationship can hardly be the presence of gas. The striking effects of varying the gas pressure must be ascribed to the result of these changes on some other fundamental cause.

Further support for this belief was found in trying the effects of very intense illumination on cell *p*. If the gas present is a real cause of the effects found, it is to be expected that if the applied voltage is held below the ionization voltage—that is, 6 or 7 volts—its influence should be minimized. By using a large nitrogen-filled tungsten lamp, illuminations up to 50,000 meter-candles were tried with only 2 volts applied, the currents being about fifty times those with the galvanometer previously employed, or up to 2×10^{-5} amperes (they were measured on a portable millivoltmeter). The illumination-current curve under these conditions was of exactly the same pronounced curvature as with the higher voltages and smaller currents. This experiment shows pretty clearly that no effects of gas ionization are active, as might have been thought from the fact that the illumination-current curves tended to flatten out with low voltages.

The greater smoothness of the curves from the high-vacua cells was suspected from the start to be in some way due to the larger dimensions of these cells, but the significance of this was not understood until several other clues were followed to their conclusion.

Before leaving the question of the behavior of high-vacua cells, an experiment, mentioned above, on multiply distilled potassium, may be described at length, because of its important bearing on the theory of photo-electricity.

Fig. 5 illustrates the manner in which this particular experiment was carried out. The apparatus used consisted of the tube *A* and bulbs *B*, *C*, *D*, *E*, *F*, *G*, *H*, and *K* in series. These were made of German glass and were about 2 inches in diameter. The bulbs *C* and *K*, while similar in size to the others, each had a small platinum wire sealed in at the bottom and a platinum loop from the side, thus making it possible to determine the photo-electric sensitiveness of the potassium, both after the first distillation as well as after the last. In addition to these electrodes, the cell *K* also had a platinum wire sealed in from the top, as indicated in the figure. The platinum wire used in making up these cells had been previously heated to near the melting-point in a good vacuum, in order to free it of dissolved gases. Between the last bulb and the molecular pump was inserted a liquid-air trap *T*. The dotted line indicates the oven in which the cells *C* to *K* were subsequently heated during exhaust. The end of the tube *A* was left open for the introduction of the potassium (in the form of balls). A wad of glass wool was fitted in rather loosely at *g*, and after the potassium balls had been introduced into *A*, another plug of glass wool was inserted at *h* and the open end closed up with a torch. The potassium used was previously washed in ether and then dried between filter-papers.

After closing up tube *A*, the pump was started and the bulbs *C* to K heated for an hour at 350° C. Meanwhile the potassium in *A* was melted and allowed to run into *B*. During the bake-out of the rest of the apparatus, the metal in *B* was heated gently, so that while it did not distil into *C*, it was yet kept hot enough for the gradual elimination of any gases that might be contained in it. At the end of the hour, the oven was raised and the metal distilled from *B* into *C*. Only about three-quarters of the metal in one bulb was distilled into the next. This permitted the gradual elimination of sodium which was undoubtedly present in the potassium.

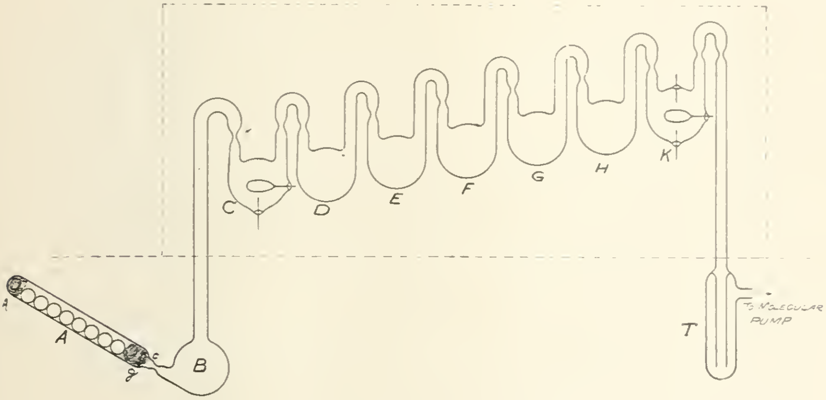

FIG. 5.—Arrangement of apparatus to investigate photo-electric effect from multiply distilled potassium in high vacuum.

After distillation into *C*, the bulb *B* was sealed off at *k*, and the photo-electric sensitiveness of the metal in *C* determined, while it was still in a molten condition. The potassium was then distilled in succession through the different bulbs, the latter being sealed off after most of their potassium content had been distilled out. The whole operation, starting from the first distillation, took about three hours.

The results of this experiment indicated, first, that the photo-electric sensitiveness of the metal in *C* was not noticeably greater that that of the final distillation product; secondly, that the *photo-electric sensitiveness does not disappear when the metal is made as gas-free as possible, and the degree of vacuum is made as high as possible.* The illumination-current curve of this cell, made after its removal from the pump, is shown by *n* in Fig. 4. It differs neither in magnitude nor in character from that of the other high-vacua cells.

This experiment was described by one of the writers at a meeting of the American Physical Society in Washington, April, 1914,[1] and while one of its objects, as stated above, was to determine the character of the illumination-current relationship at extremely high vacua, it was also intended to settle another point which is of extreme importance for the theory of the photo-electric effect itself.

For some time there had been noticed a growing skepticism, especially among German investigators, as to the existence of a pure photo-electronic emission from metals in vacua. Fredenhagen,[2] Küstner,[3] and finally Wiedemann and Hallwachs[4] had come to the conclusion that "the presence of gas is an essential condition for the existence of the photo-electric effect."

Now, while the investigations of Langmuir and his associates had shown conclusively that there exists a pure electronic emission *per ipse* from heated metals in even the highest vacua, there was every reason for believing that the photo-electric effect also would continue to persist even in the highest vacua obtainable.

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[1] See also Langmuir, "Thermionic Currents in High Vacua," *Physikalische Zeitschrift,* **15**, 524, 1914.

[2] "Das Ausbleiben des lichtelektrischen Effektes frisch geschabter Metalloberflächen bei völligem Ausschluss reaktionsfähiger Gase," *Physikalische Zeitschrift,* **15**, 65, 1914.

[3] "Das Ausbleiben des lichtelektrischen Effektes frisch geschabter Zincoberflächen bei völligem Ausschluss reaktionsfähiger Gase," *Physikalische Zeitschrift,* **15**, 68, 1914.

[4] "Dependence of the Photoelectricity of the Metal on the Gas, in Particular the Cause of the Strong Photoelectricity of Potassium," *Berichte der deutschen physikalischen Gesellschaft,* **16**, 107, 1914.

The experiment described above led us therefore to a conclusion which we had expected, but which is quite different from that arrived at by Wiedemann and Hallwachs.

Now what is the explanation of their results? The anode in their cell was a platinum wire sealed in from the top. As positive potential they applied about 8 volts; also before measuring the photo-electric current they passed a high-voltage discharge through the cell. Now we noticed in the case of cell K, when using the same positive voltages as Wiedemann and Hallwachs, *that a spark-coil brought near it caused the photo-electric current to decrease to zero.* Similar effects had been observed by Langmuir in measuring thermionic currents from heated filaments in highly evacuated bulbs, and the explanation suggested by him to the writers is as follows:

The rate at which electrons can reach the anode depends upon the voltage of the latter (space-charge effect) and upon the presence of neighboring charges. Any electrons striking the glass near the anode charge it up negatively, and in very high vacua there are not sufficient positive ions present to neutralize this charge; consequently it requires a very much higher positive potential on the anode to produce any noticeable photo-electric current. The discharge from an induction coil acts in a simlar way to charge up the glass around the anode, and with the low anode potential of 8 volts, the field near the anode is not sufficiently strong to neutralize the field, owing to the charges on the glass. However, on increasing the positive potential on the anode, or bringing a conductor near the glass so as to remove the charge, the photo-electric effect reappears and the cell acts normally.

With a given anode potential, the current to the upper electrode mentioned above in connection with the cell K was smaller than that to the loop, but the values tended to approach each other more and more as the voltage was made more and more positive.

Another point that ought to be mentioned in this connection is the observed effect of the presence of gases. It was noticed that traces of air, carbon dioxide, or water vapor sensibly decreased the photo-electronic emission from a potassium surface. On the other hand, the presence of hydrogen seemed to have no deleterious effect.

III. INVESTIGATION OF THE EFFECT OF CHANGED SURFACE CONDITIONS

Of the high-vacua cells, two were of particular interest, *p* and *s*. They both had grid anodes. Their voltage-current characteristics were the same. They were not greatly different in size, although one was approximately spherical, the other cylindrical. One of them, however, *p*, exhibited the most marked upward-bending light-current curve, the other, *s*, the most marked downward-bending curve of all the cells. They were therefore particularly suitable for crucial experiments.

They were carefully examined for differences, and the most striking one was found to be in their surface conditions. *p* had a very rough surface, evidently caused by quick cooling, while *s* had an almost mirror-like surface. The question arose whether the state of tension or annealing of the surface might not be of significance. The various shapes of the magnetization curves of iron and steel of different hardness offered counterparts for the different light-current curves, and while the phenomena are of course quite unrelated, it seemed worth while to investigate in the photo-electric cell the effect of a variable so important in the other case.

The test experiment was quite simple. Cell *p* was heated in an oil bath until its rough surface was molten, after which it was allowed to cool so slowly that it solidified as a smooth mirror. Cell *s* was similarly heated, and, when molten, was quickly taken to an open window and exposed to the outside winter temperature, being at the same time shaken so that its surface solidified into an irregular and diffusely reflecting form, closely resembling cell *p* before its transformation.

Both cells were then measured, with the very definite result that *their characteristics were absolutely unchanged.* Attention was then turned to another possible effect of surface conditions.

IV. INVESTIGATION OF THE NORMAL AND SELECTIVE EFFECTS

The known existence of two different photo-electric effects, the normal and the selective, differently localized in the spectrum, dependent on the character of the surface, the plane of polarization, and the angle of incidence, offered another suggestive field for inquiry with respect to the light-current relationship.

In order to study the light-current curves of the normal and selective effects separately, the measuring apparatus was rearranged with the object of working with molten mirror-like surfaces, using light at oblique incidence. A separate sheet-iron box was provided in which the cells in a horizontal position could be heated in an electrically warmed oil bath. From this box well-protected wires ran to the electrometer and voltage supply. The light from a point-source tungsten lamp was arranged to fall at 45° through a nicol prism upon the potassium surface. The nicol prism was mounted to turn through 90° so that the light could be polarized either in, or at right angles to, the sensitive surface. The intensity of the illumination was altered by the use of sector disks, on the employment of which a word will be said presently. By means of this apparatus the obliquely incident polarized light can be received upon a molten mirror-like potassium surface. Under these conditions with the nicol turned one way the normal effect will be obtained; turned the other way, the selective.

With regard to the use of the sector disk, it is of course essential to establish by previous experiment that Talbot's law holds. Elster and Geitel have found this to be so, but it was thought desirable to perform their experiments again. Before dealing with this, however, it must be explained why recourse to the disks was found necessary. The standard method of working with polarized light where measurements of intensity are needed is by the use of two nicols, one of which is rotated. The present work was begun in this way, but it was soon found that with the pair of nicols used (which were old and not particularly well mounted) different quadrants gave different results. Thus with the analyzer, which was nearest the potassium, unchanged in position, the light-current curve obtained by rotating the polarizer from 0 to 90° was different from that obtained between 90° and 180°. The difference, though not large, was enough to invalidate the method for the present work. It is of course due to lack of parallelism of the incident light and slightly eccentric mounting of the nicol. These matters should be carefully looked into wherever nicol prisms are used for light-variation.

The test of the sector disk was made in a somewhat roundabout fashion, necessitated by the fact that the apparatus had been rearranged before the need for recourse to the disks was found. Fortunately one of the high-vacua cells was of such an intermediate character that for a certain voltage the light-current relationship was accurately a straight line. With this cell Talbot's law was found to be accurately followed. At the same time a measurement was made of the transmission of an opaque line grating on glass in a certain position. Another cell was then taken in which the illumination-current relation was not rectilinear and a set of points obtained using both the disks and the grating. The grating point fell accurately on the curve made by the disk points, proving that Talbot's law held for this cell as well, and hence presumably for any.

The experiments with the molten potassium surface were attended with considerable experimental difficulties, chief of which was the fact that the potassium begins to distil at its melting-point. This necessitated rapid working, and an alternation of direction of making observations, to eliminate the effect of the readings decreasing in magnitude by the clouding over of the cell walls. For the final conclusive measurements, when the general character was established, observations were confined to two points, for the low point either a disk or the grating being used.

As should be the case at this angle of light-incidence, the normal current was found to be much less than the selective. But, in the case both of cell *m* and of cell *p*, *the illumination-current relation is the same in the normal and in the selective effect.*

This experiment and that described in the previous section show that the surface differences between the two cells cannot be considered as the cause of their opposite behavior to varying illumination.

V. STUDY OF FOCUSING EFFECTS

The elimination of gas and surface effects as possible causes of the non-rectilinear light-current relationship made necessary some hypothesis along quite different lines. Effects due to the electron stream itself were next considered. First to be thought of were the effects of space-charge and of reflected electrons. The former was not considered adequate, for one reason, because it would be excessively small with the currents obtained; for another reason, the most general effect of the space-charge would be to decrease the current below its normal values, as the number of electrons between the electrodes increased, while the experimental curves bend sometimes upward and sometimes downward. Reflected electrons were similarly thought to offer an inadequate explanation, for no reason was apparent why in some cases proportionately more and in other cases proportionately fewer electrons should be reflected as the number emitted was increased.

The clue which proved ultimately of most value was furnished by the idea of *focusing effects*. By this term is here meant a change of *direction* of the electron stream as the number emitted changes, whereby a different proportion of the whole number of electrons reaches the receiving electrode.

Examples of focusing effects are to be found in some of the early demonstration forms of cathode-ray tubes and in X-ray tubes. One of the former tubes, designed by Crookes to show the mutual deflection of two cathode streams, is an excellent example. Cathode rays from two adjacent, slightly inclined cathodes each normally proceed in straight paths through a common point of intersection. When, however, both streams are present together, they are mutually deflected, and their point of intersection is farther from the cathodes than the path of the individual undeflected streams would indicate. Were the current measured by a receiving anode, the current would increase more or less rapidly than the cause of the electronic emission, depending on whether the anode were inside or beyond the low-intensity focus of the two streams.

Another case of focusing is illustrated by the practice of placing the cathode of an X-ray tube back in a narrow glass stem. It has been found that this causes the cathode-ray stream to focus sharply and increase the hardness of the rays. This phenomenon is known to workers with discharge phenomena and has recently been again pointed out by T. Harris.[1] It has also been found that the position of the cathode in the neck of the discharge tubes has much to do with the intensity of the positive rays.[1]

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[1] "On the Distribution of Electric Force in the Discharge at Low Pressures," *Philosophical Magazine* (6), 30, 182, 1915.

In both cases the effect is ascribed to the presence of a strong electric field, in the first case in the cathode dark space, in the second on the glass walls of the cell, owing to the accumulation of a charge from stray electrons. These fields, varying with the strength of the current, and changing the direction or focus of the cathode-ray stream, offer satisfactory explanation of the peculiar effects found in the photo-electric cells. The ordinary photo-electric cell as constructed for photometric work, with its small dimensions, its large area of glass wall, and its straight wire or loop anode of small area, could hardly have been better designed for the purpose of producing an inextricable confusion of focused, deflected, and reflected electron streams. It is only necessary to put one of the small cells made in the earlier part of the investigation on an induction coil to get an idea, from the irregular streaks and splashes of fluorescence on the glass walls, of how complicated the conditions must be.

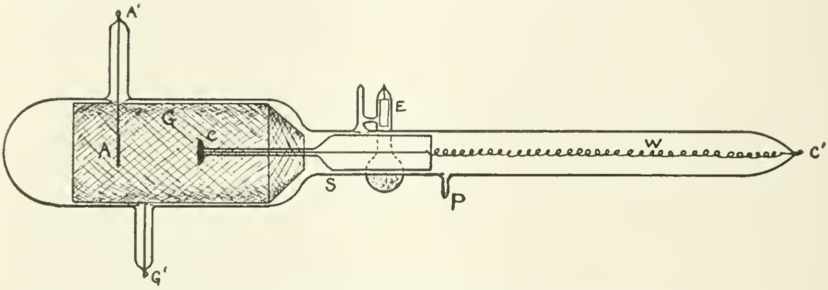

Fig. 6.—Special cell constructed to permit of various arrangements of anode and cathode. Total length, 45 cm; diameter of large part, 6 cm.

In order to test this hypothesis a special tube was constructed, shown in Fig. 6, designed to make possible a variety of different arrangements of anode, cathode, and surroundings. The cathode *C* is arranged to slide along the stem *S;* it is of glass, silvered at the end ordinarily lying in the body of the tube, the silver being connected to the sealed-in platinum wire *C′* by a short piece of platinum wire through the glass, and a long coil of copper wire. In the process of making, *C* was pulled back to the side tube *P* from which the potassium was distilled on to the silver. *G* is a fine brass wire gauze, led to an outside connection *G′* and standing free from the glass walls. *A* is the anode, an iron plate of diameter 2 cm, with a separate outside connection *A′*. *E* is a charcoal evacuator, intended to assist the Gaede mercury pump which was used in this case. This cell was thoroughly baked during exhaustion, but, probably owing to the large amount of metal used in its construction, it did not have or maintain as good a vacuum as would have been desirable. This deficiency, in the light of the results obtained from the cell and later, was not serious.

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[1] J. J. Thomson, *Rays of Positive Electricity*, p. 21.

Some nine different combinations of anode, cathode, and gauze connection, and cathode position, were tried with very interesting results.

Two of the voltage-current curves are shown in Fig. 7. Curve *a* is that obtained with gauze and anode connected to form a large inclosing anode, while the cathode is well up inside the gauze. Curve *d* is that obtained with the same connections by moving the cathode back into the stem. The latter curve was a somewhat difficult one to obtain for the reason that considerable time was required for conditions of equilibrium to be reached on deflection and return to zero. The behavior of the apparatus was entirely in accord with the idea that charging and discharging of the surrounding glass walls was taking place.

To obtain the illumination-current curves of this cell, the polarization apparatus was removed and a very short photometer track was improvised, on which was mounted a 100-watt point-source tungsten lamp. The movement of this lamp was between 12 and 35 cm from the diffusing glass, thus permitting the attainment of high illuminations and large currents, under which condition the most marked peculiarities are to be expected. At the same time the many contacts of the lamp filament and the unavoidable disturbances on moving it from point to point decreased the precision of the readings. The arrangement was therefore more suited to detecting changes of large order than for work of precision.

Fig. 8 shows four illumination-current curves. Curve *b* was obtained with *A′* and *C′* joined, and *G* as anode; current increases less rapidly than illumination. Curve *c* was obtained with *G′* and *C′* connected; current increases more rapidly than illumination. Curve *d* was obtained with cathode drawn back into stem; current increases less rapidly than illumination. Curve *e* is perhaps most interesting of all; it was obtained by letting *A* stand free, *G* serving as anode; here again the slow approach to final deflection and return to zero, and the tendency to stop and then start deflecting farther, clearly speak for the effect of charging and discharging of the isolated anode plate. The series of steps of Curve *e* are similar to those from cell *c* in previous work.

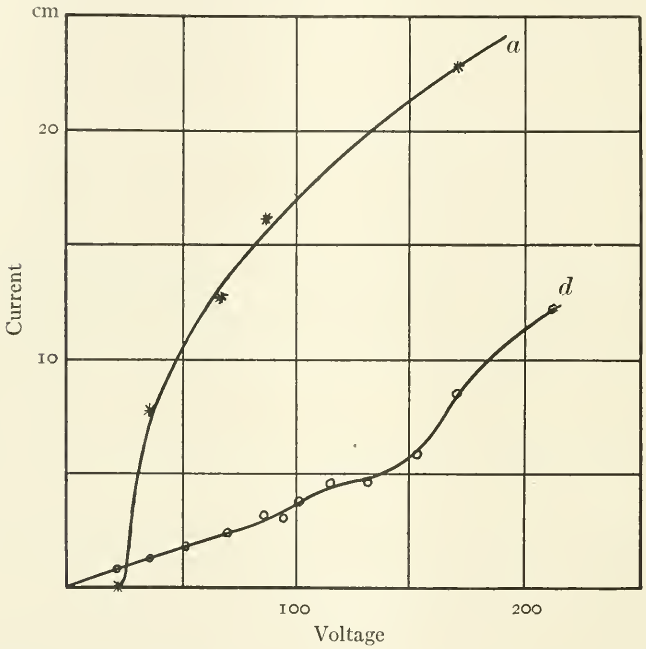

Fig. 7.—Voltage-current curves for two different positions of the cathode in special cell (Fig. 6).

These results may be summarized in the statement that every type of illumination-current curve previously obtained has been duplicated in this one cell by varying the arrangement of its parts.

Curves *b* and *c*, representing the two types found in the high-vacua cells, find a ready explanation in the accumulation of charge on the side walls of the cell. The effect of this charge is to drive a continually increasing part of the current on to the plate *A*. This means an increasing current if *A* is the anode, a decreasing one if *A* is part of the cathode. In the case of curve *d* the charged walls of the adjacent stem exert an inhibitory action on the emission of electrons. In the case of curve *e* the charged plate *A* is acting in opposition to the effect of the charged glass walls, with erratic results.

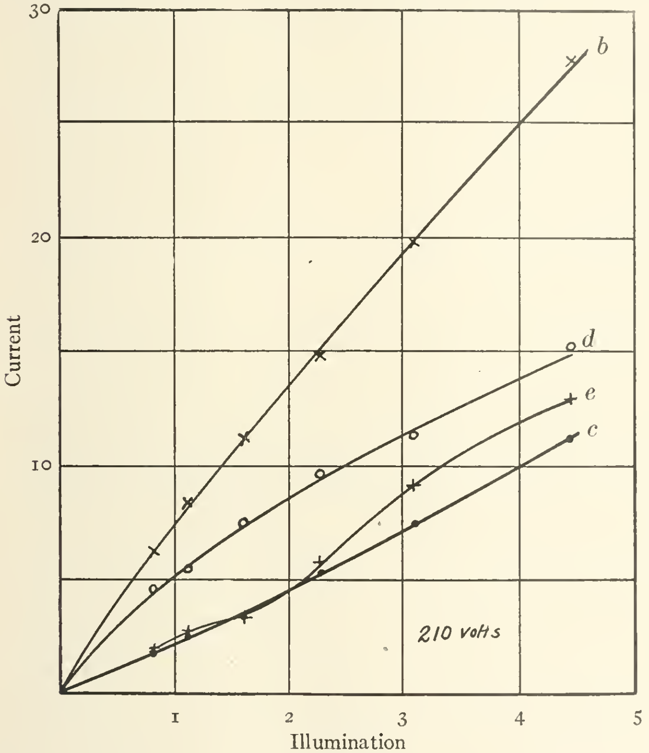

FIG. 8.—Illumination-current curves obtained with various electrode connections in special cell.

It appears, therefore, from our work that the accumulation of charges in the neighborhood of the photo-electric stream is a matter of quite vital influence. Such charges if large may inhibit the photo-electric effect, as shown above, or they may seriously disturb the illumination-current relationship.

VI. THE TYPE OF CELL NECESSARY FOR A RECTILINEAR RELATIONSHIP

After finding by these experiments the cause of the erratic relationship between illumination and current which is likely to exist in cells as usually constructed, the next step was the design

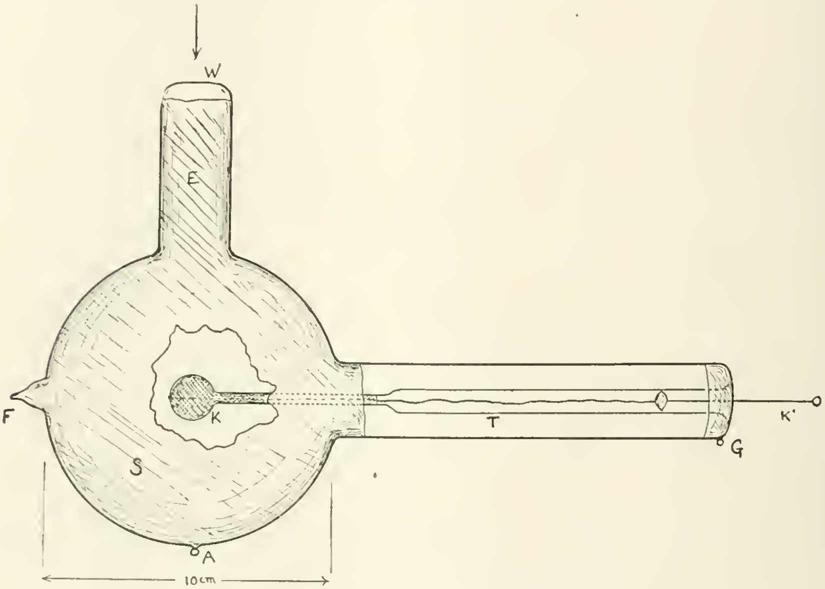

Fig. 9.—Design of cell free from focusing effects

of a cell which should be free from these disturbing factors. What is called for is a cell having absolutely no free surfaces on which electric charges can collect. This condition is practically met by the construction shown in Fig. 9. The body of the tube consists of a large bulb *S*, silvered on the inside, and provided with a small window *W*, carried on an extension tube *E*. The silvered surface serves as anode. The cathode is a small, centrally placed glass bulb *K*, which is silvered, and on which is distilled the alkali metal, which latter is first distilled on to the walls of the large surrounding bulb from a side tube. Wires at *A* and *K′* form the external points of connection to anode and cathode. *G* is the guard-ring used to prevent conduction over the glass surface.

This cell will be recognized as identical in general form with that designed by Compton and Richardson,[1] whose object, however, was the elimination of electronic reflection. It is clear that with the exception of the necessary small illuminating window the cell is entirely symmetrical with respect to the cathode. Any tendency of the electron stream to focus on one part of the surrounding anode in preference to another is not to be expected, but should it occur the total current received by the anode is unaffected. Cells of this type should exhibit the true relationship between illumination and photo-electric current.

Tests made on three such cells have shown in each case, to within the limit of accuracy of the measurements, a truly rectilinear relation between illumination and current, under conditions of illumination and size of current which in the previous cells would have produced most marked deviations from such a relation.

An interesting point in connection with this design is that it is apparently a matter of indifference whether the cell is of the high-vacuum type or of the gas-filled type. One of the three so far made had a considerable atmosphere of gas, as is evident from the voltage-current curves exhibited in Fig. 10, but its behavior was like that of the others. There is indeed no reason why the presence of ionization by collision should introduce focusing effects in this type of cell. The evidence is that the effects on the illumination-current relation of changing the vacuum as was done in the experiment on cell *d*, described in the early part of this paper, are merely variations produced in the focusing effects which are inherent in the small cell with a single wire electrode, not effects produced directly by the gas. These variations in the focusing effects were due probably in part to the change in gas pressure, in part to the alteration in the insulating properties of the glass walls, owing to the "sputtering" of the platinum wire.

Strictly speaking, the three cells so far constructed are all to some extent gas cells, since their characteristic curves show that, in spite of the use of a charcoal evacuator in one case (*X*) and the molecular pump and liquid air in the other (*Y*), the best evacuated cells exhibit a second rise of current at about 20 volts which is undoubtedly due to ionization by collision. In these cells the effect of reflected electrons is presumably eliminated, so that the

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[1] "The Photo-electric Effect," *Philosophical Magazine* (6), 24, 575–594, 1912.

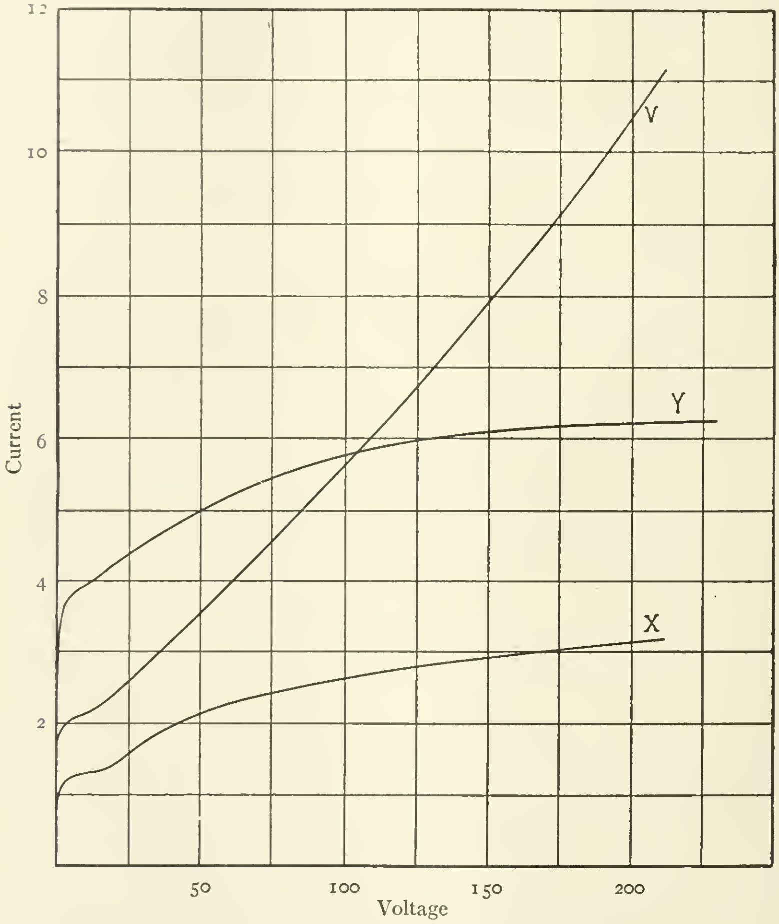

FIG. 10.—Voltage-current curves of three cells of the inclosing anode type

failure to attain a saturation current cannot be ascribed to them. Both *X* and *Y* show a faint glow on the induction coil (not shown before removal from the pump and therefore not affecting the conclusions with regard to the presence of the photo-electric effect in a gas-free cell, which are supported by measurements on the pump). It is in fact our experience that the attainment of a saturation current in alkali metal cells working with high illuminations is a task of unusual difficulty. In spite, however, of the absence of a "perfect" vacuum cell from our tests, we feel warranted in the belief that cells of all degrees of evacuation, if constructed according to the design illustrated above, will give the much-desired straight-line relation between illumination and current.

VII. DISCUSSION

A question which demands answer is why these focusing effects have passed so generally unnoticed, in the widespread belief that *any* photo-electric cell will give a current proportional to illumination. It may be pointed out in this connection that here and there careful investigators have noted deviations from strict proportionality. Nevertheless the number of cases where the rectilinear relation has apparently been established is much larger than would be expected from the practically universal divergence from this condition in the cells of the usual form which were studied in this investigation.

The explanation probably lies in two facts: first, that most of the cells in use have gaseous atmospheres of several tenths of a millimeter, which may mask or actually reduce the focusing effects; secondly, that the common "dark current" due to conduction over the glass walls may be a blessing in disguise, in so far as it prevents the accumulation of charges on the walls. A larger portion of the time spent in the first portion of this research was devoted to the elimination of the spurious dark current. It now appears that the means which succeeded in preventing the dark current were peculiarly effective in making visible the focusing effects which had heretofore escaped notice. These effects now having been found and means discovered for combating them, we are in the condition of knowing definitely, not only how to produce a rectilinear illumination-current relation, but how to secure it free from undesirable companions. The advance is from a type of cell which would perform less satisfactorily the more carefully it was made to one which is based on definite knowledge of the conditions best calculated for all the qualities desired.

VIII. BEARING OF THE WORK ON STELLAR PHOTO-ELECTRIC PHOTOMETRY

In two previous papers[1] it was pointed out that the use of the photo-electric cell in photometry should be attended with great caution. Fro one thing, the relation between illumination and current was too uncertain to warrant the use of a cell without previous calibration. For another, no two of the curves of wave-length and sensibility appear to be alike. The first defect interferes with the cell's applicability to problems of variability of light, the second to its application to comparisons of one light-source with another of different color. In spite of these defects much work has recently been done with the photo-electric cell in stellar photometry, to which the extreme sensibility of the scheme commends itself.

The work here reported has an important bearing on stellar photo-electric photometry. It offers no hope that the varying color-sensibility from cell to cell may be avoided, but it does show how the first uncertainty above noted may be overcome. Moreover, the design shown in Fig. 7 is peculiarly suitable for stellar work, since in that work there is no advantage whatever in having a large area of alkali metal; the total amount of light receivable from the star is already concentrated into a small area by the telescope, and spreading it out does not increase the amount available.

A very important point in this connection is that this design offers possibilities for great sensibility, probably much greater than any yet used. By sensitizing the surface in the manner discovered by Elster and Geitel, and introducing an atmosphere of an inert gas, all the sensibility of their type of cell should be obtained. But, in addition to this a new aid to sensibility is offered, namely, an increase in the distance between the electrodes. From the theory of ionization by collision it follows that the *current in a gas increases rapidly with the distance between the electrodes.*[2] In the Elster and

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[1] Ives, "The Illumination-Current Relationship in Potassium Photo-electric Cells, *Astrophysical Journal*, 39, 428, 1914; "Wave-Length Sensibility Curves of Potassium Photo-electric cells," *ibid.*, 40, 182, 1914.

[2] J. J. Thomson, *Conduction of Electricity through Gases*, p. 272.

Geitel form this fact cannot be utilized because of the small area of the anode. In the design here described it should be possible to make to advantage cells of very large diameter—how large would be a matter for experiment to determine. The glass stem of such a cell could easily be made of very great length, or it could be made perhaps of fused quartz, whereby, with use of a guard-ring, the dark current could be reduced by any desired amount. The complete design offers in fact possibilities in the way of sensibility and satisfactory performance which the writers cordially recommend as worthy of thorough study to those working in this field.

It is quite likely that the design of cell described by A. L. Hughes,[1] in which the walls are entirely covered with alkali metal, is equally satisfactory in its illumination-current relationship and will also increase in sensitiveness with size. Whether the increased sensitiveness of that type, due to the black-body conditions, is greater than that attainable in the inclosing anode type is a matter for experiment to decide.[2]

IX. SUMMARY

The occurrence of non-rectilinear relationships between illumination and photo-electric current is found to be due to focusing effects, caused by the accumulation of charges on the walls of the cells. A design of cell is given in which such effects are eliminated, whereby a truly rectilinear relation is obtained. It is suggested that cells of this type could be made which should exceed in sensibility and practicability any so far employed in stellar photo-electric photometry.

Physical Laboratory of the
United Gas Improvement Co.
Philadelphia
August 1915

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[1] "A Sensitive Photo-electric Cell," *Philosophical Magazine* (6), 25, 679, 1913.

[2] (Note added on correction of proof.) Experiments on one of our cells, on the walls of which considerable potassium remains, as a Hughes cell, indicate that this design gives the rectilinear illumination-current relationship and is several times more sensitive than the central cathode arrangement.

THE CHANGE OF COLOR WITH DISTANCE AND APPARENT MAGNITUDE TOGETHER WITH A NEW DETERMINATION OF THE MEAN PARALLAXES OF THE STARS OF GIVEN MAGNITUDE AND PROPER MOTION[1]

By P. J. VAN RHIJN

The present paper deals with the question of a possible relation between the color of the stars and their distances, apparent magnitude and spectral type being the same. It is found that the stars of the *Yerkes Actinometry*[2] show a relation between distance and color such that, *ceteris paribus*, the distant stars are redder than the nearer ones. Whether this phenomenon is to be ascribed to a loss of light in space or whether it is due to an influence of absolute magnitude cannot certainly be decided from the material available at present. Apart from its explanation, however, the question mentioned has an importance of its own, because its solution will afford a basis for further investigations; moreover, independently of the question as to how it is to be explained, a well-established relation between color and distance, apparent magnitudes and spectral types being the same, furnishes a valuable method for the determination of the distances of very remote objects.

For an investigation of the kind considered the *Yerkes Actinometry* contains very valuable material, viz., the visual magnitude, the color-index, and the spectral type of all the stars of the *Potsdam Photometric Durchmusterung* in the zone from 73° north declination to the pole. If the distances of these stars were also known, it would be a comparatively easy matter to derive the relation between color-index and distance. As, however, the parallaxes of only a few stars of the *Yerkes Actinometry* have been directly measured, we are obliged to use proper motion and apparent magnitude as a criterion of distance. I therefore derived the mean distance of stars of determined proper motion and magnitude, for which purpose I have made the following assumptions, in accordance with Professor Kapteyn[1] and Professor Schwarzschild:[2]

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 110.

[2] *Astrophysical Journal*, 36, 169, 1912.

a) That the mean parallax of stars of determined proper motion and magnitude can be represented by the formula

$$\bar{\pi}=a\mu_1^b\,\epsilon^{m_1} \tag{1}$$

where $\mu_1=100\,\mu$ = total angular motion per century; $m_1=m-5.0$, m being the visual magnitude on the Harvard scale; a, b, ϵ are constants to be determined from the data of observation.

b) That the quantities

$$z=\log\frac{\pi}{\pi_0} \tag{2}$$

are distributed in accordance with the law of errors, π being the true parallax of a star and π_0 the most probable parallax of stars of the same magnitude and proper motion.

If the constants a, b, ϵ and the probable error ρ of the error-curve (2) have been determined from the data of observation, the mean distance of stars of given magnitude and proper motion can be derived by the formula

$$\bar{R}=\frac{1}{\bar{\pi}}e^{+\frac{\rho^2}{(0.6745)^2\ \mathrm{mod.}^2}} \tag{3}$$

This is easily proved by means of supposition *b*).

The data from which the constants a, b, ϵ, and ρ must be determined are of two kinds: (1) the individual parallaxes; (2) the mean parallaxes of certain groups of stars, which are derived from the parallactic motions and radial velocities. As these constants may be different for stars of different spectral types, they have been derived separately for the B, A, and the second-type (F, G, K) stars.

Without entering into details as to the determination of the constants, the following may be stated:[3] The coefficients a and b were computed by means of the known individual parallaxes. For the helium stars I used those determined by Professor Kapteyn;[4]

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[1] *Publications of the Astronomical Laboratory at Groningen*, No. 8, 1901.

[2] *Astronomische Nachrichten*, **190**, 361, 1912.

[3] The details of the whole investigation have been published in my thesis under the same title as the present note.

[4] *Mt. Wilson Contr.*, No. 82; *Astrophysical Journal*, **40**, 43, 1914.

for the F, G, and K stars, the parallaxes compiled by Professor Kapteyn and Dr. Weersma in 1910,[1] and those afterward published by the Yale Observatory,[2] by Flint,[3] Slocum and Mitchell,[4] and Abetti.[5] The parallaxes of stars of the same proper motion were combined into mean values which gave the data required for the determination of a and b. For the A stars this procedure was impossible, because the number of directly measured parallaxes of this type is too small to give trustworthy results.

The constant ϵ was determined from the mean parallaxes for stars of different magnitudes, derived from the parallactic motion and radial velocity. Only those stars were used whose radial velocities have been determined by Campbell.[6] For the second-type stars a second value of ϵ was derived from the directly measured parallaxes.

Finally the values of a and b derived from the individual parallaxes were corrected by means of the final mean parallax for stars of different magnitudes and proper motions, which had been computed from the radial velocities and parallactic motions. The final mean parallax of the A stars was used to determine the constant a, the coefficients b and ϵ being supposed equal to the average of the values found for the B and F, G, K stars. The constant ρ was determined from the differences between the logarithm of the observed individual parallaxes and the logarithm of the mean parallax computed by the formula (1). As the errors of observation in the parallaxes have a large influence on the value of ρ, I used only stars whose parallaxes are well determined.

Table I gives a summary of the results for the constants appearing in the formulae (1) and (3). The results are, in general, in agreement with those derived by Professor Kapteyn,[7] but the mean parallaxes of stars of different magnitudes found by equation (1), which are given in Tables II, III, and IV, are entirely at variance with those derived by Campbell.[1] The Lick Observatory results, however, cannot be considered to be representative of the whole stellar system, as the stars with large angular motion were omitted in deriving the mean parallaxes. It can be shown that the greater part of the difference between Dr. Campbell's parallaxes and those found here will disappear if the former are multiplied by certain factors which allow for the omission of the stars of large angular motion.

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[1] *Publications of the Astronomical Laboratory at Groningen*, No. 24, 1910.

[2] Chase, Smith, and Elkin, *Transactions of the Astronomical Observatory of Yale University*, 2, 1912.

[3] *Astronomical Journal*, 27, 49, 1912.

[4] *Astrophysical Journal*, 38, 1, 1913.

[5] *Memorie del R. Osservatorio Astronomico al Collegio Romano*, Serie III, Vol. V Parte 2.

[6] *Lick Observatory Bulletins*, 6, 108, 1911; 7, 19, 1912; 7, 113, 1913.

[7] *Publications of the Astronomical Laboratory at Groningen*, No. 8, 1901.

TABLE I

Type	a	b	ϵ	ρ
B.	0″.0031	0.904	0.895	0.12
A.	0.0028	0.80	0.895	0.15
F, G, K.	0.0038	0.695	0.895	0.17

TABLE II

MEAN PARALLAX, TYPE B

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0″.00. . . .	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000
0.01. . . .	0.0054	0.0048	0.0043	0.0039	0.0035	0.0031	0.0028	0.0025	0.0022	0.0020	0.0018
0.02. . . .	0.0101	0.0090	0.0081	0.0072	0.0065	0.0058	0.0052	0.0047	0.0042	0.0037	0.0033
0.03. . . .	0.0145	0.0131	0.0116	0.0105	0.0093	0.0084	0.0075	0.0067	0.0060	0.0054	0.0048
0.04. . . .	0.0189	0.0170	0.0151	0.0136	0.0121	0.0109	0.0097	0.0087	0.0078	0.0070	0.0062
0.05. . . .	0.0232	0.0207	0.0186	0.0166	0.0148	0.0133	0.0119	0.0107	0.0096	0.0085	0.0076
0.06. . . .	0.0273	0.0244	0.0219	0.0195	0.0174	0.0157	0.0140	0.0126	0.0113	0.0100	0.0090
0.07. . . .	0.0314	0.0281	0.0251	0.0225	0.0201	0.0180	0.0161	0.0144	0.0130	0.0115	0.0104
0.08. . . .	0.0354	0.0317	0.0284	0.0254	0.0227	0.0203	0.0182	0.0163	0.0146	0.0130	0.0117
0.09. . . .	0.0394	0.0353	0.0316	0.0282	0.0253	0.0226	0.0203	0.0181	0.0163	0.0145	0.0130
0.10. . . .	0.0433	0.0388	0.0347	0.0310	0.0277	0.0248	0.0223	0.0199	0.0179	0.0159	0.0142
0.2.	0.081	0.073	0.065	0.058	0.052	0.047	0.042	0.037	0.033	0.030	0.027
0.3.	0.116	0.105	0.094	0.084	0.075	0.067	0.060	0.054	0.048	0.043	0.039

Having computed the mean distances of stars of given magnitude and proper motion, we pass on to the determination of the relation connecting color and distance. The color-index may also depend on the apparent magnitude, for we are not sure that the range of one magnitude is the same for both the visual and photographic scales. Therefore the equation for the color-index is

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[1] *Lick Observatory Bulletin,* 7, 131, 1911.

$$\sigma = a + bm + cR, \tag{4}$$

where

σ = color-index.
R = distance.
m = apparent visual magnitude.
b = change of color-index for a change of one unit in apparent magnitude.
c = increase of color-index per unit of distance (parsec).

TABLE III

Mean Parallax, Type A

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0″.00 . . .	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000
0.01	0.0049	0.0044	0.0039	0.0035	0.0031	0.0028	0.0025	0.0022	0.0020	0.0018	0.0016
0.02	0.0085	0.0076	0.0068	0.0061	0.0054	0.0049	0.0044	0.0039	0.0035	0.0031	0.0028
0.03	0.0118	0.0105	0.0094	0.0084	0.0075	0.0067	0.0060	0.0054	0.0048	0.0043	0.0039
0.04	0.0148	0.0132	0.0119	0.0106	0.0095	0.0085	0.0076	0.0068	0.0061	0.0055	0.0049
0.05	0.0177	0.0158	0.0142	0.0126	0.0113	0.0101	0.0091	0.0081	0.0073	0.0065	0.0058
0.06	0.0204	0.0183	0.0164	0.0146	0.0131	0.0117	0.0105	0.0094	0.0084	0.0075	0.0067
0.07	0.0231	0.0207	0.0185	0.0166	0.0148	0.0133	0.0119	0.0106	0.0095	0.0085	0.0076
0.08	0.0257	0.0230	0.0206	0.0184	0.0165	0.0148	0.0132	0.0118	0.0106	0.0095	0.0085
0.09	0.0283	0.0253	0.0226	0.0202	0.0181	0.0163	0.0145	0.0130	0.0117	0.0104	0.0093
0.10	0.0308	0.0275	0.0247	0.0220	0.0197	0.0177	0.0158	0.0142	0.0127	0.0114	0.0101
0.2	0.054	0.048	0.043	0.038	0.034	0.031	0.028	0.025	0.022	0.020	0.018
0.3	0.074	0.066	0.059	0.053	0.047	0.043	0.038	0.034	0.031	0.027	0.024
0.4	0.093	0.083	0.075	0.067	0.060	0.054	0.048	0.043	0.039	0.034	0.031
0.5	0.111	0.100	0.089	0.080	0.071	0.064	0.057	0.051	0.046	0.041	0.037
0.6	0.129	0.115	0.103	0.092	0.083	0.074	0.066	0.059	0.053	0.047	0.042
0.7	0.146	0.130	0.117	0.104	0.094	0.084	0.075	0.067	0.060	0.054	0.048
0.8	0.162	0.145	0.130	0.116	0.104	0.093	0.083	0.075	0.067	0.060	0.053
0.9	0.178	0.160	0.143	0.128	0.114	0.102	0.092	0.082	0.074	0.066	0.059
1.0	0.194	0.174	0.155	0.139	0.124	0.111	0.100	0.089	0.080	0.072	0.064
2.0	0.338	0.303	0.271	0.242	0.217	0.194	0.174	0.156	0.140	0.125	0.111

The constants *a*, *b*, and *c* must be determined separately for each spectral type from a number of equations of the form (4). The color-index and visual magnitude were taken from the *Yerkes Actinometry;* the distance was computed for each star by means of formulae (1) and (3).

The solution involved two steps: First, the stars of each spectral class were divided into two or more groups according to their magnitudes. Forming the mean distance, color-index, and visual magnitude for all the stars of a group, a number of equations of the form

$$a + bm = \sigma + cR$$

were obtained, from which b was derived as a function of c. As the distances of the different groups were approximately equal, the constant c entered with a small coefficient. Secondly, the stars of each spectral class were arranged according to the magnitude of the proper motion. By means of these groups c was derived as a function of b, in which b entered with a small coefficient. From these two results, the values of b and c expressed in magnitudes were easily derived.

TABLE IV

Mean Parallax, Types F, G, and K

μ	Visual Magnitude (Harvard Scale)										
	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0″.00....	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000	0″.0000
0.01....	0.0066	0.0059	0.0053	0.0047	0.0042	0.0038	0.0034	0.0030	0.0027	0.0024	0.0022
0.02....	0.0107	0.0096	0.0086	0.0077	0.0069	0.0062	0.0055	0.0049	0.0044	0.0040	0.0035
0.03....	0.0142	0.0127	0.0114	0.0102	0.0091	0.0082	0.0073	0.0065	0.0059	0.0052	0.0047
0.04....	0.0174	0.0156	0.0139	0.0124	0.0111	0.0099	0.0089	0.0080	0.0072	0.0064	0.0057
0.05....	0.0203	0.0181	0.0163	0.0145	0.0130	0.0116	0.0104	0.0093	0.0084	0.0075	0.0067
0.06....	0.0230	0.0206	0.0184	0.0165	0.0147	0.0132	0.0118	0.0105	0.0095	0.0085	0.0076
0.07....	0.0256	0.0229	0.0206	0.0184	0.0165	0.0147	0.0132	0.0118	0.0106	0.0094	0.0084
0.08....	0.0281	0.0251	0.0225	0.0201	0.0180	0.0162	0.0144	0.0130	0.0116	0.0103	0.0093
0.09....	0.0305	0.0273	0.0244	0.0218	0.0196	0.0175	0.0157	0.0140	0.0126	0.0112	0.0101
0.10....	0.0328	0.0294	0.0263	0.0235	0.0210	0.0188	0.0169	0.0151	0.0135	0.0121	0.0108
0.2.....	0.053	0.048	0.043	0.038	0.034	0.030	0.027	0.024	0.022	0.020	0.018
0.3.....	0.070	0.063	0.056	0.050	0.045	0.040	0.036	0.032	0.029	0.026	0.023
0.4.....	0.086	0.077	0.069	0.061	0.055	0.049	0.044	0.040	0.035	0.032	0.028
0.5.....	0.100	0.090	0.081	0.072	0.064	0.058	0.052	0.046	0.041	0.037	0.033
0.6.....	0.114	0.102	0.091	0.082	0.073	0.065	0.059	0.052	0.047	0.042	0.038
0.7.....	0.127	0.114	0.102	0.091	0.081	0.073	0.065	0.058	0.052	0.047	0.042
0.8.....	0.139	0.125	0.112	0.100	0.089	0.080	0.072	0.064	0.057	0.051	0.046
0.9.....	0.151	0.135	0.121	0.108	0.097	0.087	0.078	0.069	0.062	0.055	0.050
1.0.....	0.163	0.146	0.130	0.116	0.104	0.093	0.083	0.075	0.067	0.060	0.054
2.0.....	0.263	0.236	0.211	0.188	0.169	0.151	0.135	0.121	0.109	0.097	0.087
3.0.....	0.348	0.312	0.279	0.249	0.223	0.200	0.179	0.161	0.144	0.129	0.115
4.0.....	0.426	0.381	0.341	0.305	0.273	0.244	0.219	0.196	0.176	0.157	0.140
5.0.....	0.498	0.446	0.399	0.356	0.319	0.285	0.256	0.229	0.206	0.184	0.164
6.0.....	0.565	0.506	0.453	0.404	0.362	0.324	0.290	0.260	0.233	0.208	0.186
7.0.....	0.628	0.562	0.503	0.450	0.403	0.360	0.322	0.289	0.259	0.232	0.207
8.0.....	0.690	0.618	0.553	0.494	0.442	0.396	0.355	0.318	0.285	0.255	0.227
9.0.....	0.748	0.670	0.600	0.536	0.479	0.430	0.384	0.344	0.309	0.276	0.246

Before proceeding to the actual solution of c as a function of b, it was necessary to correct the second members of the equations of condition for a systematic error, which tends to diminish the value of c for the early-type stars and to increase its value for the later types. The character of this error has been fully explained by Professor Kapteyn.[1]

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[1] *Mt. Wilson Contr.*, No. 42; *Astrophysical Journal*, 30, 293, 1909.

The value of b proved to be independent of the spectral type. We may therefore adopt for b the mean of the values found for the different spectral classes, viz.,

$$b=+0^{\mathrm{m}}.025\pm0.004.$$

The values found for the constant c are given in Table V.

TABLE V

Spectrum	c	Probable Error
	Mag.	Mag.
B5 to A0	0.00000	±0.00006
A1 to A9	+0.00021	0.00006
F0 to G2	+0.00033	0.00007
G3 to K5	+0.00011	±0.00005

The weighted mean of these values is

$$c=+0^{\mathrm{m}}.00015\pm0.00003.$$

The residuals with respect to the mean of the separate values in Table V show scarcely any systematic change with the spectral type. It may be that the B and A stars show the effect to a smaller degree than the later types, as is indicated by the value $c=0$ for the B5 to A0 stars. Mr. Adams' results,[1] derived by an entirely different method, give some indications in the same direction. Omitting the value for the B5 to A0 group, the mean for the other types is

$$c=+0.000195\pm0.00003,$$

a value 6.5 times its probable error.

CONCLUSION

The distant stars of the *Yerkes Actinometry* are, *ceteris paribus*, redder than the nearer ones. The increase of color-index per parsec is:

$$c=+0.00015\pm0.00003.$$

There are some indications that this effect does not exist for the B and early A stars. Supposing this to be the case, the amount of c for the other spectral types increases to

$$c=+0.000195\pm0.00003.$$

Groningen
October 1915

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[1] *Mt. Wilson Contrs.*, Nos. 78, 89; *Astrophysical Journal*, **39**, 89; **40**, 385, 1914.

ANOMALOUS DISPERSION AND FRAUNHOFER LINES

REPLY TO OBJECTIONS

By W. H. JULIUS

If we realize the generally admitted gaseous nature of the sun, and the fact that nearly all of the solar radiation has passed through extensive, confusedly circulating masses of gas, we are inevitably led to consider the study of the propagation of light through such media as one of the necessary bases for the interpretation of the solar image and all its details. Some vigorous efforts recently made to prove that refraction has little influence on solar phenomena,[1] and that not much good should be expected from a broad application of the dispersion theory to astrophysical problems, can therefore only be regarded as a useful counterpoise against any excessive weight that might perhaps be given to the deductions from the theory of anomalous dispersion.

A thorough criticism of those deductions is certainly justified and welcome; for the remarkable versatility of the anomalous dispersion theory renders the danger of exaggerating its application by no means imaginary, and the advocates of the theory should therefore always be kept on their guard against this danger.

The principal object of the present paper is to consider carefully the *arguments* of opponents to the idea that irregular refraction and anomalous dispersion probably are very important factors in the production of a great many solar (and stellar) phenomena usually ascribed to other causes.

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[1] Anderson, *Astrophysical Journal*, 31, 166, 1910; Gouy, *Compte Rendus*, 157, 1111, 1913; Brunt, *Monthly Notices, R.A.S.*, 73, 568, 1913; Evershed, *The Observatory*, 37, 388, 1914; St. John, *Astrophysical Journal*, 41, 28, 1915; *Mt. Wilson Contr.*, No. 93; St. John, *Proc. Nat. Acad. of Sciences*, 1, 21, 1915; Adams and Burwell, *Astrophysical Journal*, 41, 116, 1915; *Mt. Wilson Contr.*, No. 95; Adams and Burwell, *Proc. Nat. Acad. of Sciences*, 1, 127, 1915.

In the present article consideration will chiefly be given to such arguments and subjects as can be discussed without entering into problems of theoretical optics; so we must defer the discussion of some of the objections raised by Gouy and Brunt to a later date.

As an introduction to this inquiry I wish to call attention to certain statements and assertions that have not the character of arguments, but are expressive of *opinions* regarding the astrophysical branch of the anomalous dispersion theory in general.

DISCUSSION OF OPINIONS

Some misapprehension seems to obtain with respect to the starting-point as well as the claims of that theory. This situation may be partly due to the use which is sometimes made of the term "optical illusion" when its fundamental notions are described. If, after having stated that the observed phenomena, such as prominences, flash spectra, flocculi, and displacements of Fraunhofer lines, are from this point of view mainly the effects of anomalous refraction in the solar atmosphere, an author continues, "so that in their study we are facing optical illusions,"[1] some readers will at once feel a little uneasy about the subject. No observer likes to be told that he is the victim of an optical illusion. The unfavorable mood is evoked, however, by an inadequate use of the term.[2]

In the case of a true "optical illusion" we are confronted with a difficulty in the appreciation of the relative dimensions, or positions, or perhaps colors of the parts of a visual image. The belief in the existence of a certain relation between the parts forces itself upon our minds, but, on closer examination, that relation proves not to exist. Our judgment thereby dealt with the *image* only; our decision greatly depended on our ability as observers; and we would rather not be led astray by the difficulties of estimation. The physical origin of the image is not involved in this question.[3]

[1] St. John, *Proc. Nat. Acad. of Sciences*, **1**, 21, 1915.

[2] I have once used the expression in the wrong sense myself (*Astron. Nachr.*, **160**, 141, 1902; *Physik. Zeitschr.*, **4**, 133, 1902), and regret the mistake.

[3] An example of a true optical illusion happens to be discernible on Plate I facing p. 19 of my paper in the *Astrophysical Journal*, **40**. The upper spectrum shows a few (five) sharp bright lines due to the carbon arc and alien to the interferential phenomenon with NO_2 under investigation. With respect to these bright lines the dark lines of the spectrum, which at first sight we believe also to be continuous straight lines, appear to be *inclined*.

The apparent dark lines, however, are built up of inclined sections, each shifted with respect to the next one (in consequence of anomalous dispersion), and we falsely impute the same inclination to the supposititious continuous straight lines.

If, on the other hand, various *physical interpretations* of a visible phenomenon are considered, the expression "optical illusion" for one of them is misplaced. Suppose two boys, both provided with some physical knowledge, had never learned anything about the sea, and are now for the first time looking from a distance at the breakers on the coast. One of them will say: "I see streaks and patches of a white liquid, like milk, appearing, moving, vanishing in blue-green water." "No," says the other, "I think it more probable that it is all water, and that the white patches are caused by reflection and refraction of the sky light in foam." As long as the boys cannot get on the beach, they will have to improve and discuss their observations and make theories; but neither of them can be rightly said to deal with an optical illusion.

Whatever may be the cause of the distribution of light in the visual images which we call sun-spots, prominences, etc., there must be a certain distribution of matter corresponding to those phenomena; so they are "real objects" in any case; and the displacement of a Fraunhofer line is a "real effect," whether it may be produced by motion in the line of sight, or by pressure, or by anomalous dispersion. The reliability of such phenomena as fundamentals of solar physics will not in the least be diminished by the necessity of considering them as dependent upon a physical process hitherto overlooked in their interpretation.

There may be some ground for presuming that the theory of anomalous dispersion "would revolutionize or render futile many of the present lines of solar and stellar observation"[1] because it might urge other questions to the front; but there need not be any fear that it "would make practically impossible the solution of many problems which confront the investigator";[1] the solutions may only turn out somewhat different from what they were expected to be from the points of view of other theories.

According to St. John there is "a degree of vagueness in the deductions from the theory, due to its extreme flexibility, that makes a quantitative examination of its claims difficult."[1] In my opinion, this theory has not "vagueness" for its own characteristic, but it has suggested that the bases of other theories may be more or

[1] St. John, *Proc. Nat. Acad. of Sciences*, 1, 21, 1915.

less vague, for it has raised doubts as to the rectilinear propagation of light (especially of the so-called R-light and V-light) through the solar gases. The anomalous dispersion theory has then led us to investigate general and *regular* consequences of *irregular* refraction, and thus to endeavor to bring some order and clearness in what, at first sight, seemed to be a hopelessly entangled condition. One of its chief claims, therefore, is that it has combated vagueness.

Are there sufficient grounds for assuming the existence of such irregular gradients of optical density as the theory postulates? Adams and Burwell say: "Most solar observers would desire some evidence tending to indicate the existence of such gradients apart from the necessity of postulating them in order to support the anomalous refraction hypothesis, more especially as they must be essentially permanent in character."[1]

I was a little surprised to read this objection, because the desired evidence has been furnished in my paper "On the Interpretation of Photospheric Phenomena."[2] The main facts, arguments, and calculations there supplied are independent of the anomalous dispersion theory; and our conclusion that, in spite of the apparent sharp boundary, the sun may be gaseous until far below the photospheric level, and that scattering is the principal cause of the gradual darkening toward the limb, has received new support from the result of a recent theoretical investigation by Schwarzschild.[3] So we may safely suppose the beams of average sunlight to come from regions where the mean density of the gaseous medium exceeds that of the atmosphere at the earth's surface. Variations of density amply sufficient to account for the degree of ray-curving assumed by our theory are sure to occur on the paths of those beams. Indeed, nobody will deny that there is motion in the sun. Even if the relative velocities of different parts of the photospheric gaseous material never exceeded, say, 0.5 km per second (an estimate which astronomers, accustomed to velocities of hundreds of kilometers, will consider to be very low)[1] such movements would be ten times as fast as terrestrial storms. They would necessarily imply pressure-gradients and local variations of density much greater than those with which our meteorologists are concerned. Besides, vertical convection currents bring into the mixture local differences of temperature and composition, which co-operate with the pressure differences in producing irregular gradients of optical density.[2] One cannot escape the conclusion that the non-existence of such irregular gradients in a medium with internal movements as assumed, would be simply a miracle, and that the idea of it even becomes an absurdity from the point of view of those who assume a state of radial circulation of vapors with varying velocities of several kilometers per second to be a permanent feature of the sun's visible layers, and who, moreover, believe in the frequent occurrence of outbursts of matter at a rate of hundreds of kilometers per second.

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[1] W. S. Adams and C. G. Burwell, *Astrophysical Journal*, 41, 140, 1915; *Mt. Wilson Contr.*, No. 95, p. 25, 1915.

[2] *Astrophysical Journal*, 38, 129, 1913.

[3] "Über Diffusion und Absorption in der Sonnenatmosphäre," *Sitzungsberichte der kgl. Akademie der Wissenschaften*, Berlin, 1914.

If it be once for all admitted that the irregular gradients required by our theory must exist, the rôle of refraction and anomalous dispersion in the production of the fundamental solar phenomena can no longer be disregarded.

A necessary consequence of our hypothesis that the distribution of light in the solar image must everywhere be influenced sensibly by refraction is that we are compelled to examine *every* solar phenomenon from this point of view first. In so doing we cannot be said to exaggerate the applicability of the principle of anomalous dispersion. If no additional hypotheses or artificial conceptions are introduced, there is no harm in our inquiring into the logical consequences of also considering anomalous dispersion pure and simple; and if then a solar phenomenon cannot be satisfactorily explained as due only to refraction and scattering there are fortunately several other physical causes at hand which we may try. As a matter of fact, the possibility that other influences such as selective emission and absorption, motion, pressure, magnetic and electric fields, temperature, radioactivity, etc., co-operate in producing the phenomena is by no means denied by the anomalous dispersion theory, and is indeed perfectly compatible with it.

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[1] The upper limit of 0.5 km per second was chosen because, according to Hansky (*Mitteil. Pulkowo,* 3, No. 25, 1908), the maximum velocity of proper motion of sun-spots is 0.4, the mean velocity only 0.15 km per second. If spots are vortices, their movements are real displacements of matter. The velocity of the material circulating round the vortices must be of the same order of magnitude. Hansky has also determined the velocities of granulations. These vary between the narrow limits 2.8 and 4.8 km. In this case, however, there is no sufficient ground for assuming that *matter* is moving at the observed rate. We suppose, in the line of our theory, that we are here witnessing the propagation of condensations and dilatations, like sound-waves. The velocity of such waves in a mixture of hydrogen, helium, and traces of other gases, at 6000°, would exactly fall between the limits mentioned.

[2] We know, from mirage effects, that even the vertical gradient of optical density in our atmosphere may be reversed by a quite reasonable temperature-gradient. Ray-curving, of the order of magnitude occurring in mirage, would produce very marked refraction effects in the sun, where the paths of the beams through the gaseous medium are so much longer, and, therefore, the probable deviations greater.

Meanwhile, the remarkable result has to be noticed, that in many cases it is scarcely necessary, at least in first approximation, to have recourse to those other principles, because the main features of a very great number of solar phenomena follow directly from the dispersion theory if we only realize the laws of propagation of light through a rather calmly circulating, extensive mixture of gases. A greater uniformity in our conception of solar phenomena than older theories were able to give is thus attained. The resultant mental image of the sun is, of course, far from complete, but it is capable of extension and improvement by the application of other physical laws.

This review of principles and claims of the anomalous dispersion theory of solar phenomena will perhaps satisfy Adams and Burwell, who felt it as a somewhat disqualifying peculiarity of that theory that its advocates "admit the existence of essentially all of the phenomena in the sun which are required by those who use the more usual explanation,"[1] a fact which, in their opinion, would render the theory rather superfluous. Qualitatively speaking, the quoted statement is quite right, but cannot be said to discredit the introduction of a new, well-established physical principle into astrophysics, nor to make its application superfluous. From a quantitative point of view, however, the statement is incorrect; for the adherents of the dispersion theory can dispense with the enormous velocities of pell-mell motion of solar matter, and with certain artificial hypotheses, and conclusions improbable from a physical point of view, that are constantly puzzling many students of solar physics.

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[1] Adams and Burwell, *Astrophysical Journal*, 41, 143, 1915; *Mt. Wilson Contr.*, No. 95, p. 28, 1915.

DISCUSSIONS OF ARGUMENTS

1. *Mutual influence of Fraunhofer lines.*—In December 1913 it occurred to me that the dispersion theory required the existence of a definite mutual influence of Fraunhofer lines separated from each other by very short distances, irrespective of the elements to which the lines are due.[1] As the current interpretation of the solar spectrum, which is based on the laws of emission and absorption, and extended by applying the Doppler effect, the Zeeman effect, and the pressure-effect, has never suggested the possibility of a similar interaction, nor seemed to be able to account for it, the actual discovery of the presumed phenomenon would furnish almost conclusive evidence in favor of the hypothesis that the darkness of Fraunhofer lines is mainly an effect of anomalous dispersion. The theory, however, could as yet only predict the general character of the phenomenon, but not the average magnitude of the expected displacements due to mutual influence.

It was hoped that suitable data for testing the foregoing deduction from the theory would be found in St. John's measurements of the Evershed effect in sun-spots.[2] Among the 506 lines of St. John's table, I found 82 members of close pairs so configurated and situated that a definite influence of their other members could reasonably be expected on the basis of the theory. The selection of the pairs and the determination of "normal" or "standard" displacements was a delicate process; I believed that I had applied it fairly. The result seemed to be in accord with the theory, and I submitted the matter with confidence to the judgment of other investigators.[3]

A few months later St. John published the elaborate article[4] in which he criticized my treatment of his observations very severely, in several respects justly. I had not overcome the difficulties of handling rightly the Mount Wilson data, nor had I entirely avoided bias. St. John made certain alterations in the method of grouping and comparing the measured displacements, added a number of omitted and of new cases, and thus reached the conclusion that there was no indication at all of a mutual influence.

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[1] Provided those elements coexist in the mixture at the same levels.

[2] *Astrophysical Journal*, **37**, 322, 1913; *Mt. Wilson Contr.*, No. 69.

[3] *Astrophysical Journal*, **40**, 1, 1914.

[4] *Ibid.*, **41**, 28, 1915; *Mt. Wilson Contr.*, No. 93.

Although I am satisfied by St. John's improved discussion of the data that, in the Evershed effect, mutual influence is not so conspicuous as my defective treatment of those measurements had made it appear, I still believe that future research will bring it to light. This conviction has recently received strong support from S. Albrecht's discovery that *in the general solar spectrum* those iron lines which have close companions are indeed systematically displaced, in perfect harmony with the requirements of the anomalous dispersion theory.[1]

For a mean separation of the interacting lines of 0.22 A the average shift was 0.007 A, if toward the violet; and 0.005 A, if toward the red.

There is no contradiction between St. John's failure as yet to find mutual influence in the Evershed effect, and Albrecht's success in disclosing the predicted phenomenon in the general solar spectrum; for we can easily show that in the latter case the effect should be sensibly greater than in the former.

To this end we must reproduce the diagram (Fig. 1) and its explanation, by which in the *Astrophysical Journal*, **40**, 12, 1914, a possible mutual influence of Fraunhofer lines was illustrated.

The value n_0 which the refractive index would have in the part of the spectrum under observation, if this were free from absorption lines, is here supposed to be >1. The effect of a line B is to reduce the indices on its violet side and to raise them on its red side, as indicated by the partly broken curves. The line A, if isolated, would produce its own anomaly in the dispersion-curve as shown in A_1. If A were situated near B, in one of the positions A_2 or A_3, that anomaly would have a somewhat different form in consequence of its being superposed upon one of the branches of the dispersion-curve due to B. The refracting properties of the medium, being determined by the value of $n-1$, will be different in the three cases represented by A_1, A_2, and A_3.

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[1] *Astrophysical Journal* **41**, 333, 1915.

Only those waves for which the absolute values of $\pm(n-1)$ exceed a certain minimum value will contribute sensibly to the formation of the Fraunhofer line, and will follow sufficiently curved paths in the outer parts of a vortex region (a sun-spot) to give rise to recognizable effects of refraction in the spectrum of the penumbra. This is indicated in the figure by means of the two broken lines drawn at equal distances above and below the line $n=1$. We may assume that only the parts of the dispersion-curve lying outside the zone between these broken lines are material to the formation of the dispersion bands enveloping the absorption lines. The R-light corresponding to the shaded area above the zone is responsible for the displacements toward the red observed at the peripheral edge of the penumbra; the V-light corresponding to the shaded area below the zone causes the displacements toward the violet at the central edge.

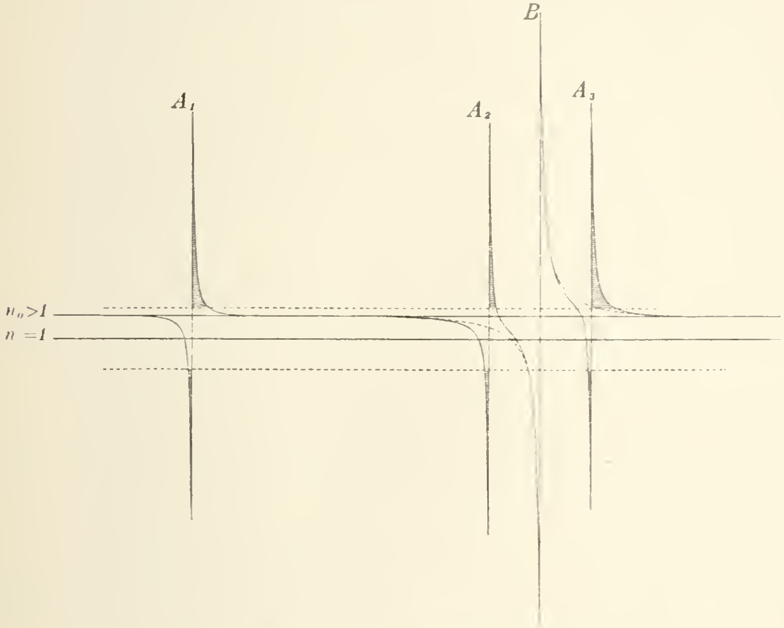

FIG. 1.—Mutual influence of Fraunhofer lines

Now comparing for the lines A_1, A_2, and A_3 the horizontal distances between the "centers of gravity"[1] of their R-area and V-area, we at once realize that A_2 will show a smaller "relative displacement" (distance) than A_1, and A_3 a greater relative displacement than A_1; but it is also evident from the figure that the difference between the cases A_3 and A_1 is not so marked as that between A_2 and A_1.

Such is the anticipated manifestation of mutual influence if we are considering the relative displacements involved in the Evershed effect. We refer to it as case α.

In the general spectrum of the disk, with the exception of spots, the R-area and V-area co-operate in forming the Fraunhofer line by irregular refraction and scattering. With respect to the uninfluenced condition A_1, A_2 will there appear displaced toward the violet, and A_3 a little less toward the red. This is the effect as studied by Albrecht. We refer to it as case β.

Now it is clear that both the R-area and the V-area of A_2 have their "centers of gravity" displaced toward the violet as compared with the condition A_1; the *mean* of the two shifts is what we observe as "mutual influence" in case β.

In case α, on the other hand, we observe only the finer detail that the center of gravity of the R-area is a little more displaced than that of the V-area; it is the *difference* of these shifts which determines the reducing influence of B on the Evershed effect of A.

Since in case β the order of magnitude of the displacements due to mutual influence proved to be 0.006 A, the mean value in case α could scarcely be expected to exceed 0.002 A, even if the separate residuals were weighted, as proposed by Albrecht, and if lines suspected of being sensibly influenced by companions on *both* sides had been omitted from the list.

This would explain why St. John obtained a negative result. As far as the available measurements of the Evershed effect were concerned, St. John's statement that the presumed mutual influence was well suited for making a "quantitative" and definitive test of the theory was premature. A positive result would have furnished a *qualitative* confirmation; the negative result left the problem in suspense; but since the existence of mutual influence has been definitely established by Albrecht, we have no doubt that it will be detected in the Evershed effect too.[1]

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[1] The term "center of gravity" is used for the sake of convenience. Properly speaking, the estimated location of the lines will of course depend on considerations somewhat different from those involved in the determination of the center of gravity of the said area.

Perhaps it is not superfluous to remark that the absolute magnitude of the displacements due to mutual influence (say 0.006 A) should not be thought to represent an approximate measure of the total share which anomalous dispersion has in the phenomena of the Fraunhofer spectrum. That share must be many times greater. Mutual influence is but a peculiar *differential* effect by which the *integral* effect (i.e., almost the total width of the Fraunhofer line) reveals its nature. Only if the Fraunhofer lines are *mainly* due to anomalous dispersion will they be able to show a mutual influence of the observed kind and magnitude.

Another form of mutual influence, a cumulative effect, is the fluctuation of n_0 along the spectrum. Many of the small irregular differences in wave-length between lines of the solar spectrum and corresponding lines observed in the laboratory will probably find their explanation in such consequences of anomalous dispersion.

Before closing this section we draw attention to the rather large number of cases which seem to be exceptions to the rule required by the theory. For companion toward the red Albrecht found that 11 per cent of the lines gave values of opposite sign from the mean, while for companion toward the violet as many as 26 per cent showed opposite sign. These need not be real exceptions. Indeed, n_0 fluctuates, and in some regions of the spectrum might be < 1, which would reverse the sign of the required effect. Besides, small uncertainties in the wave-lengths of the terrestrial comparison lines are not excluded.

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[1] Differential shifts of this kind are by no means proportional to the intensity of the influencing lines. A reasonable analysis of the various possible cases of line-shift, which diagrams of the type of Fig. 1 could illustrate, would easily bring out the error in St. John's presumption (*Mt. Wilson Contr.*, No. 93, p. 21) that the great wings of such very strong lines as H and K should be precisely the place for mutual influence (in the Evershed effect) to manifest itself. As a full treatment of this question would require more space than can here be spent on it, I leave it to the reader, and will only remark that in such large wings the slope of n_0 is rather slight.

2. *Anomalous dispersion in the laboratory and in the sun.*—Several of the objections raised by St. John in his criticisms are founded on his opinion that astrophysical deductions from the dispersion theory have been made to the neglect of, and in contradiction to, the results of laboratory work on anomalous dispersion. He says:

> It is worthy of note that Professor Julius takes no account of laboratory results when considering center and limb displacements; but to prove that very strong and very weak anomalous dispersion make the displacements small and that intermediate values give large displacements, he classifies the lines simply according to line intensity, irrespective of their known anomalous dispersion. This appears to assume that anomalous dispersion is proportional to line intensity. Likewise, there is no attempt to correlate the effects of mutual influence with what is known of the power of the influencing lines to produce anomalous dispersion phenomena. It seems again to be assumed that all lines of the same intensity are equally effective.[1]

This is all nearly true; but what St. John here represents as a procedure to be censured and an additional assumption without foundation is nothing but a paraphrase of my fundamental hypothesis that Fraunhofer lines are, in the main, dispersion bands. This hypothesis implies, of course, that in first approximation the intensity of the lines should be proportional to the anomalous dispersion—in the sun—of adjacent waves.

I think it more reasonable to try the supposition that the intensity and further properties of the Fraunhofer lines are intimately connected with the amount of anomalous dispersion effects *in the sun*, than to assert that, if dependent at all upon anomalous dispersion, they should be proportional to such effects of anomalous refraction as one has hitherto succeeded in producing *in the laboratory* under circumstances not comparable with those obtaining in the sun.

A quotation made by St. John[2] from the earliest paper ever published on the subject (1900) shows that the simple but superficial idea of expecting some close agreement between the intensity of chromospheric lines and the amount of anomalous dispersion manifested by the corresponding lines in the laboratory was also the first to occur to me; but I soon perceived that the connection between solar phenomena and this kind of laboratory results could not possibly have the simple form which St. John still continues to demand.

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[1] *Astrophysical Journal*, 41, 55, 1915; *Mt. Wilson Contr.*, No. 93, p. 28.

[2] *Astrophysical Journal*, 41, 51, 1915; *Mt. Wilson Contr.*, No. 93.

The present theory of light requires that the refracting power $\pm(n-1)$ of a medium varies rapidly for waves in the neighborhood of each of its proper periods. Anomalous dispersion may be called a general property of matter. Such expressions as "the power of the lines to produce anomalous dispersion effects,"[1] though convenient and often used, appear to be misleading. It is not the lines that produce the effects; it is the medium, and only if circumstances are favorable.

Suppose we have a homogeneous gas giving a monochromatic absorption line. Let a parallel beam of white light traverse a layer of it one meter thick and of constant density throughout: the line will show very thin and sharp (or perhaps will not show at all). Let the thickness of the layer increase to several kilometers: anomalous scattering will envelop the line in narrow, tiny wings. Let the gas become disturbed: anomalous refraction in the irregular density-gradients will widen and darken the wings.[2]

Thus, temperature, mean pressure, average density, electric and magnetic conditions remaining the same, a line may show very different aspects according as the medium is more or less extensive and disturbed.

Great extension and optically effective disturbances of density of gaseous media are, as a rule, hard to realize in the laboratory, but are sure to govern solar phenomena.

The success of a laboratory experiment on anomalous dispersion depends upon the possibility of obtaining the gas in such a quantity and condition that it presents a sufficiently great and controllable density-gradient to make the rays deviate appreciably; or, if studied by the method of the interferometer, that a sufficient number of electrons of the considered proper period are present to affect the velocity of light appreciably. A negative result of the experiment does not signify that "the line in question lacks the power of producing anomalous dispersion effects," but simply that, in the medium, the external conditions necessary for their appearance were not fulfilled.

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[1] *Ibid.*, p. 56.

[2] Phenomena of this kind might, for example, be studied by observing how the atmospheric lines of the solar spectrum vary with the sun's altitude and the conditions of our atmosphere.

Effects of anomalous refraction are much more easily obtainable with sodium than with magnesium or iron. In the solar spectrum several lines of magnesium and iron are nevertheless more conspicuous than the sodium lines. This, however, does not at all conflict with the anomalous dispersion theory of solar phenomena, for it is quite possible, for instance, that the visible layers of the sun contain much less sodium than iron and magnesium.

The foregoing remarks may suffice to refute the objections based on the absence of proportionality between properties of Fraunhofer lines and amount of dispersion effects hitherto obtained in the laboratory.

3. *General shifting of the Fraunhofer lines to the red.*—Two classes of these general displacements are usually distinguished: the sun-arc (or rather center-arc) shifts and the limb-center shifts.

The interpretation of the center-arc shifts on the basis of the Doppler principle would imply that the various constituents of the solar atmosphere are descending with specific velocities all over the solar surface. Line displacement due to this cause should, of course, decrease toward the limb. The observations, however, show an increase toward the limb; the limb-center shifts had therefore to be considered as a separate phenomenon, superposed upon the center-arc shifts. Evershed again invokes the Doppler principle for explaining them, and thus has to admit the existence of a selectively acting repulsive force exerted by the earth on the solar gases.

Another interpretation often applied to both classes of shifts is based on the pressure-effect. The limb-center shifts would on this view indicate an increasing preponderance of the pressure-influence due to the lower levels with respect to the pressure-influence due to the higher levels as we proceed from the center toward the limb. The interpretation requires that there be some parallelism between the relative displacements of the lines of a given element in the solar spectrum and those of the corresponding lines under pressure in the laboratory.[1] The fact that the agreement is far from satisfactory induced Evershed to reject the pressure theory, and to have recourse to the hypothetical repulsing force mentioned above.

According to the dispersion theory Fraunhofer lines are, as a rule, unsymmetrical; this would be the cause of their apparent displacements. Width, intensity, and degree of asymmetry will vary with the absolute quantity, the distribution of density, and the further conditions of the gases traversed by the light which comes from the interior of the sun. Quantity and condition of traversed material vary gradually from center toward limb. This produces the difference between limb lines and center lines, which lines, for the rest, must have a very similar origin. It would therefore appear safer to draw conclusions from a comparison of limb lines with center lines than from a comparison of center lines with arc lines, for in the latter case many unknown differences of condition might be involved.

It is a fact that the only broad regularity until now observed in the limb-center shifts is a dependence on line intensity of just the kind predicted from the point of view of the anomalous dispersion theory. Other interpretations of the displacements had not suggested the possible existence of a connection between the general limb-center shift and the intensity of the lines.

St. John objects[2] that the center-arc shifts appear not to be subject to the same law. I question whether there are sufficient grounds for this contention as yet. From all the data hitherto obtained in the matter by Evershed and Royds, Fabry and Buisson, and Schwarzschild we derive a result different from that given by St. John, although, it must be granted, not in *clear* agreement with the law. Further evidence must be awaited, especially because the wave-lengths of many arc lines prove to be dependent on certain still uncontrollable circumstances, which seem to make center-arc shifts less reliable than limb-center shifts.

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[1] There is more reason to demand such parallelism between solar and laboratory results when solar line displacements are to be ascribed to pressure-effects than when anomalous dispersion effects are called in for explanation, because pressure-shifts are not dependent on the *absolute* quantity of matter under observation.

[2] *Astrophysical Journal*, **41**, 61, 1915; *Mt. Wilson Contr.*, No. 93, p. 34.

A few lines of the solar spectrum are shifted to the violet instead of to the red. St. John supposes these cases to be unaccounted for by the anomalous dispersion theory; but he forgets that n_0 may occasionally be < 1, which would reverse the direction of the asymmetry of the Fraunhofer line.

The last paragraph of the section devoted by St. John to general displacement[1] should be appreciated as a friendly turn toward Mr. Adams, in which I would gladly join, since I have neither said nor thought that Mr. Adams considered pressure as the *only* effective agent concerned in limb-center shifts. Meanwhile it remains true that Adams—quite naturally from his point of view—has not foreseen the existence of a rather simple connection between displacements and line intensity.

4. *The flash spectrum and anomalous dispersion.*—The darkness of Fraunhofer lines and the brightness of flash lines are from the point of view of the anomalous dispersion theory complementary phenomena—two aspects of one and the same process. They are both caused by selective scattering of the waves whose periods differ very little from the proper periods of the medium. It is convenient to distinguish molecular scattering or diffusion from the coarser scattering due to refraction by the irregularities of optical density of the medium, although the two kinds perhaps pass gradually into each other (opalescence). Diffusion, besides depending on the refractive index, is inversely proportional to the fourth power of the wave-length, whereas refractional scattering varies only with n.

If there be a bright background, scattering will diminish its brightness. This explains the Fraunhofer lines. The scattered light itself, if seen against a dark background, constitutes the lines of the flash spectrum.

Suppose we direct a powerful telescope on a point of the sun's edge. One half of our small field of view will be a bright, the other half a dark, background. Direct light coming from the limb would have to travel a long way through the gaseous medium (20,000 km through a layer 300 km thick, 40,000 km through a layer 1200 km thick).

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[1] *Op. cit.*, pp. 64 or 37.

From a discussion of the effects of scattering in the long, narrow column of solar gases through which the light from the limbs has to pass, we should be able to deduce the principal properties of both the flash spectrum and the Fraunhofer spectrum of the limb. The task is not very simple but is, I think, quite definite; every single case is capable of being considered from this point of view, and the *general* features of those two types of spectra will be seen to follow directly from the proposed mental image.

It should first be recalled that the localization of the sun's edge depends, according to our interpretation of the photosphere,[1] upon refraction in the irregular density-gradients. The diameter of the photospheric disk is determined by the average refractional scattering of the waves not appreciably subject to anomalous dispersion. Waves suffering sensible anomalous refraction in layers near the level of the apparent limb will produce somewhat larger solar disks of their own.

To every value of $\pm(n-1)$ corresponds a definite magnitude of disk. The ring-shaped parts of those larger disks which protrude beyond the principal disk form together the chromosphere.[2]

If the slit of a spectrograph be set exactly tangent to the principal disk, it cuts the protruding rings, whose light will furnish the flash spectrum, every line of which consists of waves on both sides of the proper periods of the medium. On a flash line the corresponding Fraunhofer line may be visible ("double reversal") because, properly speaking, the slit was still *on* the disks which are formed by the waves closely approximating the proper period in question. The visibility of this double reversal requires, of course, a very accurate setting of the slit; so we at once understand why Adams and Burwell found that "double reversal seems to be a universal characteristic of all lines in the chromospheric spectrum at a low level,"[1] whereas in eclipse spectra obtained with the prism camera double reversals are restricted to the stronger lines.[2]

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[1] *Astrophysical Journal*, 38, 129, 1913.

[2] In first approximation we neglect the radial density-gradient in comparison with the irregular gradients. Refraction in the small-scale density-gradients contributes to both the R-light and the V-light of the chromospheric lines. By the term "small-scale gradients" I refer to a condition of differing density in spaces so small that they cannot separately be seen at the distance of the earth. The objection made by Adams and Burwell at the top of p. 26 of *Mt. Wilson Contr.*, No. 95, viz., that the wave-lengths of the chromospheric bright lines do not show any marked systematic variations, will thus be met. We are dealing with an average effect. Large-scale density-gradients in higher levels manifest themselves as prominences; in their spectrum it is often observed that R-light and V-light are locally separated.

Important results of recent observations of the flash spectrum (by Hale and Adams, Mitchell, Adams and Burwell) are:

1. There is a regular progression of the lengths of flash spectrum arcs with intensities of the Fraunhofer lines.[3]

2. The difference in intensity between flash lines and Fraunhofer lines is positive for very weak solar lines, diminishes gradually as stronger lines are considered, passes through zero, and attains increasing negative values[4] for solar lines up to intensity 6 (which law might also be expressed thus: flash line intensity increases with solar line intensity, but at a slower rate).

3. Enhanced lines are of an exceptionally great intensity in the flash spectrum, and nearly always appear as double reversals. The same class of lines generally are weakened and almost deprived of their wings in the spectrum of the disk close to the limb.

4. According to Adams and Burwell flash lines are displaced to the red relative to the Fraunhofer lines at the center of the sun by an average amount of about 0.006 A. For the mean of the limb-center shifts the same observers found 0.008 A, but they do not feel justified in considering the slight difference between those two mean values as certainly real.[5] Earlier measurements by Hale and Adams had given an equal small difference of opposite sign. So we may in first approximation admit that flash lines are not displaced appreciably with respect to Fraunhofer lines of the limb spectrum.

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[1] *Astrophysical Journal*, **41**, 136, 1915; *Mt. Wilson Contr.*, No. 95, p. 21.

[2] The fact that the general duplicity of chromospheric lines, which was predicted in the earliest paper on anomalous dispersion in the sun (1900), did not clearly appear on the best eclipse plates has been adduced as an argument against the interpretation of the chromosphere on the basis of the dispersion theory. Now that duplicity seems after all to be a universal property of those lines, the subject might perhaps again serve as an argument, but now in favor of the theory.

[3] St. John, *Astrophysical Journal*, **40**, 358, 1914; *Mt. Wilson Contr.*, No. 88, p. 3.

[4] *Astrophysical Journal*, **40**, 368, 1914; *Mt. Wilson Contr.*, No. 88, p. 13.

[5] *Astrophysical Journal*, **41**, 143, 1915; *Mt. Wilson Contr.*, No. 95, p. 28.

These four points will now be briefly discussed from the point of view of our theory.

1. We refer to the concept, mentioned above,[1] of the long column of gases through which a small field of view surrounding a point of the limb is observed. To every line there corresponds, within the column, a certain number of carriers of a given definite proper period. An immense stream of light goes across the full length of the column. The amount of selective scattering (molecular and refractional) will depend upon that number of carriers. Evidently there must be a close relation between the intensity of the scattered light which in the dark half of the field of view produces the flash line, and the loss by scattering which in the bright half causes the darkness of the Fraunhofer dispersion line. A regular progression of flash intensity with Fraunhofer intensity is thus easily accounted for. At the same time, increasing intensity of the flash lines will make the flash-arcs, obtained with the prism camera, show longer; and as the distance from the sun's edge at which scattered light is still visible will also increase with the number of carriers in our column, we have an additional cause for the arcs of the flash spectrum to increase in length with the intensity of the solar lines.

This is the place to notice a difference of some importance between the conclusions to which the current and the new interpretation of flash-arcs lead.

If the flash-arcs be considered as due to selective emission, the length of an arc would furnish us with an indication of the height to which the element in question generally rises in the solar atmosphere. Elements with high atomic weights show, as a rule, short arcs, and it seems reasonable that they should be found only at low levels. But carbon arcs being equally short, a low level has also to be assigned to carbon. There are other discrepancies, or at least difficulties, of this kind. Different lines of one and the same element (Ca, Sr, Ti, Fe, etc.) often indicate, on this view, very different levels, a fact requiring for its explanation the additional hypothesis that the emissive properties of such elements vary with height.

[1] Cf. p. 58.

If, on the other hand, we explain the flash light as due to scattering, the lengths of the arcs (depending on the distances outside the limb at which scattered light is still visible) are determined by the concentration with which each kind of carriers of proper periods is represented in the mixture. It is not denied that the composition of the mixture will probably change with height; but *even* if it remained the same at all heights, carriers weak in number would give short flash-arcs, carriers strong in number would produce long arcs. On this view, the short flash lines of carbon do not indicate low level of that element, but feeble concentration; and as the different carriers corresponding to the various lines of an element need not be equally numerous nor equally effective, there is no difficulty in understanding why an element should give flash lines of very different lengths.

2. The peculiarity mentioned in point 2 is very interesting, but it seems hazardous to derive any positive conclusion from it as yet, because the scale of flash intensities is not easily comparable with the scale of intensities assigned to Fraunhofer lines. I would only venture to suggest that, according to the anomalous dispersion theory, the relatively great flash intensity which corresponds to the weakest Fraunhofer lines may find its explanation in the conception that flash light is always a part of the strong photospheric light, however scanty the scattering carriers may be, and that a weak light on a dark background is more conspicuous and will also give stronger contrast on the photographic plate than a slight obscuration on a bright background.

3. The following hypothesis would seem useful for elucidating the special behavior of the *enhanced lines* in the spectra of flash and limb by means of the notions derived from our theory.

We simply suppose those solar lines to be due to carriers whose percentages in the gaseous mixture are relatively great.

All scattering effects produced by such carriers will then be very strong. The "solar disks" peculiar to their various R-light and V-light waves are much greater than the principal solar disk (a fact which, in the language of the current theories, would be expressed by saying that the corresponding elements occupy a thick layer in the chromosphere). From this consideration it follows: (*a*) that, if the slit of a spectrograph be set tangent to the principal disk or at a small distance outside, those chromospheric lines will appear very bright and long, and nearly always as double reversals; (*b*) that in eclipse photographs taken with the prism camera the arcs will be long and broad; (*c*) that, if the slit of a spectrograph be set *on* the disk at a certain short distance from the limb, it will be at *greater* distances from the edges of the peculiar disks which correspond to the strongly refracted waves environing the proper periods of the carriers. This involves weakening of the dark wings in the limb spectrum. For in each monochromatic disk there is a rapid increase of brightness from the edge inward, so that at the selected place of the slit, large-disk waves will have gained more in intensity than average-disk waves, and therefore will show relatively bright. Large-disk waves are those corresponding to the wings; they will be brighter, as compared with the rest of the spectrum, than they are in the spectrum of the central parts of the disk. It is true that for all kinds of light the *absolute* brightness decreases from center toward limb; but for the strongly scattered and refracted light the region of rapid decrease is shifted a little outward; consequently such light is less weakened, at a given point near the limb, than the average light. Its relatively greater strength in the spectrum of the limb manifests itself as a narrowing of the line or a weakening of its wings.

There is an additional cause, dependent on refractional scattering, by which lines, due to principal constituents, will have their wings gradually weakened toward the limb.

For comparison we first consider the rôle of molecular scattering. According to Rayleigh's well known formula the coefficient of molecular scattering increases with decreasing wave-length, and also with decreasing distance from each proper frequency of the medium. The "fogginess" of the sun therefore varies very strongly with the kind of light used in the observations; it is greater with violet than with red light, and greater with R-light and V-light than with waves of adjacent blank parts of the spectrum. Fogginess hinders us from discerning structural particulars at lower levels (hence the varying character of spectroheliograms as the selected wave-length approaches the center of a line), and causes weakening of the transmitted light (hence, in substance, the darkness of the Fraunhofer lines).

Now, besides molecular scattering we have scattering by refraction in irregular gradients, which is especially strong with the R-light and V-light of lines of principal constituents. By this kind of scattering the peripheral parts of the disk will appear illuminated at the expense of the central parts (compare the case of a globe of frosted glass inclosing a source of light); the result is a relative weakening and narrowing of the winged lines in the limb spectrum.

4. The rather close agreement of the wave-lengths of flash lines with the wave-lengths of the Fraunhofer lines at the limb is in perfect harmony with the requirements of the anomalous dispersion theory. Indeed, both classes of lines originate from the same scattering process in almost the same long column of gases. Any small increase in the degree of scattering would, as a rule, make the Fraunhofer line show darker, the flash line brighter. If, therefore, scattering effects be somewhat greater for the R-light than for the V-light, this will manifest itself as a displacement of the flash line as well as of the Fraunhofer line toward the red, by nearly the same amount. It is not[1] difficult to understand, from this point of view, why double reversals should nearly always be symmetrical, although the refracting power of the solar atmosphere is, as a rule, greater on the red side of an absorption line.

5. *Supplemental remarks.*—A few of the objections made by St. John are left which have not been given explicit or implicit consideration in the preceding pages.

On p. 39 of *Mt. Wilson Contr.*, No. 93, St. John refers to observations showing displacements when the slit of the spectrograph is perpendicular to the radius of the solar disk passing through the center of the umbra of a spot. From earlier publications by Evershed and St. John, I understood that such displacements were rather exceptional and not systematic; so I suggested that they might be accounted for by admitting unequal refraction at opposite edges of the spot. Since observations by more refined methods have shown that such displacements "occur and persist for weeks in regular and symmetrical spots," I am inclined with St. John to ascribe them in part to the Doppler effect connected with vortex motion. The rotary motion round a vortex is more likely to be sufficiently rapid to be observed by its Doppler effect than the radial movement in the spot; and Hansky's above-mentioned observations (cf. p. 47, n. 1) on proper motions of sun-spots seem to prove that real currents of matter at the rate of 0.1 to 0.4 km per second may occur in the sun.

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[1] Cf. Adams and Burwell, *Mt. Wilson Contr.*, No. 95, pp. 26–28, and *Communications to the Nat. Acad. of Sciences*, No. 4. It will be noticed that our argument leads to conclusions directly opposite to those which Adams and Burwell deduced from the anomalous dispersion theory as they apprehended it.

On p. 40, *op. cit.*, St. John again emphasizes the fact that in the spectra of the penumbrae some strong lines show relative displacements in a sense opposite to the direction of the general Evershed effect, and says that up to the present the anomalous dispersion theory has been unable to suggest an explanation. I have twice drawn attention to this point myself (on p. 25 and p. 30 of the paper criticized)[1]; I suggested the cause of the difficulty of the question, and reserved the subject for further investigation. So I do now, since a satisfactory solution has not yet been found.

A third point is that the dispersion theory has until now furnished no explanation of the differences characteristic of the elements such as those shown by the displacements of the lines of iron, titanium, and the heavy elements when lines of the same region and intensity are compared. Thus far no special attention has really been given to this important subject, because, from our point of view, it seemed necessary first to investigate general optical solar phenomena irrespective of the separate elements concerned. But in future the matter should of course also be considered. For the present I would only suggest that the varying composition of the gaseous mixture with increasing height will probably furnish the key of the problem.

We owe to the rigorous critique exercised by St. John and Adams and Burwell on some of my recent papers the fact that attention has been called to various deductions from the anomalous dispersion theory which required ampler discussion than had been given them up to the present, and also that some cases of misapprehension regarding the claims of the theory have come to light. These weak points could now be corrected or explained. I see no reason, however, for modifying the principal conclusions arrived at in those former papers, except in so far as clear evidence of mutual influence of Fraunhofer lines has not yet been obtained from observations on the Evershed effect in sun-spots, but seems to be available in the results of Albrecht's investigations on the wave-lengths of iron lines in the solar spectrum.

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[1] *Astrophysical Journal*, **40**, 1, 1914.

UTRECHT
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THE ULTRA-VIOLET SPECTRUM OF KRYPTON

By E. P. LEWIS

The most complete investigation of the spectrum of krypton has been made by Baly,[1] but he did not carry his measurements in the ultra-violet beyond λ 2418. Recent photographs, taken by the writer, of the spectra of rare gases in vacuum tubes prepared by Hilger showed that the spectra of both krypton and xenon extend to a great distance in the ultra-violet. The intensity of the lines of both these gases diminishes rapidly in approaching λ 2100 to the same degree as the spectra of zinc, cadmium, and other metals known to contain strong lines in this region. The inference is that the apparent termination of their spectra at this point is due to the absorption of the thick quartz system, and that the spectra actually extend into the Schumann region.

The spectrograph used was the large two-prism instrument previously described.[2] Between the limits λ 2100 and λ 2400 the dispersion lies between 1.5 and 3 units per millimeter—a dispersion considerably greater than that in the first order of the 15-foot grating in this laboratory. The light-power is such that three minutes was a sufficient exposure with condenser and spark-gap and ten minutes with self-induction added.

The wave-lengths of the new krypton lines are given in Table I. Some comparisons with Baly's results indicate the probable accuracy of the measurements. Beyond λ 2200 the results are not so reliable, on account of the limited number of standard lines in this region. The wave-lengths were calculated from Hartmann's interpolation formula, corrected by comparison with iron and cadmium lines.

Comparison of the spectra of helium, neon, krypton, and xenon indicated that all these gases were nearly free from contamination with each other. Helium has no lines and neon but few in the region investigated; and but few lines were observed which may

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[1] *Phil. Trans.*, A **202**, 183, 1903.

[2] *Astrophysical Journal*, **23**, 390, 1906.

TABLE I

λ	C.	S.-I.	λ	C.	S.-I.	λ	C.	S.-I.
2145.9	1	......	2265.70	1	......	2341.97	1	......
2149.3	1	......	2266.25	1	......	2343.11	3	......
2155.6	1	......	2267.03	1	......	2344.55	8	5
2159.5	1	......	2268.06	1	......	2345.62	6	......
2162.7	2	......	2270.59	2	......	2348.27	1	......
2164.6	2	......	2271.90	1	......	2350.08	1	......
2166.1	1	......	2272.68	1	......	2353.12	2	......
2169.2	1	......	2273.15	6	2	2353.95	10	9
2170.8	2	......	2273.63	3	......	2355.65	1	......
2172.2	2	......	2274.70	3	......	2356.60	1	......
2173.7	1	......	2277.50	7	......	2358.80	3	......
2177.7	2	......	2279.79	5	......	2360.40	3	......
2179.3	1	......	2280.54	1	......	2362.18	4	......
2185.4	2	......	2281.26	1	......	2363.01	8	......
2186.8	1	......	2282.92	10	3	2363.74	1	......
2188.3	1	......	2284.43	1	......	2365.03	1	......
2192.1	2	......	2285.61	2	......	2365.80	5	......
2193.6	1	......	2287.75	10	3	2366.35	1	......
2197.4	2	......	2289.31	1	......	2368.30	4	......
2206.6	2	......	2290.30	1	......	2369.25	4	......
2211.88	2	......	2291.26	6	......	2370.27	2	......
2213.05	3	......	2292.45	1	......	2371.60	8	......
2215.70	2	......	2296.02	2	......	2372.93	1	......
2216.08	1	......	2297.98	1	......	2373.81	6	......
2216.72	1	......	2299.02	6	......	2375.68	10	6
2217.59	1	......	2300.35	8	......	2376.85	3	......
2218.26	1	......	2301.69	8	......	2382.99	1	......
2219.19	2	......	2302.88	6	......	2383.91	1	......
2221.87	1	......	2304.37	1	......	2384.80	1	......
2225.10	2	......	2304.78	1	......	2385.97	1	......
2227.96	6	2	2305.33	1	......	2387.23	1	......
2230.70	1	......	2307.19	1	......	2388.05	2	......
2232.26	1	......	2309.42	1	......	2389.62	2	......
2232.90	1	......	2311.97	8	......	2390.65	4	......
2233.76	1	......	2314.21	8	3	2392.92	7	3
2234.30	2	......	2315.45	9	4	2394.06	8	......
2237.07	5	......	2316.29	10	4	2394.90	1	......
2243.10	1	......	2317.81	1	......	2397.09	5	......
2244.09	1	......	2319.01	1	......	2398.38	10	......
2244.38	1	......	2319.70	1	......	2399.21	2	......
2245.34	6	......	2320.92	6	......	2400.30	5	......
2247.19	1	......	2322.40	2	......	2401.66	1	......
2248.58	2	......	2323.85	4	......	2402.62	2	......
2250.22	3	......	2324.85	3	......	2403.14	3	......
2251.13	2	......	2326.55	5	......	2403.80	3	......
2251.96	2	......	2328.21	1	......	2406.42	6	......
2252.82	1	......	2329.30	8	......	2407.28	5	......
2253.47	1	......	2330.35	1	......	2408.59	7	3
2254.14	1	......	2331.55	2	......	2409.15	8	3
2255.10	3	......	2332.28	1	......	2412.11	3	......
2259.83	2	......	2333.33	1	......	2413.93	9	3
2260.72	2	......	2336.75	1	......	2415.06	9	3
2262.94	1	......	2339.15	1	......	2416.31	3	......
2263.71	2	......	2340.05	2	......	2416.90	3	......
2264.69	1	......	2340.93	5	......			

possibly be common to the spectra of krypton and xenon. Argon is a possible impurity in the krypton, and some of the observed lines lie very near argon lines given in Eder and Valenta's table; but in most cases the observed intensities make their identity improbable.

The addition of a small inductance to the circuit previously containing spark-gap and condenser caused a marked change in the spectrum. There was a general reduction in intensity, but if the time of exposure was increased until some of the stronger lines in the two spectra were of about the same intensity, it was found that a number of other strong lines were completely suppressed, while a smaller number persisted, but with greatly reduced intensity. The intensities are compared in Table I under the heads C. (condenser and spark-gap) and S.-I. (inductance added).

TABLE II

λ	C.	S.-I.	Baly	λ	C.	S.-I.	Baly
2418.26.....	10	2	2418.13	2446.54.....	8	......	2446.56
2420.31.....	10	2	2420.30	2452.43.....	6	......	2452.38
2425.15.....	5	......	2425.15	2453.39.....	6	......	2453.37
2426.43.....	9	8	2426.46	2454.15.....	4	......	2454.19
2428.43.....	10	9	2428.44	2455.35.....	2	......	2455.42
2439.33.....	8	......	2439.32	2456.18.....	8	4	2456.16
....	......	......	2439.64	2457.78.....	8	......	2457.79
2440.11.....	5	......	....	2459.68.....	7	......	2459.74
2442.67.....	7	......	2442.68	2464.86.....	8	7	2464.87

Tables I and II show that many strong lines are completely suppressed by self-induction, some affected only slightly, and others reduced in intensity in varying degrees. The study of the effect of self-induction was carried into the field of Baly's measurements up to about λ 3700. Beyond this point it did not seem worth while to go, on account of the small dispersion of the spectrograph in this region. In general, it may be said that the suppressive effect of self-induction appeared to be less pronounced in the visible region than in the ultra-violet. Table III gives the lines (additional to those contained in the preceding table) which were only slightly affected in relative intensity by self-induction (all being reduced in approximately the same degree). The interpolation formula was used merely to identify the lines, the wave-lengths being those given by Baly.

The table of new lines shows that a large number are completely suppressed by self-induction. The same is true for the region examined by Baly. In Table IV is given a list of the stronger lines only in the latter region which are so affected. In some cases (indicated by an asterisk) the suppression is not complete, an exceedingly faint trace of the line being seen when self-induction is used.

TABLE III

λ	C.	S.-I.	λ	C.	S.-I.
2503.97	8	6	3446.66	7	6
2506.66	9	6	3460.24	7	6
2589.19	7	5	3470.19	6	5
2592.57	8	8	3488.74	8	7
2620.54	7	6	3503.38	7	5
2643.18	5	4	3535.48	8	7
2648.26	8	7	3544.69	9	9
2712.50	8	8	3589.79	7	6
2733.38	5	4	3600.05	8	8
2742.67	6	5	3608.02	8	8
2795.92	7	6	3623.74	4	3
2816.58	8	8	3632.02	9	9
2833.11	7	7	3637.63	6	5
2967.37	6	5	3654.11	9	9
3151.06	8	8	3669.16	8	8
3200.53	6	5	3680.52	8	8
3207.91	8	8	3686.30	8	8
3405.28	8	8	3718.17	10	9
3427.84	6	5			

Between these extreme groups are a number of lines of different intensities which are not suppressed by self-induction, but which are greatly reduced in intensity.

The first spectrum of krypton (that of the simple uncondensed discharge), as given by Baly, terminates at λ 3500. This contains many lines which are not found in the second spectrum (that of the condensed discharge). The writer has found that in the region of the former, as given by Baly, the spectrum of the simple discharge includes apparently all the lines which survive with self-induction, as well as other lines which presumably belong to the first spectrum only. In the region beyond λ 3500 the spectrum of the simple discharge and that of the discharge with self-induction appear to be identical. The lines which survive with self-induction thus appear in the spectra of all forms of discharge, and appear to be of a persistent type. It seems likely that if any numerical relationships exist, such as constant differences of wave-number, they might best be looked for among these persistent lines. A number of such approximately constant differences were actually found, and are given in Table V. The wave-lengths given in the first column do not belong to lines actually observed, but are strong lines in Baly's table. No attempt was made to ascertain the effect of self-induction upon them, on account of difficulties of identification in this region; but the constant differences between them and groups of persistent lines actually observed indicate probable relationship. The wave-numbers of these lines are given in the second column, and are followed by successive differences between the wave-numbers of lines belonging to each group. The actual discrepancies exceed the limits of accuracy of measurements, and this gives ground

TABLE IV

λ	C.	λ	C.	λ	C.
2483.71	9	2696.71	6	3191.33	8
2486.40	8	2697.41	6	3220.76	6
2487.75	6	2772.73	6	3224.99	6
2494.10	9	2817.00	8	3240.55	10
2497.81	10	2839.92	6	*3245.82	10
2511.83	6	2844.59	7	*3264.94	8
2513.03	6	2851.29	7	3268.61	7
2515.50	7	2870.73	8	3271.77	6
2519.38	6	2892.30	7	3286.01	7
2525.07	7	2893.81	6	3311.59	7
2528.51	6	2900.19	6	*3325.84	8
2538.43	7	2909.30	6	3330.88	8
2554.35	6	2952.69	8	3342.59	8
2555.23	6	2968.44	6	3352.07	10
2563.32	8	2992.36	9	3260.22	6
2565.72	6	3022.43	8	3375.09	6
2604.59	7	3024.57	9	3389.06	6
2630.76	7	3047.07	8	3396.72	6
*2639.86	10	*3056.86	9	*3439.60	9
2643.18	6	3063.26	8	3474.79	7
2648.26	6	3097.27	10	*3497.29	6
2670.78	7	3112.36	8	*3507.58	10
2680.44	7	3120.73	6	3514.68	6
2681.29	8	3124.52	7	*3549.57	8
2683.66	6	3141.48	10	3564.38	8
2690.35	7	3171.06	6	3641.48	6
2695.81	6	3189.23	8	3690.80	8

for some hesitation; but the constant recurrence of differences so near to 95, 130, 165, and 722 seems significant when it is considered that account is taken of nearly all the strong persistent lines, and not arbitrarily selected groups. There are also a number of repetitions of one or another of these differences, or two of them, between isolated pairs or triplets.

TABLE V

λ	n	Δ_1	Δ_2	Δ_3	Δ_4
4059.02..........	2463.66	95.26	129.68	164.89	722.25
3947.76..........	2533.08	95.68	131.33	164.51	721.98
3920.29..........	2550.80	95.94	131.00	....	885.83 (164.50+721.33)
3906.37..........	2586.65	....	130.36	164.69	722.30
3850.23..........	2597.24	94.41	129.46	164.32	721.84
3836.64..........	2606.45	....	130.19	164.72	722.93
3817.23..........	2619.70	....	129.35	164.51	721.31
3783.28..........	2643.21	....	128.39	165.32	721.85
3754.35..........	2663.58	96.01	129.38	....	886.09 (164.50+721.59)

It is possible that further search in the region of greater wavelengths might reveal other lines having similar relationships; and they might also be found among the lines suppressed by self-induction. But the labor involved in such a search seems to be hardly justified in our present ignorance regarding the significance of such relationships.

Physical Laboratory
University of California
October 10, 1915

THE PRINCIPAL SERIES OF SODIUM

By R. W. WOOD and R. FORTRAT

In a paper published by one of us[1] in this *Journal*, it has been shown that the principal series of sodium is much more complete in the absorption than in the emission spectrum. But seven members of the series had been observed up to the time of the publication of this paper. The absorption method raised the number to forty-eight, a greater number than that found even in the case of chromospheric and stellar hydrogen. In this paper the opinion was expressed that higher resolving powers would bring more lines into view, since the head of the band, though clearly indicated on the photographic plate, was not resolvable into lines.

In the present paper we shall give the results of a repetition of this work made in the laboratory of Professor P. Weiss, at Zurich, with the largest and most powerful quartz spectrograph in the world, which he most kindly placed at our disposal. This instrument is designed for double transmission, through a train of six large quartz 60° prisms and one 30° prism backed by mercury (the equivalent of 13 prisms). In the vicinity of the head of the sodium series ($\lambda=2414$) it is capable of resolving lines separated by only 0.03 A and has a dispersion such that one angstrom is represented by 3.5 mm on the plate. This enormous dispersion and separating power, while advantageous for the observation of lines, is very unfavorable for the study of variations of intensity along the continuous spectrum, and it is perhaps on this account that we have failed to detect the sudden decrease in the transmitting power of the vapor after the head of the band has been passed, which was recorded in the earlier paper.

The results which have been accomplished in the repetition of the work may be briefly summarized as follows: (*a*) The number of members of the series has been raised from forty-eight to fifty-eight, ten new lines at the head coming into view, and we are of the opinion that this marks the limit and that no further modification of the experiment or improvement of apparatus will enable any notable advance to be made. Possibly the use of a very long tube (say 15 or 20 m) would enable us to record a few more lines. (*b*) Previous investigations have shown as *doublets* only the first three members of the series, namely, the D lines, and the ultra-violet lines at λλ 3302 and 2853. We have distinctly resolved four more, so that we now have the wave-lengths of the components of the first seven doublets of the series. (*c*) The wave-lengths of all the lines of the series have been determined to the third place of the decimals with reference to the new secondary standards of Fabry and Buisson.

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[1] R. W. Wood, *Astrophysical Journal*, **29**, 97, 1909.

APPARATUS AND METHODS

The metallic sodium was vaporized in a tube of steel 2.8 m in length and 4 cm in diameter, heated electrically by a spiral of nickel wire insulated with asbestos and wound directly on the metal tube. The tube thus wound was packed in a thick layer of infusorial earth contained in a long wooden box; this prevented loss of heat, and enabled us to keep the tube at a red heat with an expenditure of current of about 1.3 kw. The ends of the tube, which protruded from the box, were closed with thin quartz lenses attached with sealing-wax, and kept cool by means of coils of lead tubing (soldered around the ends of the steel tube) through which a current of water circulated. Previous to heating, the tube was exhausted by a Gaede pump, and the pressure kept at a few millimeters throughout the experiment, by the occasional operation of the pump, which was necessary on account of the evolution of hydrogen by the sodium. As a source of light we required something which would give us a continuous spectrum of great intensity in the ultra-violet. The spark under water was first tried, but we finally adopted a quartz mercury arc of special construction which was placed at our disposal by Professor A. Cotton of Paris. In this lamp the arc passes through a heavy tube of quartz 3 cm in length and 4 mm in diameter. The lamp is constructed in such a way that the light from the narrow tube (end-on) passes out through a window of optically worked, fused quartz, and it operates at a full red heat.[1] The spectrum is quite different from that given out by the usual type of quartz mercury arc, for the lines are enormously broadened and only slightly brighter than the continuous background.

This lamp was mounted end-on at the principal focus of the first quartz lens (for the wave-length in question) and the parallel beam, after traversing the tube, was focused on the slit of the spectrograph. The temperature of the tube and the resulting vapor-density of the sodium were regulated according to the portion of the series under investigation. The D lines probably appear when the temperature is as low as 100° C. (the senior writer has seen them with an echelon with a glass bulb of vapor only 8 cm in diameter at a temperature of 120°), while to get the last members near the head of the bands a density corresponding to a red heat is necessary. One sacrifices the sharpness of the lines somewhat by this augmentation of density, and it is possible that slightly better results could be obtained with a longer tube (say 20 m) at a lower temperature. With a little practice one soon learns how to control the temperature for any portion of the series by observing the width of the D lines, and the condition of the channeled spectra in the red and green, in the transmitted light, by means of a small direct-vision spectroscope.

RESULTS

To secure the complete spectrum, it was of course necessary to expose a large number of plates, on account of the enormous dispersion of the spectrograph. In each case a comparison spectrum of iron was impressed on the plate in coincidence with the absorption spectrum. The values of λ for the D lines are those obtained by the recent measurements by means of interference fringes (*étalons secondaires* of Buisson and Fabry). For the lines at λ 3302 we have taken the values given by Kayser and Runge, correcting them by comparing their values for the iron lines with the new standards of Buisson and Fabry. For the value of the lines at λ 2853 we have taken that of Fortrat for the emission lines, determined with reference to the same standards. The other lines were determined from the photographs of the absorption spectrum. The wave-lengths, correct to a few thousandths of an angstrom unit, are given in Table I.

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[1] The lamp, "type Pistolet," was constructed by the Westinghouse Cooper-Hewitt Company, Paris Branch.

REGULARITY OF THE SERIES

It is well known that Ritz has given a formula for the principal and subordinate series which is closely related to the formula of Balmer, which expresses the frequency ν as follows:

$$\nu = N\left(\frac{1}{4} - \frac{1}{m^2}\right).$$

Though this formula represents the series of hydrogen very exactly ($N = 109{,}678.4$ and $m = 3, 4, 5, \ldots, 32$), no other series has been found which is well represented by so simple an expression.

The formula which Ritz has proposed for the principal series[1] is the following:

$$\nu = N\left[\frac{1}{A^2} - \frac{1}{\left(m + a - \frac{b}{m^2}\right)^2}\right],$$

in which N is the universal constant 109,674.8, A, a, and b are constants depending on the element which emits the series, and m takes the successive values 2, 3, 4, ... , etc.

The form of A is an essential point in the theory.

$$\frac{1}{A^2} = \frac{1}{\left(1.5 + a' + \frac{b'}{(1.5)^2}\right)^2}.$$

The principal series of sodium is a series of doublets, the two components of the successive members approaching as the wave-length decreases. In the Ritz formula there are two values for each of the constants a and b, but single values only for the constants a' and b'. For the calculation of the constants we have employed the three first doublets of the series, i.e., the D lines and the ultra-violet lines at wave-lengths 3302 and 2853, reducing them to the new standards as previously explained.

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[1] *Collected Works*, pp. 52 and 100; *Annalen der Physik*, **25**, 662, 1908.

TABLE I

m	Observed Wave-Length	Observed Frequency	Difference between Calculated Frequency and Observed Frequency
2 D_1......	5895.930	16,955.88	0.00
2 D_2......	5889.963	16,973.02	−0.04
3 D_1......	3302.94	30,266.66	−2.80
3 D_2......	3302.34	30,271.10	−1.29
4 D_1......	2853.031	35,039.67	0.00
4 D_2......	2852.828	35,043.10	+0.94
5 D_1......	2680.443	37,295.71	+0.38
5 D_2......	2680.335	37,297.67	−0.45
6 D_1......	2593.927	38,539.68	+0.27
6 D_2......	2593.828	38,541.07	+0.07
7 D_1......	2543.875	39,297.86	+0.21
7 D_2......	2543.817	39,298.76	+0.03
8 D_1......	2512.210	39,793.16	+0.97
8 D_2......	2512.128	39,794.46	+0.15
9 D_2......	2490.733	40,136.27	+0.36
10 D_2......	2475.533	40,382.73	−0.04
11........	2464.397	40,565.19	+0.25
12........	2455.915	40,705.28	−0.36
13........	2449.393	40,813.65	+0.13
14........	2444.195	40,900.45	−0.07
15........	2440.046	40,970.02	+0.37
16........	2436.627	41,027.48	+0.43
17........	2433.824	41,074.69	+0.77
18........	2431.433	41,115.11	+0.35
19........	2429.428	41,149.05	+0.30
20........	2427.705	41,178.24	+0.09
21........	2426.217	41,203.51	−0.23
22........	2424.937	41,225.25	−0.31
23........	2423.838	41,243.90	−0.24
24........	2422.856	41,260.61	−0.16
25........	2421.997	41,275.28	−0.16
26........	2421.233	41,288.29	−0.15
27........	2420.520	41,300.45	−0.71
28........	2419.922	41,310.67	−0.54
29........	2419.380	41,319.92	−0.45
30........	2418.893	41,328.25	−0.35
31........	2418.454	41,335.72	−0.20
32........	2418.062	41,342.43	+0.02
33........	2417.695	41,348.70	+0.06
34........	2417.362	41,354.41	+0.11
35........	2417.058	41,359.60	+0.20
36........	2416.779	41,364.39	+0.25
37........	2416.518	41,368.87	+0.23
38........	2416.271	41,373.08	+0.16
39........	2416.046	41,376.94	+0.08
40........	2415.838	41,380.51	+0.03

TABLE I—*Continued*

m	Observed Wave-Length	Observed Frequency	Difference between Calculated Frequency and Observed Frequency
41	2415.651	41,383.70	+0.10
42	2415.474	41,386.74	+0.10
43	2415.305	41,389.64	+0.03
44	2415.147	41,392.34	−0.03
45	2415.006	41,394.77	+0.01
46	2414.872	41,397.07	+0.01
47	2414.746	41,399.22	+0.03
48	2414.627	41,401.27	0.00
49	2414.518	41,403.13	+0.05
50	2414.411	41,404.97	0.00
51	2414.313	41,406.65	+0.01
52	2414.218	41,408.28	−0.03
53	2414.131	41,409.77	−0.05
54	2414.050	41,411.16	+0.02
55	2413.971	41,412.52	+0.01
56	2413.910	41,413.57	+0.23
57	2413.873	41,414.20	+0.81
58	2413.837	41,414.81	+1.34

For the lines below the last of the resolvable doublets (λλ 2512.210 and 2512.128) we have compared the measured values with the values calculated from the constants of the D_2 series, since the line D_2 is more intense than D_1. For D_1 we found:

$$\nu = 41{,}448.59 - \frac{109{,}678.4}{\left(m + 0.14373 - \frac{0.11043}{m^2}\right)^2}.$$

For D_2:

$$\nu = 41{,}448.59 - \frac{109{,}678.4}{\left(m + 0.14497 - \frac{0.11240}{m^2}\right)^2}.$$

In the last column of Table I will be found the differences between the calculated and observed frequencies. The calculated position of the head of the series is

$$\lambda = 2412.627,$$

and we have therefore reached a point only 1.21 A from the theoretical end.

The last thirteen members of the series are plotted from the wave-length determinations in Fig. 1. The calculated head of the series lies to the left of the fifty-eighth line at a distance equal to the length of the portion of the series figured (i.e., the distance between the forty-sixth and fifty-eighth members). If we continue plotting the series on the same scale, the D lines will lie on our right at a distance of 348.3 m, or more than a thousand feet! The last thirty-two members, or more than half of the entire series, cover a spectral range not wider than the distance between the D lines.

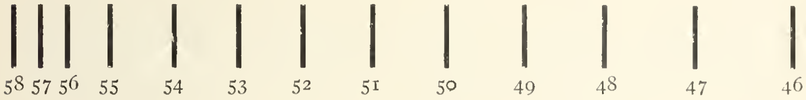

Fig. 1

If we examine the last column of the table, which gives the differences between the measured and calculated frequencies, we see that the formula of Ritz represents the series with a surprising degree of accuracy. The *differences* of frequency become much less when we convert them into differences of wave-length. For example, in the case of the last line measured ($m=58$), which could not be very accurately determined on account of its faintness, the difference amounts to only 0.08 of an angstrom. This is the largest difference except that for $m=3$, which is -2.8, and Ritz has already pointed out the disagreement which occurs at the *beginning* of the series, which seems to be a general phenomenon.

The disagreement which we have found at the *end* of the series may result from slight errors of measurement. Neglecting the first and last three members, the differences vary in a systematic manner and indicate the degree of imperfection of the formula. In the case of the largest, where the error of measurement probably adds itself to the error of formula, the difference is less than 0.05 A.

As is well known, only the first seven of these sixty-eight members have been found in the emission spectrum. The explanation of this circumstance seems to be not difficult. In the first place, Ritz has supposed that "turbulent motions," such as occur in the spark, are unfavorable to the appearance of the higher members, i.e., more lines appear in the arc than in the spark. On this hypothesis we should expect the comparatively cool and quiescent vapor used in absorption experiments to be particularly favorable to the development of the groupings that give the higher members (Ritz's theory of magnetic atoms). Moreover, if we assume that the groupings which form the higher members occur only in small proportion, we can bring them out only by using an immense volume of vapor, which can be done to better advantage by the absorption method.

ZURICH
1914[1]

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[1] Publication delayed on account of the war.

MINOR CONTRIBUTIONS AND NOTES

NOTES ON THE USE OF THE CONCAVE GRATING

THE EFFECT OF TEMPERATURE ON THE CONCAVE GRATING

All investigators agree that in order to get perfect definition with a concave grating, the temperature during the exposure must be constant, barometric pressure being assumed constant. However, there still appears to be some uncertainty as to the exact nature and extent of the effects of varying temperature.

The author,[1] after a rather careful investigation, convinced himself that the only important effect of changing temperature is the change in the width of the grating space. The dispersion is directly proportional to this width, and hence also the distance of all lines from the slit. It follows immediately from this that on any one plate the shift of all lines is approximately the same. It is to this point that Grebe,[2] in an article on this same subject, devotes most of his attention. He deals entirely with the effect of temperature on the accuracy of wave-length measurement, and concludes with the statement that, even for the most accurate work, variations of temperature of not more than 1° C. are non-injurious.

This statement is not at all true when the definition of the grating is under discussion, a change of 1° C. shifting λ 5000 the relatively large amount of 0.13 A. This is several times the resolving power of a large grating. For band spectra, with which the author is primarily concerned, the best possible definition is essential, for even the largest gratings, used under the best conditions, have far too small dispersion and resolving power to separate many lines. For such work it is necessary to keep the temperature, after suitable correction for barometric pressure, within a range of 0°1 C., if one is to secure the best results.

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[1] *Astrophysical Journal*, 39, 55, 1914.

[2] *Zeitschrift für wissenschaftliche Photographie*, 13, 264, 1914.

The author wishes, in addition, to call particular attention to another effect of temperature, which up to the present appears to have been overlooked. This is the effect upon the definition of *temperature-gradients* across the face of the grating. The magnitude of such gradients depends, of course, primarily upon the location of the grating. The effects which the author has observed, using the large concave grating of the University of Wisconsin, may be quite lacking on other gratings more favorably situated. This grating, as has been mentioned before (*loc. cit.*), is mounted inside a double-walled room built entirely within an ordinary room. The situation of the outer room is such, however, that almost the entire flow of heat takes place naturally through the west side, and unfortunately the rail carrying the grating carriage is located close to this side. Even when no attempt is made to control temperature, the grating's temperature never changes at a rate of more than 0°1 per hour. The effect of such a change on short exposures is negligible. But the transfer of heat entirely through the west wall sets up temperature-gradients in the grating which have been found experimentally to produce a loss of definition equal to that caused by a direct change of temperature of fully 1° C. This effect is so serious that the author made a careful investigation to convince himself that it was really due to temperature-gradients. Even after the temperature becomes constant, it is necessary to wait for one or two hours, before these gradients are entirely "smoothed out."

In conclusion, then, I desire to repeat that in order to obtain the best definition with gratings subject to unequal heating from different sides, it is necessary, in the case of *short* as well as of long exposures, for the grating to have a constant temperature not only during the exposure but for an hour or two previous to that time.

THE EFFECT OF TEMPERATURE UPON THE COINCIDENCE OF ORDERS OF THE CONCAVE GRATING

Miss Howell[1] has shown, for the particular pair of lines λ 5640 and λ 2820, that there is a shift of 0.005 A in the coincidence of orders of a concave grating, when the temperature is varied from

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[1] *Astrophysical Journal*, 39, 230, 1914.

0° to 10° C. She does not, however, explain why Kayser[1] found no such shift (15°–25°), nor does she show how the shift varies with wave-length. Instead, she advises reducing all wave-lengths to standard conditions (15° C. and 760 mm) when using the coincidence of orders. This, however, involves a considerable amount of quite unnecessary work.

Using the ordinary formula for change of index of refraction with temperature:

$$n_t - 1 = \frac{n_0 - 1}{1 + at}, \tag{1}$$

where

n_t = index of refraction at t° C.
n_0 = index of refraction at 0° C.
$a = 0.00367$ (air)
t = temperature Centigrade

it can easily be shown that if any two lines coincide at t_1, the amount of non-coincidence (i.e., shift) at any other temperature t_2 is

$$x = \frac{\lambda_1 a(t_2 - t_1)\Delta n}{1 + a(t_2 + t_1)}, \tag{2}$$

where

Δn = difference of the indices of refraction of the two lines at 0° C. (a *negative* quantity)
λ_1 = line of longer wave-length (i.e., smaller order)
x = shift from coincidence in terms of λ_1

In deriving this expression only terms of the first order were retained. It is, however, quite accurate for all practical conditions.

Formula (2) shows that there will always be a shift in coincidence, with changing temperature, if air has any dispersion (Δn) whatever. Kayser, computing the shift by the longer direct method, evidently failed to carry out a sufficient number of places in the calculations, and so got in some cases a negative result.

Δn depends upon the curve of dispersion of air. The latest work on this is by Miss Howell.[2] She also gives the results of all previous investigators (*op. cit.*, p. 87). These different results give different values of Δn, but the difference is not great enough to affect the work appreciably.

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[2] *Handbuch der Spectroscopie*, 1, 720.

[1] *Physical Review* (2), 6, 81, 1915.

As an example of the use of formula (2) I take Miss Howell's figures (1):

λ 5640 and λ 2820 coincide at 0° C. What is the shift at 10°? Here $\lambda_1=5640$, $a=0.00367$, $t_2-t_1=10°$, $\Delta n=1.0002924-1.0003091=-0.0000167$, $t_2+t_1=10°$.

$\therefore x=-0.00334$ A. That is, λ 5640 will fall 0.003 A to the *violet* of λ 2820. Miss Howell gives 0.005 A, showing again how easily large errors can creep into a result in a calculation involving small differences of large quantities, such as she uses.

It may be said in general that, for second- and third-order coincidence, the shift is negligible above $\lambda_1=4500$ and $t_2-t_1<5°$. For the second- and first-order coincidence the shift is always more than 0.001 A for $t_2-t_1>5°$. This shows that, for summer temperatures of 10° or more above the standard 15° C., it is necessary to make a correction for the effect of temperature upon the coincidence of orders.

ABSORBING SOLUTIONS FOR THE CONCAVE GRATING

In using an absorbing solution to cut out certain orders of a concave grating, the spectroscopist desires a solution as transparent as possible in one definite region, and as opaque as possible in another. The photographs and accompanying description in atlases of absorption spectra are not usually explicit enough in this regard. The author in some cases has found 10 per cent transmission at a wave-length apparently within the region of total absorption, while 1 per cent to 2 per cent transmission is common. The following figures may therefore be of use to other spectroscopists. It is hoped that they are accurate enough for all practical purposes. The solutions given, with the addition of glass, are sufficient for most ordinary work on the grating.

1. $CuCl_2$, aqueous solution, 2×normal:
    No absorption λ 4380–6000
    Increasing absorption to red
    Transmission 40 per cent at λ 4110

Transmission 6 per cent at λ 3950
Transmission 2 per cent at λ 3800
Transmission 0.2 per cent at λ 3590
Absorption increasing rapidly at λ 3590

2. K_2CrO_4, aqueous solution, $\frac{1}{2}\times$normal:
No absorption above λ 5100
Transmission 10 per cent at λ 4880
Transmission 1 per cent at λ 4750
Transmission less than 0.1 per cent at λ 4500
Transmission less than 0.02 per cent at λ 3500–4000
Transmission less than 1 per cent at λ 3000–3400

(NOTE.—All the solutions were contained in the ordinary glass U-shaped absorption cell, with parallel sides. Soret[1] gives for K_2CrO_4 a region of comparative transparency at λ 3000–3300. In this region the iron lines are relatively faint and 1 per cent is the best ratio that I have.)

3. $K_4Cr_2O_7$, aqueous solution, $\frac{1}{16}\times$normal:
No absorption above λ 5450
Transmission 10 per cent at λ 5120
Transmission less than 0.5 per cent at λ 4900
Transmission less than 0.05 per cent at λ 4400
Transmission less than 0.1 per cent at λ 3400–3900
Transmission less than 1 per cent at λ 3000–3400

The foregoing results were obtained with exposures of different lengths of the iron arc. I used the Pfund type, with one electrode carbon, an arrangement giving an arc of extremely constant intensity, with respect both to time and to the particular area used.

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[1] *Arch. sc. phys. et nat.* (2), **61**, 322, 1878.

REVIEWS

X-Rays. An Introduction to the Study of Röntgen Rays. By C. W. C. KAYE. London and New York: Longmans, Green, & Co., 1914. Pp. xix+251. $1.25.

This is a most delightful and unpretentious little book which contains much more sound learning than the modesty of the author permits him to claim for it. Indeed I know of no other book which can take its place either for the student of pure physics or for the practical X-ray worker. There are other books on X-rays, but no others which condense into so small a compass so comprehensive and intelligible a summary of what is now known of the subject. Approximately the first half of the book is concerned with the simple statement of the most important of the facts having to do with the technique of X-ray production. Then come the chapters which are all-important for the theorist. The most notable of these are chap. ix, on "Secondary Rays," pp. 108–144, and chap. xii, pp. 168–205, on "Interference and Reflection of X-Rays." These chapters contain an illuminating and able account of the three great discoveries of the past decade: (1) the discovery by Barkla of characteristic X-rays, (2) the discovery by Laue of the crystal grating, and (3) the discovery by Moseley of the relations between the wave-lengths of the characteristic X-rays of the different elements. Although the subject treated is growing rapidly, Kaye's little book will not quickly be outgrown.

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An Introduction to the Study of Variable Stars. By CAROLINE E. FURNESS. Boston: Houghton Mifflin Co., 1915. 12mo, pp. xv+327, with 14 plates and 36 figures in the text. $1.75 net (weight 30 oz.).

The phenomena of stellar variation are taking on new astrophysical meanings from recent investigations and discussions, and offer unexcelled opportunities for exploring cosmic mysteries. The data for these discussions have been furnished by amateur observers in greater proportion than in any other important branch of astronomy, yet there has been no comprehensive book to instruct the observer and warn him against pitfalls. The present work, which is one of the volumes issued in commemoration of the semi-centennial of Vassar College, meets this need very completely. The author's experience as a teacher and as director of the Vassar College Observatory is shown in the quality of the book. In addition to the historical and practical treatment, descriptions of underlying principles and also of the latest methods and instruments which have been used in astrophysical investigations of the phenomena of stellar variation are given in considerable detail.

The first chapter is introductory, and gives a bird's-eye view of the whole subject. The following fourteen chapters go into details and fall naturally into four divisions: (1) the observer's equipment (chaps. ii–v), maps, charts, catalogues, meaning of the term magnitude (this corresponds to Part I in Hagen's work); (2) photometry of variables (chaps. vi–viii), visual, photographic, photo-electric; (3) use of the observations (chaps. ix–xi), light-scale, light-curves, elements of variation, and predictions from elements; (4) deductions from the data in the preceding chapters (chaps. xii–xv), eclipsing binary variables, long-period variables, statistics of stellar variation, hints to observers, tables.

The professional as well as the amateur observer will read with profit the practical hints given in chap. xv on the use of the telescope, time, identification of variables, methods of recording, and precautions. The author shows herself to be possessed of the spirit of both the teacher and investigator, in thus combining the description of underlying principles with that of the latest extensions of work on variable stars.

From the standpoint of the astrophysicist a few points would seem open to modification; for example, the failure in chap. i to give credit to Mrs. Fleming for the Harvard classification of spectra. The Zöllner photometer is described on p. 118 as provided with the historical petroleum lamp instead of the incandescent electric lamp which is now used. On p. 141 halation circles on the photographs are spoken of as unavoidable. On p. 142, in the description of Fig. 21, "Color-Curve of the 40-Inch Objective," the larger ordinates are made to lie in the wrong direction. On p. 221, in the description of the correcting lens, the statement is made that "such a lens is always necessary when the telescope which is intended for visual work is used for photographic purposes," while in fact such a correcting lens is useful only with the spectroscope. No mention is made of the important fact that most classes of variables show a change in color along with the change in light, in the direction of increasing color-intensity with decreasing light. This is perhaps the only important factor in the modern theory of stellar variation that has been omitted by the author.

The press work, binding, and illustrations are very good; but the plates of stellar spectra, Nos. II, X, XI, XIII, and XIV, show no wave-lengths or anything to indicate the range included; also Plate XI is printed with the violet end to the right instead of in the usual direction. Plate II, typical spectra, is printed as a negative, while the rest show as positives. It would seem better that all should be positives.

J. A. P.

Collected List of Lunar Formations. Compiled by Mary A. Blagg under the direction of the late S. A. Saunder. Published by the Lunar Nomenclature Committee of the International Association of Academies. Edinburgh, 1913. 8vo. Pp. 182.

A recent example of successful international co-operation in scientific work is furnished by the Lunar Nomenclature Committee of the International Association of Academies. The Committee, which originally consisted of Loewy, Franz, Newcomb, Saunder, Weiss, and Turner, suffered greatly by the death of members especially interested in the moon, but the plans laid down are largely completed.

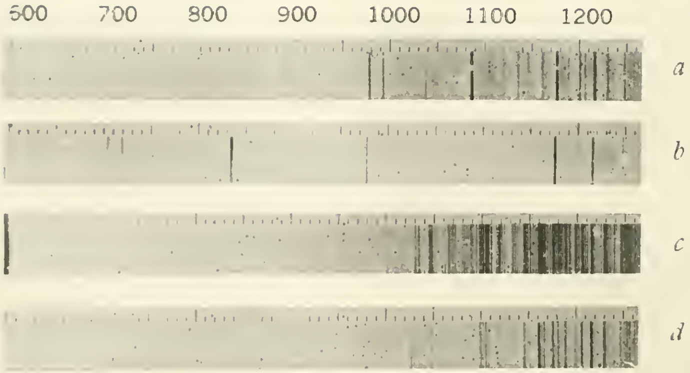
Measures for an accurate map of the moon were made by Franz at Breslau and Saunder in England. The drawing of the central portions has been nearly completed in England by W. H. Wesley, and it is expected that the outer portions will yet be drawn in Breslau.

A collection of the names of lunar formations was planned by Saunder and has been carefully completed by Miss Blagg. The funds for printing the list were provided by the Paris Academy of Sciences. The work is a long step in the problem of lunar nomenclature and will be the basis from which a uniform system may be devised.

The list gives in parallel columns the names given by Neison, Schmidt, and Beer and Mädler, with references, together with the descriptive position and nature of the object. At the end of the list are many pages of useful notes concerning the individual formations. There are 4,789 numbers in the list, which includes practically every lunar feature. The list will prove most useful to observers of the moon, for it allows the different authorities to be compared with certainty and without delay.

A. H. Joy

PLATE III.



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THE EXTENSION OF THE SPECTRUM BEYOND THE SCHUMANN REGION

By THEODORE LYMAN

I have recently published a brief statement of the result of my investigations on the extension of the spectrum.[1] Since the appearance of that article my researches in this field have been directed toward clearing up certain doubtful points. Although these experiments have not resulted in pushing the spectrum beyond the limit already announced, and although the investigation is by no means at an end, it now seems time to make a fuller statement than was possible in a preliminary notice. Such a statement is all the more desirable since an understanding of the difficulties which have been encountered is necessary if one is to estimate the probability of future progress. The first pages of this article, therefore, will be found to contain substantially the same material as the earlier notice, while the later pages are devoted to a rather detailed description of the experimental procedure.

The violet limit of the spectrum determined by direct eye-observation lies in the neighborhood of λ 4000; with a glass prism and lenses, the spectrum may be followed photographically to λ 3000 or thereabout; with a quartz system or with a reflecting grating, the limit may be pushed to λ 1850. Victor Schumann showed that the absorption of the air and of the gelatine of the photographic plate were responsible for the abrupt termination of the ultra-violet spectrum near λ 1850. By employing a vacuum spectroscope and a special photographic plate, the emulsion of which was very poor in gelatine, he was able to push his observations to λ 1230. At this point he was stopped by the opacity of the fluorite of which his lenses were made. I have employed a concave diffraction grating arranged in such a manner that the light-path from the source to the photographic plate is wholly in gas. Briefly, the containing apparatus may be described as a brass tube about 11 cm in diameter and rather over a meter long.[1] This tube is closed at each end by brass plates ground to fit suitable flanges. In my most recent experiments the light is generated electrically in a discharge tube of quartz provided with tungsten electrodes. This discharge tube fits air-tight on one of the two brass plates just mentioned; light from it, having passed through a slit, traverses the length of the apparatus and falls upon the diffraction grating by which, having been analyzed into its component colors, it is brought to focus on a special photographic plate placed in close proximity to the slit.

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[1] *Proceedings of the National Academy of Sciences*, **1**, 368, 1915; *Nature*, **95**, 343, 1915.

As the discharge tube is in no way separated from the body of the spectroscope, it is obviously necessary to choose for the experiment some gas which will not only yield radiations in the region under investigation but which will be transparent to these radiations. My earlier experiments were conducted with hydrogen, since it had shown the necessary characteristics in that part of the spectrum investigated by Schumann. With this gas at a pressure of 2 or 3 mm and by employing a strong disruptive discharge, in May 1914, I was able to extend the spectrum[2] to λ 900.

A rather tedious investigation having convinced me that nothing more was to be expected from the use of hydrogen, at least in this neighborhood, I turned my attention to helium, being guided by some of my earlier experiments which had proved that this gas possesses the necessary transparency.[3] At the same time I made some improvements in my apparatus which, though they left its general form unchanged, resulted in making it far more air-tight than ever before. The success of this improvement may be judged by the fact that I have been able recently to leave the spectroscope for over fourteen hours at a pressure of about 3 mm without being able to detect any leak, either by a McLeod gauge reading to 0.007 mm or by the appearance of impurities in the spectrum of the gaseous content. I also took particular pains to purify the helium which I employed.

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[1] Lyman, *Spectroscopy of the Extreme Ultra-Violet* (Longmans, Green, & Co.), p. 34.

[2] *Ibid.*, p. 78; *Nature*, **93**, 241, 1914.

[3] *Ibid.*, p. 69.

I have been rewarded for my trouble by a very considerable extension of the spectrum, for with helium free from nitrogen, at a pressure of 2 or 3 mm, by the use of the disruptive discharge, and with an exposure of about ten minutes, I have repeatedly observed a number of new lines the most refrangible of which has a wavelength of 600 A. All this has been accomplished with a grating ruled on speculum and with photographic plates prepared exactly as recommended by Schumann. The line λ 600 represents an extension beyond Schumann's limit greater than that which Schumann himself achieved beyond the region of λ 1850.

The relations of the spectra of hydrogen and helium have recently come into prominence through the theoretical researches of Bohr, Nicholson, and others. It so happens that the region on the more refrangible side of λ 1250 offers an important ground for the study of these relations. In order that the conclusions may be of value, however, it is necessary that the gases under observation should be free from impurities. In the best form of closed discharge tubes the difficulties, arising from the contamination of the gas by the material of the tube or by the electrodes, are very considerable, especially when a disruptive discharge is used. In my apparatus, where the brass spectroscope and quartz discharge tube communicate directly, these difficulties are considerably increased. I trust in time to overcome them, but for the present I must confine myself to the following statements.

Prominent in the spectrum of hydrogen is the line at λ 1216 which forms the first member of a series predicted, on theoretical grounds, by Ritz.[1] I have also found the two next members near λ 1026 and λ 972. With pure hydrogen, λ 972 is not visible and λ 1026 is best seen when a disruptive discharge is used, but singularly enough, with helium containing a trace of hydrogen, both lines occur quite strongly with a simple alternating current from a 60-cycle transformer. This may be an illustration of the curious behavior of helium, for an atmosphere of this gas seems to facilitate the production of the spectra of other gaseous substances which occur in it as impurities.

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[1] W. Ritz, *Gesammelte Werke*, p. 105.

In addition to these lines predicted by Ritz there are a number of others of equal prominence which always appear with a disruptive discharge in hydrogen. Two of these are of special interest, for their frequencies may be calculated rather exactly from the formula:

$$v = N \left\{ 1 - \frac{1}{(n+0.5)^2} \right\} ;$$

they persistently occur at λ 1086 and λ 992. Though the matter is perhaps not entirely settled, all the evidence seems to indicate that the two lines in question are due to an impurity.

The radiations from atmospheric nitrogen and from argon have also been examined. With both substances the disruptive discharge is necessary to produce the best results. The spectrum of the former gas with this type of excitation possesses some strong lines between λ 1050 and λ 1250; the spectrum terminates, however, near λ 975. With a simple 60-cycle alternating current, argon shows a spectrum terminating near λ 915. The extent of its spectrum is thus superior to that of hydrogen or of helium when subjected to similar electric conditions. With a disruptive discharge argon gives a strong spectrum of many lines terminating abruptly near λ 800.

As far as I am able to observe, with one exception all the lines seen with helium when excited by a non-disruptive discharge are also found in hydrogen; a disruptive discharge is necessary for the production of the new lines of extremely short wave-length. The intensity of these lines seems to depend in a critical way on the electrical conditions in the exciting circuit, for it is only when the pressure in the discharge tube, the external spark-gap, and the capacity are all nicely adjusted that the extreme lines appear to the best advantage. The nature of the adjustments suggests that the radiations of the shortest wave-lengths require for their production that the atoms or molecules within the gas possess a very high velocity at the moment of collision. Some of the points in the preceding paragraphs are illustrated in Plate III.

Spectrum *a* is the spectrum obtained from hydrogen when a spark-gap of about 3 mm is placed in series with the tube and when a capacity of 0.014 microfarad is shunted across the terminals of the transformer. Pressure about 1 mm. Time of exposure about 12 minutes.

Spectrum *b* is the spectrum of helium containing hydrogen under the same electrical conditions as for spectrum *a*. Pressure 1.7 mm. Time of exposure 8 minutes.

Spectrum *c* is the spectrum of hydrogen excited by a 60-cycle alternating current, there is no capacity in the system beyond that of the leads; current about 15 milliamperes. Pressure 2–3 mm. Time of exposure one hour.

Spectrum *d* is the spectrum of helium under the same electrical conditions as for spectrum *c*. Pressure 2.2 mm. Time of exposure 30 minutes.

The extreme lines in the helium spectrum are probably not visible in the reproduction. The scales are intended only to give the position of the lines approximately. I crave the reader's indulgence for the blemishes which disfigure the spectra; they are almost unavoidable under the conditions of the experiment.

The wave-length of the X-rays, as determined by the Braggs, is of the order of one angstrom. There is therefore a gap of some six hundred units between the region of the ordinary Roentgen rays and the limit which I have reached. Several attempts have been made recently to produce less refrangible X-rays, but there is no satisfactory way of estimating their wave-length unless perhaps one relies on the Planck-Einstein formula $Ve=hv$. Taking V as the potential drop which gives the cathode particle the velocity necessary to produce a very soft X-ray, e as the charge on the particle, and giving to h the usual value ascribed to Planck's constant, the experiments of Dember[1] were made to yield a wave-length for the X-ray longer than that which I have determined.

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[1] Lyman, *Spectroscopy*, p. 108.

Such speculations, though they are very interesting, cannot be given any great weight. There is still a considerable region between λ 600 and wave-length 1 angstrom which must be experimentally explored. My present limit is probably due rather to difficulties of technique than to any fundamental fault in method. Now that we know that a Schumann plate can be used and that the speculum grating is efficient, I see no insuperable difficulty to a still further extension by purely spectroscopic means.

The following pages contain a detailed description of my experimental procedure. A table of the wave-lengths of the new lines is given near the end of the paper.

The vacuum grating spectroscope with which this research has been carried out has been already fully described.[1] The changes on which the success of the present research largely rest are simple in character but of fundamental importance. Their object was to improve the air-tightness of the containing vessel and to guard the purity of the gas content. To this end, the plate which closed the working end of the container was of the simpler form used in my earlier work. It was of brass 1.3 cm thick ground to fit the flange which it was destined to cover; of its two windows, one was permanently provided with a quartz disk through which comparison spectra were obtained, the other was fitted with a cup into which a brass cone, destined to carry the discharge tube, was ground. The plate was attached to the flange of the spectroscope in the following way: A string of pure beeswax, some 2 mm in diameter, was bent into a circle of the required size; the flange of the receiver was then slightly warmed and the wax was lightly pressed against it; a photographic plate having been put in place, the face plate, also slightly warmed, was pressed against the wax. On the receiver being exhausted, the pressure of the air forced the face plate against the flange and thus squeezed the wax into a thin gasket. This device was suggested to me by Professor R. J. Strutt; by its means it has been possible to reduce the leak in the apparatus to a value far smaller than I have been able to attain by any other method. An example of its efficiency has already been given. As a high degree of air-tightness is fundamental to the success of the whole research, the importance of the wax gasket is evident. It is obvious that the nature of the arrangement just described necessitates the removal of the face plate whenever the plate-holder is to be withdrawn from the apparatus. At present there seems no escape from this rather inconvenient procedure.

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[1] Lyman, *op. cit.*, p. 34.

In order to secure radiations of the shortest wave-length, it is necessary to employ a disruptive discharge; under these electrical conditions the gas-filling is always more or less contaminated by material torn from the discharge tube. To reduce this cause of error, quartz has been employed in place of glass. A change of form as well as of material has also been made, for the internal capillary type, so well suited to some previous investigations,[1] cannot be employed when a violent disruptive discharge is used, because the propinquity of the end of the capillary to the inside of the spectroscope encourages a spreading of the discharge with a consequent fatal fogging of the photographic plate. The form of discharge tube finally arrived at was of the simple "end-on" type with a capillary 4.5 cm long and about 3 mm in diameter. The tungsten electrodes, provided by the kindness of the General Electric Company, were in the form of cylinders 1.5 cm long and about 3 mm in diameter. They were bound with fine tungsten wire to stouter wires of the same material which, in turn, were clamped in brass rods. These rods, covered with quartz sleeves, were contained in legs of the discharge tube and were sealed into these legs with Khotinski cement, the seals being kept hard by baths of mercury. The object of this arrangement was to restrict the electric discharge to the tungsten alone in the hope that thereby the contamination of the gas might be reduced to a minimum. The whole discharge tube thus constituted was sealed into a brass cone which, in turn, fitted air-tight into the cup on the face plate.

It is well known that tungsten if properly treated can be freed from occluded gases; this material, therefore, if employed for electrodes in a discharge tube, affords an escape from the contamination of the gas by the hydrogen which is so freely given off by most metals. In practice, I have found tungsten far more satisfactory than any other material which I have employed, always provided that it has been freshly treated.

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[1] Lyman, *op. cit.*, p. 41.

Atmospheric nitrogen, argon, hydrogen, and helium have been used in this investigation. I am indebted to my colleague, Professor Baxter, for the first-named gas. It was free from impurities save for those rare gases whose presence its atmospheric origin rendered inevitable.

I am indebted to the General Electric Company for my sample of argon. They informed me that the gas contained less than 1 per cent of impurity. As judged by spectroscopic observations in the visible spectrum, some hydrogen and a trace of nitrogen were present.

The hydrogen employed was prepared by electrolysis from amalgamated zinc and hydrochloric acid in an automatic generator of special type.[1] This method was recommended to me as one of the best available. After passing through towers of fused potassium hydroxide, the gas was collected over water. It was found necessary to prepare the hydrogen immediately before it was required for use, for, by standing over water, it is sure to become contaminated with air. Before being introduced into the phosphorous-pentoxide drying system, the gas was passed over platinized asbestos heated to redness in a quartz tube to insure the removal of the least trace of oxygen. A delicate test for the presence of a trace of air in the spectroscope is afforded by the appearance of the nitrogen lines near λ 5005 with disruptive excitation. Judged by the appearance of its visible spectrum the hydrogen seemed free from impurities.

The helium employed has been obtained from three sources. The first sample was prepared from clevite in the usual way and was purified over charcoal plunged in liquid air; the second was obtained from Professor Cady, of the University of Kansas; the third I owe to the generosity of Professor Boltwood. This last specimen I treated with pyrogallic acid and then passed over hot copper, then over hot calcium, and finally it was exposed to charcoal and liquid air. All the specimens were free from nitrogen and oxides of carbon, but contained a trace of hydrogen. Charcoal cannot be trusted to remove the last remnant of this gas, but as the helium was bound to be exposed to some admixture of hydrogen from the walls of the spectroscope and from the discharge tube, it seemed useless to go into the greatest refinement of purification at the present stage of the investigation. It must be noted that after the gas had been admitted to the spectroscope it sometimes showed a trace of oxides of carbon, but it appeared to be free from nitrogen.

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[1] Cook and Richards, *Proceedings of the American Academy*, 23, 149, 1887.

About 100 cc of helium are necessary for a single experiment; part of this gas is used in washing the spectroscope and part in the final observations.

In order to guard the spectroscope from mercury and to reduce the presence of hydrocarbons as far as possible, a U-tube with legs, each about 35 cm long, was plunged in liquid air and interposed between the spectroscope receiver and the rest of the system; this latter consisted of the gas reservoir, the McLeod gauge, and the Trimount oil pump.

When a disruptive discharge was required, the current was obtained from a $\frac{1}{2}$ kw Clapp-Eastham transformer run from a 60-cycle, 110-volt circuit. A capacity of 0.014 microfarad was placed across the terminals of the secondary, while a spark-gap of about 3 mm was used in series with the discharge tube. Under these circumstances, the equivalent spark-gap between needle-points at the discharge tube was 8.5 mm. When the non-disruptive discharge was employed, the current through the tube was of the order of 15 milliamperes.

With the idea of eliminating altogether the difficulties which arise from the introduction of electrodes into a radiating gas, some attempts have been made to use the electrodeless discharge. In this experiment a Chaffee gap[1] was connected in series with the primary of a Tesla coil, the arrangement thus formed being shunted by a condenser which, in turn, was placed across the leads of a 500-volt circuit, one of the lines of which contained a choking coil. The capillary discharge tube was provided with armatures of tinfoil at either end and these were connected to the secondary of the

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[1] E. L. Chaffee, *Proceedings of the American Academy*, 47, 270, 1911.

Tesla. Thanks to the regular behavior of the Chaffee gap, a very brilliant and steady illumination resulted.

The successful reconnoissance of Saunders[1] into the region of extremely short wave-lengths suggested the advisability of trying a calcium arc in quartz. The arrangement was very similar to that described by Saunders except that the lamp was operated from the 500-volt circuit. The chief technical difficulty arises from the rapid deterioration of the quartz. In these experiments magnesium as well as calcium electrodes were employed.

If of the many experimental difficulties of the subject there is one which seems at present more troublesome than the others, and to which the limit of the spectrum may be directly traced, it is a luminosity in the gas content of the spectroscope excited by the disruptive discharge. The phenomenon makes its appearance when the pressure is of the order of 1 mm and produces a background of fog on the photographic plate in which faint lines are lost. This luminescence is especially aggravating since the low pressure at which it appears seems particularly favorable to the production of the lines of the very shortest wave-length. As a result of this difficulty long exposures are out of the question. It seems probable that if a window could be interposed between the discharge tube and the spectroscope, the excited luminosity would vanish.

Unfortunately, material for such a window is not at hand. A test with a bit of fluorite 1 mm thick, of proved transparency in the Schumann region, showed that this substance was opaque from λ 1230 to the extreme end of the spectrum at λ 600. A similar test with quartz 0.5 mm thick demonstrated that this substance shows no signs of transparency in the extreme ultra-violet.

Lenard[2] has interpreted some of his experiments on the ionizing power of light from an aluminum spark as proving the existence of radiations from this source lying on the more refrangible side of the quartz absorption band and transmitted to some extent by air. I have made an attempt to find these radiations by placing an aluminum spark in air outside my spectroscope and about 2 mm distant from a quartz window 0.2 mm thick which closed the apparatus. The result of the experiment was entirely negative, the adjustment of the apparatus being such that the field covered by the photographic plate extended to λ 250.

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[1] *Astrophysical Journal*, 40, 377, 1914.

[2] Lyman, *op. cit.*, p. 107.

After this account of the apparatus and the methods employed, it seems important to discuss the various spectra in some detail, paying particular attention to the circumstances under which they were produced.

The electrodeless discharge is easily dismissed; no radiation on the more refrangible side of λ 800 has been obtained with it. The arc discharge in quartz, both when calcium electrodes were used and when magnesium was employed, showed no lines more refrangible than λ 1000 which could be certainly attributed to these metals. The spectra in both cases consisted mainly of secondary hydrogen lines; the primary lines at λ 1216 and λ 1026 were, however, quite strong. The spectrum, which is intense, terminates near λ 905.

The absence of metallic lines in this region is also confirmed by experiments with the spark-discharge at reduced pressures. Pointed terminals of both aluminum and iron, about 1 cm apart, were arranged in a small globular vessel communicating directly with the spectroscope. A spark-gap in air, in series with the terminals, insured the disruptive nature of the discharge. At a pressure of about 1 cm of hydrogen the spectra obtained with both metals were characteristic of the gas-filling. They contained no lines which could be ascribed with certainty to either aluminum or iron. The spectra terminated near λ 1030. The amount of gas present in the light-path was about equivalent to that in a column of hydrogen at atmospheric pressure 2.5 cm long. The experiment, therefore, is of some interest as giving an idea of the order of transparency of hydrogen for these short wavelengths.

The relation between the spectra of helium and of hydrogen forms a fascinating subject for speculation, and the spectral region now under consideration is an excellent field for the test of hypotheses. However, as has already been pointed out, the data at hand cannot be made to yield conclusive answers to the questions involved. The difficulty is inherent in the nature of the problem since the type of the apparatus which must be employed, if the region in question is to be studied at all, precludes the separation of the effects of absorption from those produced by radiation, and at the same time renders the elimination of traces of hydrogen from an atmosphere of helium extremely difficult. In considering the spectra obtained from the discharge tube, therefore, it must suffice for the present at least, to confine the attention to their general character.

On comparing the spectra obtained from hydrogen excited with and without capacity, it becomes evident that when the lines obtained in the latter case are subtracted from those produced with a disruptive discharge, some seventeen strong lines remain. Of these, λλ 1216, 1026, and 972 form the Balmer analogue predicted by Ritz; λ 1216 is one of the strongest lines in the spectrum; the second member, however, is so feeble as to be hardly visible in spectrum *a*, Plate III, but it is easily seen in spectrum *d*. This illustrates the curious fact, to which reference has been already made, namely, that the extreme lines of the Ritz series appear to be produced more strongly in helium with a non-disruptive discharge than in hydrogen when a condenser and spark-gap are employed. The line λ 972 is not visible in spectrum *a* but may be seen quite clearly on the original negative from which spectrum *d* was taken. In connection with Bohr's speculations it is important to observe that λ 1216, which forms the first member of the Ritz series, occupies exactly the same position when obtained from helium as when it is produced in hydrogen.

The striking pair of strong lines near λ 1086 and the wider pair near λ 992 have already been attributed to an impurity. They occur in both quartz and glass discharge tubes and their appearance is independent of the nature of the electrodes. They occur very strongly in nitrogen and may perhaps be attributed to this gas, though not with perfect certainty. They may possibly be produced by an oxide of carbon, a trace of which has been occasionally detected in the visible spectrum of helium and which probably takes its origin from the wax used to seal the spectroscope. An inspection of spectra *a* and *b* will show that these lines are less intense in helium than in hydrogen; they cannot, therefore, be ascribed to helium, though Bohr has hinted that they belong to this gas.[1] The line at λ 1176 is perhaps the strongest in the whole hydrogen spectrum; it is equally strong in helium and is very strong in nitrogen; of its origin nothing positive can be said. All the remaining lines to λ 977 are stronger in nitrogen than in either helium or hydrogen; all occur in argon. From λ 997 to λ 904 all the lines, with one exception, occur in hydrogen, helium, and argon, but with relative intensities depending on the gas in which they are produced. All the lines on the more refrangible side of λ 900 are obtained only when helium is employed, with the exception of λ 833 which occurs in argon. The strength of the pair near λ 835 is striking. An examination of the print from which Plate III was made showed in spectrum *b*, if a magnifying glass was used, the extreme line near λ 600 A; however, this line is probably lost in the reproduction.

Nitrogen yields a few strong lines with a disruptive discharge besides those which it appears to contribute to the hydrogen spectra; with a non-disruptive discharge it yields but two or three weak lines near λ 1200. As its spectrum does not extend beyond λ 977, I have been chiefly interested in the gas in its character of an impurity and have made no measurements upon its lines.

As has been mentioned already, argon gives a spectrum containing many lines terminating only near λ 800, but here, again, I have not delayed the progress of this research in order to make measurements. A careful study of the argon spectrum in the future, however, will probably well repay the trouble.

The wave-lengths which are to be found in Table I were obtained by the two-slit method which I have frequently employed.[2] They rest on the hydrogen line λ 1216.0 and upon the shifted spectra of iron and aluminum. They make no claim to extreme accuracy, but I hope that, when standard wave-lengths shall have been established in this region, the values given in the table will not be found to depart from the standards by more than one unit.

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[1] *Philosophical Magazine*, **30**, 401, 1915, note.

[2] Lyman, *op. cit.*, p. 45.

The numbers in the first column indicate the intensities of the lines as they occur in helium.

It must be remembered that λλ 1216, 1026, and 972 represent the only strong radiations on the less refrangible side of λ 900 which can be attributed to helium or to hydrogen with any degree of certainty. Even the extreme lines produced in helium alone may owe their appearance on the photographic plate to the superior transparency of the gas and may be produced by some subtle impurity.

TABLE I

Strong Lines in the Extreme Ultra-Violet

Intensity in Helium	λ	Intensity in Helium	λ
1	599.0	3	992.0
1	643.7	4	1010.6
2	702.4	4	1026.0
3	703.5	5	1037.0
5	718.2	2	1084.9
2	796.8	5	1086.1
8	833.4	1	1134.7
7	834.8	8	1175.5
4	904.6	10	1176.3
2	916.7	5	1199.8
1	972.7	10	1216.0
2	976.8	1	1236.0
6	977.6	5	1247.9
1	990.2		

The result of this investigation is easily stated: the spectrum has been extended to λ 600 A.

I cannot conclude this article without expressing my appreciation of the skill and patience which my assistant, Dr. Paul Sabine, has shown during the whole course of this research.

The Jefferson Laboratory
Cambridge
December 1915

THE STELLAR MAGNITUDES OF THE SUN, MOON, AND PLANETS

By HENRY NORRIS RUSSELL

An examination of the data concerning the brightness and albedo of the various members of the solar system, begun with the object of obtaining trustworthy values for textbook purposes, has shown the desirability of a general revision of the data, and discussion of the theory. The resulting conclusions regarding the brightness of the sun, moon, and planets are presented below. The data bearing on the albedo of the earth, and the theoretical discussion of the whole will form a later paper.

I. THE STELLAR MAGNITUDE OF THE SUN

Several determinations of this fundamental constant have been made in the last few years, and its value is now known with considerable accuracy.

a) Of the older determinations,[1] the only one which now deserves serious consideration is that by Zöllner,[2] who used a photometer of the type still called by his name, and compared the starlike image of the sun formed by a suitable combination of lenses and shade-glasses, and the direct image of Capella formed by a larger objective, with the same "artificial star." The corrections for the loss of light in the optical systems and shade-glasses were very carefully determined. The sun was observed on 6 days, and the star on 11 nights, with remarkably accordant results, the average deviation of a single day's or night's result from the general mean being only $0^{\mathrm{M}}.05$, which is all the more notable because the light of the artificial star was assumed to be absolutely constant throughout the interval of four months covered by the observations, and any variations in it would tend to increase the deviations. He concludes that the difference of magnitude between the sun and Capella is

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[1] See Müller, *Die Photometrie der Gestirne* (Leipzig, 1897), p. 316.

[2] *Photometrische Untersuchungen* (Leipzig, 1865), pp. 107, 124–25.

26.87 ± 0.05, the uncertainties of the instrumental constants being included in the assigned probable error. With the Harvard magnitude of Capella, 0.21, the magnitude of the sun would be −26.66.

b) Next in order of time is the determination by C. Fabry,[1] who compared the light of a standard electric lamp, shining through a blue screen, with that of the sun weakened by divergence through a lens of short focal length by means of a Lummer-Brodhun photometer, and later compared the light of the same standard, reduced to a stellar image by means of a microscope objective and reflected from a glass plate at an angle of 45°, with Vega, directly seen by the naked eye. Under clear skies and for bodies near the zenith the results obtained on different days or nights differed only by *quelques centièmes.* Fabry also determined the candle-power of his lamp and the percentage of its light transmitted by the blue solution, and gives as his results that the sun in the zenith and at mean distance gives light equal to 100,000 *bougies décimales* at 1 m, while Vega in the zenith is equal to a candle at 780 m. This makes the sun $26^{\mathrm{M}}.94$ brighter than Vega, or of magnitude −26.80 on the Harvard scale.

c) Ceraski,[2] observing at Moscow in 1903, compared with a Zöllner photometer the image of the sun, reflected from the surface of a small lens at a distance of 152 m, with Venus, which was near elongation and visible in full daylight. Later auxiliary comparisons were made between Venus and various bright stars. The constants of reduction were very carefully determined. Observations on seven days made the sun $22^{\mathrm{M}}.93$ brighter than Venus 112 days after inferior conjunction, with an average deviation of $0^{\mathrm{M}}.10$. The supplementary observations, made more than a year later, showed that Venus, 42 days before inferior conjunction (when, according to Müller, it was $0^{\mathrm{M}}.66$ brighter than on the former occasion), was $4^{\mathrm{M}}.86$ brighter than Procyon, $6^{\mathrm{M}}.30$ brighter than Polaris, and $3^{\mathrm{M}}.22$ brighter than Sirius. With the Harvard magnitudes of these stars, and weights 4, 4, 1, proportional to the number of nights of observation, the sun's magnitude comes out −26.60 on the Harvard scale.

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[1] *Comptes Rendus,* **137,** 973 and 1242, 1903. The more detailed account (*Assoc. française pour l'Avancement de Science, Congrès d'Angers,* **1903,** p. 255) is inaccessible to the writer.

[2] *Annales de l'Observatoire de Moscou,* 2d series, **5,** 1–30, 1911.

d) W. H. Pickering,[1] observing in Jamaica in 1901, with a 12-inch telescope of 135 feet focal length, compared, by means of a shadow photometer, the brightness of the center of the focal image of the sun, shining through a small aperture, about 1 mm in diameter, in front of the objective, and of the extra-focal images of stars, 2 or 3 mm in diameter. The comparison light was a pentane lamp shining through a cell filled with a blue solution. Ten independent determinations, on as many days and nights, using Capella, Arcturus, Vega, and Sirius as reference stars, gave the sun's magnitude as −26.83, with an average deviation of $0^{\mathrm{M}}.19$ for a single determination.

These four determinations, made in altogether different ways, are in remarkably good agreement, especially when it is considered what enormous instrumental constants, representing the effects of the various methods of weakening the sun's light, have to be determined in every case. The results on the Harvard scale are:

Zöllner	$-26^{\mathrm{M}}.66$
Fabry	−26.80
Pickering	−26.83
Ceraski	−26.60
Simple mean	−26.72±0.04

The probable error of one determination, assuming them to be of equal weight, comes out $\pm 0^{\mathrm{M}}.075$, showing that the systematic errors of the various methods of observation must be very small, for it is highly improbable that four such radically different processes should all be affected with errors of the same sign and magnitude.

Two determinations of the sun's photographic magnitude have also been made, each based on a considerable amount of careful work.

King,[2] comparing the sun with Polaris, Arcturus, and Capella by his well-known extra-focal method, finds a mean value of $-25^{\mathrm{M}}.83$, with an average deviation of only $0^{\mathrm{M}}.07$ for his 11 determinations. The probable error of the mean is presumably somewhat larger than this agreement would indicate, on account of the uncertainty of the measurements of the absorption of the screens which were employed.

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[1] *Harvard Annals*, **61**, 56–71.

[2] *Ibid.*, **59**, 248.

Birck,[1] working extra-focally by a method essentially similar to that of Pickering's visual observations, finds differences in photographic magnitude between the sun and stars which, with King's photographic magnitudes for the latter, give the following showing:

Star	Difference of Magnitude	Color-Index		Photographic Magnitude	Photographic Magnitude of Sun	
	Birck	Birck	King	King	Uncorrected	Corrected
Vega	25.95	0.00	0.00	0.14	−25.81	−26.15
Capella	27.04	1.02	0.82	1.03	−26.01	−26.01
Arcturus	27.76	1.71	1.09	1.33	−26.43	−26.20

The discordance between the results found from the three stars arises from a difference in color-equation between King's and Birck's methods of observation. The photographic difference in magnitude between Vega and the other stars can be deduced directly from Birck's observations, and comparison with the Harvard visual magnitudes gives the color-indices in the third column, which indicate a variation with spectral class about 50 per cent greater than for King. If Birck's results are altered by one-third of the differences of his color-indices from that of Capella, and so approximately reduced to the Harvard photographic scale and the solar type of spectrum, the figures given in the last column are obtained. These are in good agreement and give a mean of $-26^{\mathrm{M}}.12$. On account of the uncertainty of this correction, King's determination may be given double weight in taking the mean, and the value $-25^{\mathrm{M}}.93$ may be adopted for the sun's photographic magnitude on the Harvard scale.

The color-index of the sun, resulting from the visual and photographic magnitudes here adopted, is $+0^{\mathrm{M}}.79$, which differs from

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[1] Inaugural dissertation (Göttingen, 1909).

King's mean value[1] for stars of Class G, $+0^{\mathrm{M}}.72$, by less than its probable error.

II. THE MAGNITUDES OF THE PLANETS

By far the most extensive and homogeneous series of observations of the brightness of the planets are those of Müller,[2] which will be adopted as the main basis of the present discussion. The scale of magnitudes to which these observations are referred is defined by Table IV, p. 235, of his memoir, and is intermediate between those of the *Revised Harvard Photometry* and the *Potsdam Durchmusterung*. Müller's magnitudes for his 14 fundamental stars, which range in brightness from Sirius to Polaris, are all brighter by $0^{\mathrm{M}}.19$ than those of the final Potsdam catalogue, and the differences for the fainter stars, though more irregular, average about the same. The mean values of the difference Müller minus Harvard, grouped according to spectral class and magnitude, are given in Table I. The relative color-equation of the two systems is small, being approximately $0^{\mathrm{M}}.035$ for each spectral class, Müller making the red stars the brighter. If the observed differences are reduced to spectrum G on this assumption, and grouped according to magnitude, the means given in the lower part of Table I are obtained. There may be a difference in scale between the two

TABLE I

	SPECTRAL CLASS						
	B	A	F	G	K	K2 to M	All
Brighter than $4^{\mathrm{M}}.0$							
Number	3	6	3	3	3	2	20
Mean M−H	$+0^{\mathrm{M}}.08$	$+0^{\mathrm{M}}.18$	$+0^{\mathrm{M}}.07$	$-0^{\mathrm{M}}.02$	$+0^{\mathrm{M}}.02$	$-0^{\mathrm{M}}.13$	$+0^{\mathrm{M}}.06$
Fainter than $4^{\mathrm{M}}.0$							
Number	3	5	3	2	8	5	26
Mean M−H	$+0^{\mathrm{M}}.04$	$+0^{\mathrm{M}}.01$	$-0^{\mathrm{M}}.08$	$-0^{\mathrm{M}}.15$	$-0^{\mathrm{M}}.10$	$-0^{\mathrm{M}}.06$	$-0^{\mathrm{M}}.06$
Magnitude	$0^{\mathrm{M}}.3$	$1^{\mathrm{M}}.9$	$3^{\mathrm{M}}.7$	$5^{\mathrm{M}}.0$	$5^{\mathrm{M}}.5$	$6^{\mathrm{M}}.2$	$7^{\mathrm{M}}.2$
Number	8	8	8	9	8	8	9
Mean M−H	$+0^{\mathrm{M}}.05$	$+0^{\mathrm{M}}.07$	$-0^{\mathrm{M}}.05$	$-0^{\mathrm{M}}.02$	$-0^{\mathrm{M}}.08$	$-0^{\mathrm{M}}.10$	$-0^{\mathrm{M}}.01$

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[1] *Harvard Annals*, **59**, 179.

[2] *Potsdam Publications*, **8**, Part IV, pp. 197–371, 1893, with some additions in *Die Photometrie der Gestirne*, 1897.

systems, but it is very small. In what follows it will be assumed that the brighter planets are measured $0^{\mathrm{M}}.06$ fainter by Müller than on the Harvard system, and that for Uranus, Neptune, and the asteroids no correction is necessary.

For those planets which show conspicuous phases, Müller's formulae for the stellar magnitude corresponding to the phase-angle, and reduced to mean distance from the sun and unit distance from the earth, in the case of Mercury and Venus, and to mean opposition for Mars, are:

$$\text{Mercury}\ldots\begin{cases}\text{I.} & \text{Mag.}=-1.04+0.0368(\alpha-50^\circ)\\ \text{II.} & \text{Mag.}=-0.90+0.0284(\alpha-50^\circ)+0.000102(\alpha-50^\circ)^2\end{cases}$$

$$\text{Venus}\ldots\ldots\ldots\ldots\text{Mag.}=-4.71+0.01322\alpha+0.000000425\alpha^3$$

$$\text{Mars}\ldots\ldots\ldots\ldots\text{Mag.}=-1.79+0.0149\alpha$$

Mercury can be observed only between phase-angles 50° and 120°, and within this interval the two formulae represent the observations almost equally well, the probable error of the unit of weight being $\pm 0^{\mathrm{M}}.137$ with I, and $\pm 0^{\mathrm{M}}.135$ with II. An observation by Jost[1] during the total solar eclipse of May 28, 1900, at phase-angle $6^\circ.8$, gave the value -2.81 for the magnitude of the planet, reduced to standard distance. Formulae I and II give -2.63 and -1.94, respectively. Since, however, formula I leads to the rather improbable conclusion that Mercury is 21 times brighter at the full than at the half phase, both formulae will be retained. As Müller points out, formula II indicates a phase-variation almost exactly like that of the moon.

The formula for Venus represents the observations very satisfactorily over the wide range from 24° to 156°, and that for Mars is equally good from 0° to 47°, the maximum phase observable. Within the range of phase in which both these planets have been observed, their variation is almost identical. Jupiter can be observed only up to about $\alpha=12^\circ$. Within this region its brightness varies far less than that of any of the inner planets would. Müller's mean results for groups of about 30 observations are as follows:

Phase-angle	$1^\circ.0$	$3^\circ.1$	$5^\circ.5$	$7^\circ.5$	$9^\circ.0$	$10^\circ.3$	$11^\circ.1$
Magnitude	-2.27	-2.23	-2.23	-2.24	-2.20	-2.26	-2.20

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[1] *Heidelberg Mittheilungen*, No. 1 (1901).

The whole variation within these limits can hardly much exceed $0\overset{\mathrm{M}}{.}05$, whereas if the planet behaved like Mars it would be $0\overset{\mathrm{M}}{.}15$, and if like the moon, $0\overset{\mathrm{M}}{.}22$. Lambert's law gives a variation of $0\overset{\mathrm{M}}{.}02$ and the Lommel-Seeliger law $0\overset{\mathrm{M}}{.}03$.

Saturn, when the ring is invisible, likewise shows no sensible variation with phase, according to Baldwin's observations.[1] The phase-variations which appear when the ring is visible are explicable by Seeliger's theory, which takes account of the meteoric constitution of the rings. Müller, applying this theory to his 252 observations,[2] finds for the magnitude at mean opposition with ring invisible the value +0.95, in exact agreement with the results of Baldwin's direct observations. The correction for the light of the rings is approximately $0\overset{\mathrm{M}}{.}04$ for every degree of elevation of the earth above their plane.[3]

For the magnitude of Uranus at mean opposition, Müller finds the value 5.86 from 93 observations in 8 different years, while Pickering,[4] from 31 nights' measures in 5 years, obtains 5.51, a surprising discordance. The value 5.74 will be adopted here, giving Müller's observations double weight on account of their greater number. For Neptune, Müller finds 7.66 as the mean of 138 observations, Pickering[5] 7.65 from 66 observations, and Baldwin[6] 7.99 on the Potsdam scale, or 7.64 on the Harvard scale, from 32 observations—all in remarkable agreement.

For the four brightest asteroids, whose diameters have been determined by Barnard, the magnitudes determined by Pickering[7] from the results of several observers have been adopted. The rates of change in magnitude per degree of phase-angle are from Müller's summary,[8] taking again the mean result of all observers. They are 0.043 for Ceres, 0.038 for Pallas, 0.030 for Juno, and 0.022 for Vesta. The mean value of this constant for 34 asteroids is found by Müller to be $0\overset{\mathrm{M}}{.}030$ per degree.

[1] *Monthly Notices*, **69**, 458, 1909.

[2] *Op. cit.*, p. 348.

[3] *Photometrie der Gestirne*, p. 347.

[4] *Harvard Annals*, **46**, 204.

[5] *Ibid.*, **24**, 265; **46**, 203.

[6] *Monthly Notices*, **68**, 620, 1908.

[7] *Harvard Circular*, No. 169, 1911.

[8] *Photometrie der Gestirne*, p. 378.

Guthnick's magnitudes[1] for the four Galilean satellites of Jupiter have been adopted, and for Titan the mean of the results of Guthnick and Wendell,[2] which differ by only $0^{\mathrm{M}}.03$. Nothing at all is known about the change of brightness with phase for any of these satellites.

The adopted values of the stellar magnitude at mean opposition, on the Harvard scale, for the bodies more remote than Mars, are therefore:

Jupiter	−2.29	Ceres	+7.15	Jupiter I	+5.54
Saturn	+0.89	Pallas	+7.84	Jupiter II	+5.69
Uranus	+5.74	Juno	+8.95	Jupiter III	+5.08
Neptune	+7.65	Vesta	+6.04	Jupiter IV	+6.26
				Titan	+8.30

For the photographic magnitudes of the planets, King's extra-focal observations[3] appear to be the only published material. King has derived empirical formulae from his own observations to represent the variation with phase in the case of Venus, Mars, and Saturn, and derived color-indices by comparing these magnitudes, extrapolated to zero phase, with Müller's visual magnitudes, similarly extrapolated, applying to the latter a correction of $-0^{\mathrm{M}}.12$ to reduce from the Potsdam to the Harvard scale. It appears to the writer that it would be preferable to compare directly the visual and photographic observations at the phases at which they were actually made. As Müller's observations are much the more numerous, and his empirical formulae represent their course very closely, these were adopted as a standard of reduction, and the differences between King's photographic magnitudes and these formulae were grouped according to the phase-angle, giving each night's observations equal weight, with the results shown in Table II. The magnitudes of Saturn computed by Müller according to Seeliger's theory were employed. There is no appearance of change in the color-index with phase, except in the case of the first group of observations on Venus, which are very discordant

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[1] *Astronomische Nachrichten*, **198**, 251, 1914.

[2] *Harvard Annals*, **69**, 223, 1914.

[3] *Ibid.*, **59**, 261–64.

inter se. Further observations here are evidently desirable; in their absence the simple mean of all the observations is taken.

TABLE II

VENUS				MARS			
Phase-Angle	Mean K−M	Obs.	Average Deviation	Phase-Angle	Mean K−M	Obs.	Average Deviation
64°.6........	+1$^{\mathrm{M}}$.10	8	0$^{\mathrm{M}}$.35	4°.9.........	+1$^{\mathrm{M}}$.30	8	0$^{\mathrm{M}}$.21
75.4........	+0.67	7	0.33	15.6.........	+1.34	10	0.18
84.9........	+0.46	5	0.29	25.0.........	+1.33	10	0.12
116.6........	+0.52	7	0.14	32.7.........	+1.31	8	0.26
JUPITER				SATURN			
2.6.........	+0.49	9	0.12	0.6.........	+1.07	6	0.06
8.6.........	+0.38	10	0.12	2.7.........	+1.05	6	0.06

After correction by 0$^{\mathrm{M}}$.06 to reduce to the Harvard visual scale (as found above), the color-indices of the four planets become: Venus, +0.78; Mars, +1.38; Jupiter, +0.50; Saturn, +1.12.

III. THE VARIATIONS OF THE MOON'S BRIGHTNESS WITH PHASE

Seven series of observations of the relative brightness of the moon at different phases demand consideration. Though the light-curves derived by the various observers differ very considerably, their actual observations can all be satisfactorily represented by the same mean curve, as is shown below.

The earliest reliable observations are those of Sir John Herschel,[1] which consist of direct comparisons of many stars with the reduced image of the moon given by his "astrometer." Since they were inconsistent with Euler's "law," which Herschel supposed to be true, he concluded that they were affected by systematic errors, and their true value was first pointed out by Bond and Zöllner. Zöllner's reduction[2] of these observations was used in deriving the provisional curve described below, but in the final discussion the values are taken from the new reduction of Herschel's observations which is described in the next section of this paper.

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[1] *Results of Astronomical Observations Made at the Cape of Good Hope* (London, 1847), pp. 353–374.

[2] From the plot in Tafel IV of his *Photometrische Untersuchungen*, with the corrections given in the footnote to p. 175.

Bond,[1] who first showed that the actual phase-variation of the moon was very different from that predicted by the existing theories, compared the image of the moon reflected from a silvered glass globe with that of a lamp, shining through a small diaphragm and reflected in the same way, and varied in distance from the globe to obtain equality of the images. Of his 13 nights of observation (in the spring of 1860), 3 are described as "hazy," and the results then obtained have received half-weight in the present discussion, as have also a set of observations made when the moon's altitude was only 8°.

Zöllner's observations[2] were made with two polarizing photometers, and with exemplary care. He was the first to introduce a blue screen to make the artificial light similar in color to the moon. As in the case of his observations of the sun, the accuracy of his results depends on the constancy of his standard lamp for weeks at a time, but the agreement of his results shows that in this respect his apparatus was surprisingly successful.

The visual observations of W. H. Pickering[3] were made by comparing direct moonlight by means of a shadow photometer with a pentane lamp shining through a blue solution. Of the 26 nights' observations, those on 2 nights have been rejected by Professor Pickering, one on account of known instrumental defects, the other for a very large but unexplained discordance.

King's extensive series of observations[4] consist of photographic comparisons of moonlight with the extra-focal images of stars, without the intervention of artificial light. Their large number, 83, on 64 nights, adds to the value of the series.

Stebbins and Brown,[5] using selenium cells, compared the intensity of direct moonlight with that of an amyl-acetate standard lamp, the latter being put at such a distance as to secure equality of effect, thus eliminating the question of the proportionality of the electrical effect to the intensity of illumination. These observers were the first to call attention to the difference in brightness of the moon at equal phases before and after the full. The mean of all the results obtained on each night has been taken, and all nights given the same weight.

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[1] *Memoirs of the American Academy of Arts and Sciences*, N.S. **8**, 221–266, 1861.

[2] *Photometrische Untersuchungen*, p. 102.

[3] *Harvard Annals*, **61**, 62–69.

[4] *Ibid.*, **59**, 63–94.

[5] *Astrophysical Journal*, **26**, 326, 1907.

Finally, just in time for inclusion in the present discussion, has appeared Wirtz's reduction of the observations of Wislicenus,[1] the most extensive series so far published. They consist of comparisons with a Zöllner photometer of a reduced point image of the moon with Polaris. The nórmals given by Wirtz have been taken as they stand.

In reducing these observations to a common standard, each observer's results (reduced when necessary to differences of stellar magnitude from his assumed brightness of the full moon) were compared with a provisional light-curve (the mean of the curves given by Pickering, King, and Stebbins). It appeared at once that much the greater part of the discrepancies between the light-curves given by the various observers arose, not from contradictions in the observations, but from the way in which their curves had been drawn, especially in the immediate neighborhood of the full, and that the different series of observations could be brought into better agreement by applying corrections to the values of the full brightness assumed by each observer. As these values were determined independently in each case, from freehand curves or empirical formulae, and as the relation between the brightness of the moon and the ordinary standards of stellar magnitude, when determined at all, is known with much less accuracy than the relative brightness of the different phases, the application of these zero-point corrections to improve the agreement of the various series is entirely permissible.

After several approximations, during the course of which Wislicenus' important series of observations appeared and was included in the discussion, the light-curve given in Table III was adopted as definitive. In this table the first column gives the phase-angle; the second, the difference of the moon's stellar magnitude at this phase, before the full, and that of the full moon, the third, the corresponding intensity of moonlight, taking the full moon as 1000; and the fourth, the mean surface brightness of the visible portion of the moon's disk, expressed in stellar magnitudes and referred to the full as standard. The next three columns give similar data for the moon after the full; the following column, the mean magnitude for equal phases on both sides of the full; and that succeeding it, magnitudes computed on the empirical assumption that the brightness of the moon varies as the cube of its elongation from the sun. The residuals in the last column show that this relation is very nearly true except within 30° of the full, where it gives too small a value.

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[1] *Astronomische Nachrichten,* **201,** 289–331, 1915.

TABLE III

LIGHT-CURVE OF THE MOON

PHASE-ANGLE	BEFORE FULL			AFTER FULL			MEAN		
	Mag.	Light	Surface Brightness	Mag.	Light	Surface Brightness	Obs.	Comp.	O.−C.
0°	$0^{\mathrm{M}}.00$	1000	$0^{\mathrm{M}}.00$	$0^{\mathrm{M}}.00$	1000	$0^{\mathrm{M}}.00$	$0^{\mathrm{M}}.00$	$0^{\mathrm{M}}.16$	$-0^{\mathrm{M}}.16$
10	0.22	816	0.21	0.22	816	0.21	0.22	0.34	− .12
20	0.44	666	0.41	0.48	642	0.45	0.46	0.54	− .08
30	0.67	540	0.59	0.74	505	0.66	0.72	0.75	− .03
40	0.90	436	0.77	1.03	387	0.90	0.96	0.97	− .01
50	1.13	353	0.92	1.31	299	1.10	1.22	1.23	− .01
60	1.37	283	1.06	1.58	234	1.27	1.47	1.47	.00
70	1.65	218	1.22	1.86	180	1.43	1.76	1.76	.00
80	1.98	161	1.40	2.17	136	1.59	2.07	2.07	.00
90	2.35	115	1.60	2.50	100	1.75	2.42	2.41	+ .01
100	2.78	77	1.82	2.86	72	1.90	2.82	2.79	+ .03
110	3.22	51	2.01	3.27	49	2.06	3.25	3.23	+ .02
120	3.77	31	2.26	3.74	32	2.23	3.75	3.74	+ .01
130	4.39	18	2.52	4.30	19	2.43	4.35	4.35	.00
140	5.14	9	2.81	4.98	10	2.65	5.06	5.06	.00
150	6.09	4	3.15	5.89	4	2.95	5.99	5.99	0.00

The manner in which this curve represents the observations of the various series is shown in Table IV, which gives, for means formed from small groups of the observations, the mean phase, the observer's initial, the number or weight of observations combined into the mean, and the residual from the curve. The agreement of the results of the different observers is further illustrated in Table V, in which are given the mean residuals for the observations near first quarter, full moon, and last quarter, and also the zero correction, described above, which has been applied to the magnitudes as given by each observer, the average number of observations combined into a mean in each case, and the average of the residuals given in Table IV, after reduction to this mean weight. There is very little evidence of any systematic difference between the results of the different observers, though King's photographic observations may perhaps indicate a smaller range between the full and half moon than the others do. It seems legitimate,

TABLE IV

Observed Means

Phase	Obs.	No.	O.−C.	Phase	Obs.	No.	O.−C.	Phase	Obs.	No.	O.−C.
−152°..	B	½	+1$^{\mathrm{M}}$04	− 34°..	B	2	−0$^{\mathrm{M}}$04	+ 34°..	Z	4	−0$^{\mathrm{M}}$05
−146...	W	3	+0.06	32...	S	3	− .10	40...	H	3	.00
−139...	K	3	− .11	32...	H	3	+ .11	41...	W	4	+ .11
−138...	P	2	− .16	31...	K	5	+ .07				
								+ 55...	K	6	− .04
−134...	W	5	− .13	− 23...	Z	3	− .08	57...	W	5	+ .02
124...	B	3	− .07	18...	W	5	− .03	60...	S	3	+ .13
126...	K	5	− .19	14...	K	6	+ .15				
118...	W	4	+ .17					+ 61...	P	4	+ .01
				− 12...	K	8	+ .02	61...	Z	3	+ .12
−103...	W	4	− .12	11...	H	3	− .04	66...	W	3	− .19
101...	K	7	− .01	9...	W	3	− .06				
98...	S	2	+ .13					+ 78...	W	4	+ .05
98...	W	3	+ .13	− 8...	S	1	− .10	80...	K	4	− .16
				7...	Z	3	− .06	82...	H	1	− .15
− 95...	P	1	+ .15	5...	B	1½	− .01	84...	S	2	− .14
83...	B	2½	+ .05	4...	K	6	− .04	93...	W	3	+ .02
82...	W	5	+ .06	− 2...	P	1	+ .22				
79...	K	7	.00					+ 95...	P	3	+ .22
71...	W	4	.00	+ 3...	W	6	− .02	100...	W	5	+ .04
				6...	P	3	− .07	102...	S	2	− .12
− 67...	S	3	− .11	7...	S	1	− .03	103...	K	4	+ .15
65...	K	6	− .08								
64...	H	3	− .13	+ 8...	K	4	+ .33	+111...	W	5	− .07
				8...	Z	2	− .01	124...	W	3	− .21
− 58...	Z	3	+ .09	13...	B	1½	− .18	125...	P	1	+ .18
58...	W	6	+ .03	14...	P	2	− .02	127...	S	1	+ .42
50...	K	6	− .01	16...	W	5	− .01				
								+133...	K	1	− .66
− 49...	H	4	+ .04	+ 21...	P	3	− .13	139...	W	6	+0.04
40...	W	6	− .04	22...	H	2	− .03				
35...	Z	4	0.00	26...	K	5	+ .05				
				+ 28...	S	3	+ .06				
				28...	W	5	+ .16				
				30...	P	4	−0.05				

therefore, to combine the results of the different observers into more comprehensive normals. Unit weight was given to each individual observation by Bond, King, or Wislicenus, and weight 2 to each observation by the others, these weights being based on the average residuals given in Table V, and also on the general reliability of the methods of observation. The grouping of the observed means is shown by the spacing of Table III, and the resulting normals are given in Table VI. The residuals from the adopted light-curve are apparently accidental in character. Their average value, regardless of sign, is only $\pm 0^{\rm M}.035$.

TABLE V

OBSERVER	ZERO COR-RECTION	MEAN RESIDUALS (ALGEBRAIC)						AVERAGE DEVIATION	
		First Quarter		Full		Last Quarter			
		Mag.	Obs.	Mag.	Obs.	Mag.	Obs.	Mag.	Obs.
Herschel........	$+0^{\rm M}.00$	$-0^{\rm M}.04$	7	$+0^{\rm M}.02$	8	$-0^{\rm M}.04$	4	$\pm 0^{\rm M}.07$	2.7
Bond...........	+ .24	+ .08	6	− .07	5	......	......	± .14	1.9
Zöllner..........	+ .01	+ .04	7	− .06	8	+ .02	7	± .06	3.1
Pickering........	+ .01	− .06	3	− .05	13	+ .11	8	± .11	2.4
King............	+ .25	− .06	34	+ .08	40	− .06	15	± .09	6.5
Stebbins.........	− .02	− .01	5	− .03	8	+ .04	8	± .12	2.1
Wislicenus.......	0.00	+0.02	34	+0.01	34	−0.02	34	±0.07	4.4

TABLE VI

NORMAL MAGNITUDES

Phase	Magnitude	O.−C.	Weight	Phase	Magnitude	O.−C.	Weight
−142°.....	$5^{\rm M}.32$	$+0^{\rm M}.01$	8½	+ 5°.....	$0^{\rm M}.07$	$-0^{\rm M}.04$	14
−126......	4.07	− .06	17	+ 12......	0.28	+ .01	18½
−100......	2.80	+ .02	18	+ 23......	0.52	− .04	15
− 80......	2.02	+ .04	20½	+ 29......	0.75	+ .04	19
− 65......	1.40	− .11	18	+ 38......	0.97	.00	18
− 55......	1.29	+ .04	18	+ 57......	1.54	+ .04	17
− 42......	0.95	.00	22	+ 62......	1.65	+ .01	17
− 32......	0.74	+ .02	19	+ 83......	2.20	− .07	17
− 18......	0.41	+ .01	17	+100......	2.95	+ .09	19
− 11......	0.22	− .02	17	+119......	3.71	+ .02	12
− 5......	0.09	−0.02	17½	+138......	4.76	−0.06	7

Tables III and IV are illustrated graphically in Fig. 1. It is clear that the transition from increase to decrease of brightness in passing through the full is almost abrupt. The visual stellar magnitude of the moon at any phase may be read directly from this figure.

The variation of the moon's brightness with phase is now among the best determined of photometric data, and the values given in Table III are probably within $0^{M}.05$ of the truth up to phase-angles of about 135°. Extension of the curve toward the new moon is necessarily uncertain, but the assumption that the moon's light continues to vary nearly as the cube of the elongation from the sun is probably as good a guide as any. According to this table, the full moon is 8.7 times as bright as the half moon at the first quarter, and 10.0 times as bright as the last quarter. The difference of the brightness at the opposite quadratures, as Stebbins points out, is immediately explained by the greater extent of the dark *maria* in the eastern half of the visible disk. The great difference between the light of the full and half phases arises, as is well known, from the rough character of the lunar surface, which, except at the full, is largely darkened by the shadows of its own irregularities.

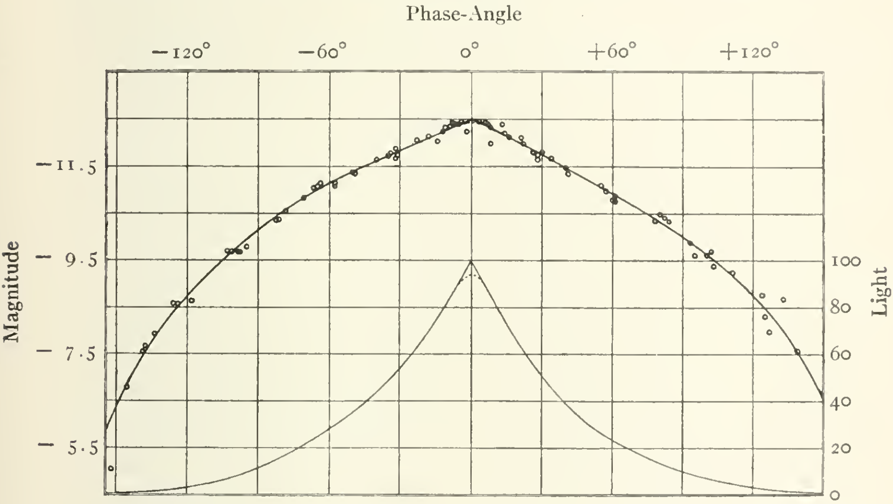

FIG. 1.—Light-curve of the moon. The upper curve shows the variations in stellar magnitude; the lower, those of the actual intensity of moonlight.

IV. THE STELLAR MAGNITUDE OF THE FULL MOON

While the results of different observers agree closely with regard to the relative brightness of the moon's phases, their determinations of the absolute brightness of the mean full moon are extraordinarily discordant, and this important constant is still decidedly uncertain. The various series of observations will here be discussed in historical order.

a) *Herschel's observations—new reduction*.—These observations,[1] made at the Cape of Good Hope in 1836, consist of 165 comparisons, on 19 nights, of a reduced image of the moon with the brighter stars, and were made primarily with the intention of determining the relative brightness of these stars. They form, however, the most extensive and direct comparisons of the relative brightness of the moon and stars which had ever been published before the appearance of Wislicenus' work in 1915. Herschel derived from his comparisons with α Centauri a value of the ratio of the brightness of the moon to that of this star, and Bond and Zöllner have derived from his observations data concerning the variations of moonlight with phase; but no general reduction of the measures, based on modern photometric magnitudes of the comparison stars, appears ever to have been made. Such a reduction (which involved no serious labor) has been carried out by the writer, and its results are briefly presented here.

Herschel's "astrometer," with which these observations were made, consisted essentially of a long staff which could be pointed at any star, on which was mounted a movable slider, carrying a lens of short focal length, into which the moon's light was reflected through a right-angled prism. By varying the distance of the slider from the eye, the starlike image of the moon produced by the lens was made equal in brightness to the star, viewed without optical aid. This simple method of observation yielded remarkably accurate results, as Zöllner has shown,[2] 9 nights' measures of α and β Centauri giving their difference of magnitude with an average residual of $\pm 0^{\mathrm{M}}.06$ from the mean.

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[1] *Results of Astronomical Observations Made at the Cape of Good Hope* (1847), pp. 353–372.

[2] *Photometrische Untersuchungen*, p. 176.

From each measured distance d (in inches) at which the moon's image appeared equal to a star of magnitude m' the apparent magnitude m of this image as seen at the standard distance of 100 inches was derived by the equation

$$m=m'+10+5\log d$$

and corrections were applied for the atmospheric extinction (which were usually small, as the observations were made at high altitudes). These corrections were taken from a nomogram prepared in the manner described by Birck,[1] and based on the Potsdam extinction table, which made their determination very rapid.

The details of the reductions for a typical night are given in Table VII. The magnitudes of the stars are taken from the *Revised Harvard Photometry*. The observations of α Lyrae, which was at an altitude of only 16°, are excluded from the mean. The average deviation of the observations from their mean is rather smaller than usual, the mean value of this quantity for the 14 nights on which five or more stars were observed being $\pm 0\overset{\mathrm{M}}{.}14$. The moon's declination was $-17°$, which explains why the extinction was usually smaller for it than for the stars.

TABLE VII

REDUCTION OF HERSCHEL'S SERIES K

STAR	d	OBSERVATION	m'	EXTINCTION		m	RESIDUAL
				Moon	Star		
α Centauri	33.3	6	0.06	0.01	0.02	2.46	+0.17
α Bootis	38.7	4	0.24	.01	.16	2.45	+ .16
α Lyrae	51.7	3	0.12	.05	.59	2.00	(−0.29)
β Centauri	55.0	5	0.86	.01	.02	2.17	− .12
α Aquilae	61.5	2	0.89	.05	.19	2.09	− .20
α Triang. Aust	80.7	4	1.88	.05	.05	2.35	+ .06
α Pavonis	94.0	3	2.12	.05	.07	2.28	− .01
γ Centauri	107.3	3	2.38	.01	.03	2.24	− .05
β Corvi	131.0	2	2.84	0.01	0.04	2.28	− .01
Mean (excluding α Lyrae)	....	....	....	0.02	0.07	2.29	±0.10

The results thus obtained for each night must be reduced to mean distance, and then to full moon. Herschel gives, for each night, the theoretical brightness of the moon according to Euler's law, computed by the formula

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[1] Inaugural dissertation (Göttingen, 1909), p. 54.

$$M = \frac{1000\Delta^2}{(R \times 933''.5)^2} \sin^2 \tfrac{1}{2} e \qquad (1)$$

where Δ is the moon's apparent semi-diameter, including the augmentation, R the earth's radius vector, $933''.5$ the moon's mean semi-diameter, and e its elongation from the sun. If the last factor be omitted, this gives the correction to mean distance.

TABLE VIII

Date	Stars	Average Deviation	m	Phase-Angle	Correction to		Image of Full Moon	Wt.	Residual
					Mean Distance	Full Moon			
1836									
March 28.	17	$\pm 0^{\mathrm{M}}.17$	$2^{\mathrm{M}}.12$	−50°	$-0^{\mathrm{M}}.05$	$-1^{\mathrm{M}}.14$	$0^{\mathrm{M}}.93$	4	$+0^{\mathrm{M}}.22$
30.	17	.16	1.56	−25	+ .01	−0.57	1.00	4	+ .29
31.	4	.12	0.93	−13	+ .04	−0.29	0.68	2	− .03
April 1.	5	.08	0.45	− 1	+ .06	−0.01	0.50	2	− .21
3.	13	.11	1.19	+28	+ .10	−0.69	0.60	3	− .11
4.	12	.15	1.61	+41	+ .09	−1.07	0.63	3	− .08
7.	5	.16	2.71	+82	+ .08	−2.23	0.56	2	− .15
26.	13	.13	2.06	−59	− .03	−1.35	0.68	3	− .03
27.	19	.12	1.73	−46	+ .01	−1.03	0.71	4	.00
June 29.	5	.17	1.00	+13	+ .11	−0.30	0.81	2	+ .10
July 22.	8	.10	2.29	−75	+ .03	−1.81	0.51	2	− .20
24.	8	.14	1.58	−47	+ .09	−1.06	0.61	2	− .10
26.	13	.16	1.10	−19	+ .10	−0.42	0.78	3	+ .07
Aug. 22.	3	.22	1.52	−50	+ .08	−1.13	0.47	1	− .24
23.	5	.19	1.35	−37	+ .10	−0.83	0.62	2	− .09
Nov. 19.	12	.11	1.53	−38	.00	−0.85	0.68	3	− .03
25.	1	. . . . . .	1.77	+31	+ .07	−0.77	1.07	1	+ .36
Dec. 17.	3	.04	1.85	−57	− .11	−1.30	0.44	1	− .27
26.	2	0.03	1.56	+44	+0.14	−1.13	0.57	1	−0.14
				Weighted mean			0.71		

Table VIII gives the results obtained from Herschel's 19 nights of observation. The first column gives the date; the second, the number of stars compared with the moon; the third, the average deviation of the individual determinations of the moon's brightness from the mean; the fourth, the mean stellar magnitude of the moon's image at 100 inches distance; the fifth, the phase-angle, negative before the full; the sixth, the correction to mean distance; the seventh, the correction to full moon; the eighth, the deduced magnitude of the image of the mean full moon; the ninth, the weight assigned, and the tenth, the residual from the final mean. The observations of the variable stars α Orionis and η Argus were excluded from these calculations, and also one observation each of α Bootis, β Centauri, and α Lyrae, for reasons given by Herschel.

Some sort of "night error," affecting all the observations of one night in the same direction, is evidently present. Weights have therefore been assigned which are roughly proportional to the square root of the number of observations. The probable error of the unit of weight, computed from the residuals, is $\pm 0^{\mathrm{M}}\!.17$, which is not large, in view of the character of the observations.

Herschel's observations thus indicate that the image of the mean full moon, as seen in his astrometer at a distance of 100 inches, was of magnitude 0.71 ± 0.03 on the Harvard scale. To find the magnitude of the moon itself, it is necessary to correct for the influence of the small lens, and for the loss of light in passing through the lens and prism. Herschel states that the focal length of the lens was 0.2253 inch. He gives no details regarding the way in which this was determined, but it is probable that this precise statement by so careful an observer represents the result of accurate measurements. This would make the moon brighter than its image in the ratio $(100)^2:(0.2253)^2$, or by 13.24 magnitudes, to which must be added the instrumental losses. The latter can be estimated from Herschel's data. The prism was of crown glass (by Fraunhofer, and of fine workmanship) and of refractive index 1.571. By Fresnel's formula,[1] the loss of light by reflection from the surface at perpendicular incidence should be $0^{\mathrm{M}}\!.055$. Assuming the lens to be similar to the prism, the total loss by reflection at the four surfaces would be $0^{\mathrm{M}}\!.22$. Herschel does not give the dimensions of the prism, but from his figure of the astrometer, which appears to be drawn to scale, it would seem to have been about one inch on the rectangular faces. As the lens was only 0.12 inch in diameter, the whole thickness of glass traversed by the light was probably about an inch. Now, according to Vogel,[2] the transmission of light through 100 mm of ordinary crown glass is 0.85

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[1] Preston, *Theory of Light*, pp. 346–347.

[2] *Astrophysical Journal*, **5**, 80, 1897.

for the visual rays. The loss by absorption in Herschel's apparatus may therefore be estimated at $0^{M}.04$, and the whole instrumental loss as $0^{M}.26$. The value of the stellar magnitude of the mean full moon resulting from his observations is therefore -12.79 on the Harvard scale.

This makes the moon $12^{M}.85$, or 138,000 times brighter than *α* Centauri. Herschel[1] himself, from his comparisons with this star alone, deduced the very different ratio of 27,400. The discrepancy is, however, easily explained. In the first place, Herschel reduced his observations to full phase by means of Euler's formula, which gives far too small a correction. The observations near full, taken separately, according to his statements, give a ratio of 45,000, and if the true corrections for phase had been applied, this would have been over 50,000. Secondly, Herschel expressly states, in the footnote in which he explains his method of reduction, that the quantity called M in his original memoir, and defined by equation (1) above, is "500 times the actual illuminating power of the moon at the time of observation, that of the mean full moon being unity." But, according to equation (1), M is 1000 for the mean full moon, and Herschel's numerical calculations bear this out. There seems, therefore, no escape from the conclusion that, in this later addition to his work, Herschel inadvertently confused the brightness of the mean full moon and the mean brightness of the moon at all phases, which, according to Euler's formula, is just half as great, and that his numerical result must be doubled to give the value of the former. The introduction of these two corrections would raise Herschel's value of the ratio to about 110,000. The outstanding difference is probably due to differences in the methods of reduction, and in the estimates of the instrumental losses.

b) Bond's observations.—These consist of two independent series of comparisons, one with Venus and Jupiter, the other with the sun. In the first,[2] the images of the planets, reflected from a silvered glass globe, were compared with those of a standard lamp at adjustable distances, exactly as in the observations on the moon, and often on the same nights. From observations on 6 nights he concludes that Venus, at phase-angle 68°.8, and reduced to mean distance from the sun and unit distance from the earth, was $8^{\mathrm{M}}.08$ fainter than his lamp at a distance of one foot, and from 4 nights' observations (rejecting one night for good cause) he concludes that Jupiter at mean opposition was $9^{\mathrm{M}}.51$ fainter than the same standard. The average residual for a night was $\pm 0^{\mathrm{M}}.14$ for Venus and $\pm 0^{\mathrm{M}}.12$ for Jupiter. Bond's assumed value for the light of the mean full moon is $0^{\mathrm{M}}.06$ brighter than the lamp at one foot, and the zero correction given in Table V means that, if reduced anew with the light-curve of Table III, this would be $0^{\mathrm{M}}.24$ brighter still. Hence Bond's observations make the mean full moon $8^{\mathrm{M}}.38$ brighter than Venus, and $9^{\mathrm{M}}.81$ brighter than Jupiter at the given phases. Müller's magnitudes for the planets at these times, reduced to the Harvard scale, are −3.71 and −2.29; so that the moon's magnitude comes out −12.09 from the comparisons with Venus, and −12.10 from those with Jupiter.

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[1] *Outlines of Astronomy* (New York, ed. 1876), p. 595.

[2] *Memoirs of the American Academy of Arts and Sciences*, N.S., **8**, 250, 1861.

In spite of the agreement of the two, this result deserves little weight. Bond did not employ a blue screen to make the light of his lamp similar in color to that of the moon or of the planets, and he states that the lamp was decidedly red in comparison with the moon. The same reflecting globe was used in observing the moon and the planets, and the image of the lamp appeared to the eye more than a thousand times fainter in the second case than in the first. The Purkinje effect would therefore come into play to a high degree, and would operate to make the lamp appear relatively faint at the lower intensity, and hence to make the measured difference in magnitude between the moon and the planets too small, as appears from comparison with other observations to be actually the case.

Bond's comparisons of the moon and sun[1] were made by a very similar method. The light of the sun, reflected from a silvered globe, was again reflected in a second, and compared with the reflection of a "Bengola light" in the latter, and the moon was subsequently observed in exactly the same way. The obvious weakness of the method, severely criticized by Zöllner,[2] is the uncertainty whether the light of these fireworks could be considered as even roughly constant, even if "those of the same size and manufacture were used throughout, and a fresh one burned for each comparison." Some idea of the magnitude of the errors involved can, however, be derived from the fact that the five observations of the sun, on two days, show an average deviation from the mean ratio of sunlight to Bengola light of $\pm 0^{\mathrm{M}}.20$, and the six of the moon (all on one night, when its phase was $7^{\circ}.6$ and its altitude 24°) a deviation of $\pm 0^{\mathrm{M}}.18$. From these comparisons Bond concludes that the sun is 471,000 times, or $14^{\mathrm{M}}.18$, brighter than the mean full moon. The correction to full moon from the observed phase is $0^{\mathrm{M}}.03$ according to Bond's curve, and $0^{\mathrm{M}}.15$ by Table III, so that the difference of magnitude, on the system of the present paper, is 14.06, which, with the value of the sun's magnitude already found, makes that of the full moon -12.66. The probable error of this value, deduced from the observations, is $\pm 0^{\mathrm{M}}.12$; but it may really be considerably greater.

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[1] *Ibid.*, p. 287, 1861.

[2] *Photometrische Untersuchungen*, p. 116.

c) *Zöllner's observations.*[1]—These fall again into two groups. In those made by his "second method" a point image of the moon was compared with an artificial star, and this again with Capella (using a different optical train). Eight nights on the moon and 11 on the star (separated by an average interval of some three months) make the difference in magnitude between the star and the mean full moon $12^{\mathrm{M}}.39$ when reduced with Zöllner's formula. Applying the zero correction found above, this becomes $12^{\mathrm{M}}.38$ and the moon's magnitude -12.17 on the Harvard scale.

By his "first method" the moon and sun were compared with a standard lamp, with a polarizing photometer of different type. Ten days' observations on the sun, and 6 nights' on the moon, when reduced with the light-curve here adopted, give for the difference of magnitude $14^{\mathrm{M}}.45$, and for the magnitude of the moon -12.27.

d) *The observations of W. H. Pickering.*—These observations, which have already been described,[2] were referred to the stars by removing the pentane lamp, with blue screen as used in the measures of moonlight, to such a distance that it appeared of the same brightness as Arcturus (which seemed nearly of the same color). Observations were unfortunately made on only 4 nights, and their average deviation from the mean was $\pm 0^{M}.24$, so that the probable error of the determination of the relative brightness of the star and lamp from the mean of the four is $\pm 0^{M}.12$, while that of the relative brightness of the lamp and the mean full moon (reducing the 24 comparisons by means of Table III) is only $\pm 0^{M}.024$. Pickering finds that the magnitude of the latter is −12.50, which becomes −12.51 on applying the zero correction.

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[1] *Photometrische Untersuchungen*, pp. 90–115, 124.

[2] *Harvard Annals*, **61**, 63–68.

e) From the comparisons of sunlight and moonlight with the standard candle (see below) the ratio of the two is found to be 428,000, corresponding to a difference of magnitude of 14.08.

Rejecting Bond's comparisons with the planets, for reasons already stated, the mean results of each observer for the magnitude of the mean full moon are as follows:

		Residual
Herschel	−12.79	−0.24
Bond	−12.66	− .11
Zöllner	−12.22	+ .33
Pickering	−12.51	+ .04
Comparison with candle	−12.64	−0.09

If Bond's determination is given half the weight of the others, the general mean is −12.55, with a probable error of ±0.07, as computed from the residuals. The actual uncertainty of this result is probably greater, as some of the determinations may be affected by systematic errors.

From the values here found for the magnitudes of the sun and moon, it appears that the sun exceeds the mean full moon in brightness by 14.17 magnitudes, or 465,000 times, with an uncertainty of fully 10 per cent.

f) The photographic magnitude of the moon is well determined by King's long series of observations. The value $-11^{M}.20$, which he originally gave, requires a correction of $+0^{M}.07$ to reduce it to his definitive system of magnitudes,[1] and a further one of $-0^{M}.24$ for the difference between his light-curve and that of Table III, making the final value $-11^{M}.37$. This makes the color-index of moonlight $+1^{M}.18$, about that of a star of Class K0, and $0^{M}.39$ greater than that of the sun. At first glance it might be doubted whether the moon is really as red as this; but the spectrophotometric observations of Wilsing and Scheiner[1] proved that her surface is strongly colored. Their measurements, made on a bright and a dark region, showed sensibly the same behavior for both, the relative reflecting power increasing from 0.61 at λ 4480 to 0.64 at λ 5130, 0.87 at λ 5840, and 1.00 at λ 6380. This indicates a reflecting power about 20 per cent less in the photographic than in the visual region, and hence a color-index about $0^{\mathrm{M}}.25$ greater than that of the sun, in fair agreement with the results of the photometric measures.

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[1] *Ibid.*, 59, 153.

V. THE STELLAR MAGNITUDE OF THE STANDARD CANDLE

Though this quantity is not directly related to the question of albedo, it may appropriately be discussed here.

a) *Sunlight and terrestrial standards.*—The older comparisons,[2] summarized by Müller, must have been systematically in error, for their results are all much too low. The first approximately correct value appears to have been Fabry's, already quoted, which made the sun in the zenith equal to 100,000 meter-candles. Much the most extensive and precise series of observations appears to be the very recent one by Kimball,[3] at Mount Weather, Virginia. With a photometer of precision, and carefully tested standards of light, he measured the intensity of illumination of a horizontal surface by the combined light of the sun and sky, and also by the sky when direct sunlight was cut off, and thus deduced the intensity of normal illumination by the latter over a large range of zenith distances on many days.

From the data given for various air-masses in his Table IV, it is possible on many days to extrapolate to the zenith (air-mass unity) and determine the transmission of light by the atmosphere on each day. If the results are grouped according to the brightness of the light of the sky when the sun was hidden, that is, according to the haziness of the weather, the following mean values are obtained, the intensity of the light being given in foot-candles, which units Kimball uses throughout:

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[1] *Publikationen des astrophysikalischen Observatoriums zu Potsdam,* **20,** No. 61, 1909.

[2] *Photometrie der Gestirne,* pp. 309–312.

[3] *Monthly Weather Review,* **43,** 650, 1914.

Days	Light of Sky	Zenith Sun	Atmospheric Transmission
5	880	9750	0.782
5	1035	9550	0.780
5	1155	9550	0.736
3	1420	8650	0.675
4	2400	6700	0.677

As might be expected, the brightness of direct sunlight falls off seriously in hazy weather. For comparison with other results of astronomical photometry, it is probably desirable to take only the observations made in fairly clear weather, the first three groups in the table. Observations on seven more clear days (with sky light at noon less than 1200 foot-candles) can also be employed by reducing them to the zenith with the mean transmission 0.766 found for the three groups above.

The mean of the 22 days' observations in clear weather gives, for the intensity of sunlight from the zenith, 9600 foot-candles, or 103,000 meter-candles, with an average deviation for the individual determinations of ±5.4 per cent. According to Kimball's statements, the probable error of the mean, including systematic errors, should not exceed 5 per cent.

This determination makes the zenith sun 12.53 magnitudes brighter than a standard candle at 1 m distance. With the atmospheric transmission found above, the sun outside our atmosphere would be $0^{M}.29$ brighter, and its light equal to 134,500 meter-candles.

b) *Moonlight and standard candle.*—The latest and probably the best determination of the intensity of moonlight compared with artificial standards is that by Graff,[1] who compared, with a Weber photometer, the light of the moon with that of an amyl-acetate lamp shining through pale-blue glass, and this with a Hefner lamp. Observations on 5 nights gave for the light of the mean full moon in the zenith 0.269±0.014 Hefner-meter-candles. The light-curve used in reducing the observations, all of which were made

[1] *Astronomische Nachrichten*, **198**, 14, 1914.

within 24° of the full, to the latter phase was so nearly identical with that of Table III that no correction is required. Graff also states that the mean of a considerable number of older determinations, most of which are satisfactorily accordant, is 0.251 English normal candle, or 0.267 Hefner candle, at 1 m. The ratio of moonlight to the meter-candle appears therefore to be very well determined. Since the Hefner candle is equal to 0.90 international standard candle, full moonlight may be taken as 0.241 meter-candle on this scale, and the international meter-candle as 1.55 magnitudes brighter than the mean full moon.

c) *Standard candle and starlight.*—Direct comparisons of laboratory standards of light with the stars are complicated by the great difference in color, and depend upon the accuracy with which the transmission of the blue screens which are necessary can be determined. Fabry's result,[1] that a *bougie décimale* at a distance of 780 m would appear as bright as Vega, appears to be the best available direct comparison, as he determined the transmission of his screen from the mean of a large number of laboratory measurements. This makes the meter-candle $14^{\mathrm{M}}_{.}46$ brighter than Vega.

W. H. Pickering's result,[2] that a standard candle at 1 m is of magnitude −13.60 on the Harvard scale, depends on a transmission coefficient determined from comparisons of moonlight with the lamp, with and without the blue screen, on but a single night and is necessarily of little weight, because of the small number of comparisons, and especially since the intensity of the illumination of the screen of the shadow photometer with which the observations were made—that of moonlight—was so low that the Purkinje phenomenon is to be feared. It would operate to make the measured brightness of the unscreened lamp lower, and the transmission of the screen higher, than would have been found had the observations been made with illumination of sufficient intensity to lie outside the dangerous region, and so to give too low a value for the brightness of the standard candle. As the result is actually much lower than that of any other observer, it may be excluded from the mean, for the reasons stated.

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[1] *Comptes Rendus*, **137**, 1242, 1903.

[2] *Harvard Annals*, **61**, 69, 1908.

The other determinations give the following values for the stellar magnitude of the standard candle at a distance of 1 m.

		Weight
Kimball (sun)	−14.19	3
Graff and others (moon)	−14.09	2
Fabry (Vega)	−14.32	1

With the weights assigned, the mean is −14.18, with an apparent probable error of about $\pm 0\overset{\mathrm{M}}{.}05$.

Photographically, E. S. King has found[1] that a standard 2-candle electric lamp, supplied by the Bureau of Standards, was of magnitude −12.06 at a distance of 1 m. The color-index of this lamp was therefore $+2\overset{\mathrm{M}}{.}87$, so that, compared with stars of Class A, the lamp was 14 times fainter photographically than visually. Other standards of light might, however, have very different color-indices.

It follows from the foregoing that a light of the same mean color as that of the stars, and of intensity which, measured under high illumination, was of one candle-power, would appear to be of stellar magnitude 0.82 at a distance of 1 km, and 1.85 at a distance of a mile. An actual standard candle, of the ordinary varieties, if removed to these distances, would undoubtedly appear much fainter, on account of the Purkinje effect. Pickering's measures indicate that the difference would probably exceed half a magnitude.

It is evident from the foregoing summary that the precise determination of the relative brightness of the full moon and of the stars, and the direct comparison of the stars and terrestrial standards of light, by trustworthy visual and photo-visual methods, are urgently desirable. No direct visual measurements of the ratio of sunlight to moonlight, and but one of that of moonlight to starlight, appear to have been made in the past fifty years. Most of the existing determinations are derived from incidental observations by astronomers whose main attention was directed elsewhere. A determination of these fundamental photometric constants with modern precision would well repay the undivided attention of a competent observer for a considerable time.

PRINCETON UNIVERSITY OBSERVATORY
January 20, 1016

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[1] *Ibid.*, **59**, 275.

PHOTOMETRIC STUDY OF THE ECLIPSING VARIABLE RV OPHIUCHI

By RAYMOND S. DUGAN

The variability of this star (B.D.$+7^\circ3404$) was discovered by Mrs. Fleming,[1] and independently by Mme Ceraski,[2] in comparing photographic plates. It has been further observed by Blajko,[2] Hartwig,[3] and Wendell.[4] In 1906, after quite a long watch, I finally observed the star entering eclipse on April 18. In May, I was able to announce provisional elements, which agreed with those given by Wendell at the same time.[5] The length of the period, the position of the star, and many interruptions delayed the completion of the observation of the light-curve until 1914. The position of the star is (1900) $\alpha=17^h29^m.8$, $\delta=+7^\circ19'$, and its provisional designation as a variable was 136.1904.

The 14,128 measures were made with the polarizing photometer in the usual manner and reduced in duplicate.[6] The tables in *Harvard Annals*, **33**, have been extended to three decimal places. Atmospheric absorption was allowed for and the times reduced to the sun. The usual comparison star was $a=$B.D.$+7^\circ3403$. A second star, $b=$B.D.$+7^\circ3405$, was also used for comparison on a few nights.

From an inspection of the observations by Wendell and by myself of eclipses scattered over 994 periods I assumed the preliminary elements

$$\text{J.D. } 2420269.70597+3^d.687126\ E. \quad \text{Elements 1.}$$

With these approximate elements all my observations taken during primary minima were collected and formed into a mean curve.

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[1] *Harvard Circular*, No. 80.

[2] *Astronomische Nachrichten*, **166**, 155, 1904.

[3] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **40**, 329, 1905.

[4] *Harvard Annals*, **69**.

[5] *Harvard Bulletin*, No. 244.

[6] *Contributions of Princeton Observatory*, No. 1.

This was drawn on transparent paper and used to determine the times of minima from the individual plots of minima observed by Wendell and by myself. In Table I, the fourth column contains these observed times. The weights are assigned in accordance with the accuracy with which the observations, from their number and distribution, apparently determine the time of mid-eclipse.

The residuals $O.-C._1$, when plotted, indicate, with considerable clearness, the need of a sine term of small amplitude. One-half the amplitude is apparently about 2 minutes, the period about 1280 E, while the residual becomes zero at about −125 E. The following new elements were, therefore, assumed:

J.D. $2419808.81513 + 3\overset{\mathrm{d}}{.}687126\ E - 0\overset{\mathrm{d}}{.}0014 \sin 0\overset{\circ}{.}28\ E$. Elements 2.

A least-squares solution was made, regarding the amplitude and phase of the sine term as known. The resulting corrections were

$$\begin{aligned} dT &= -0\overset{\mathrm{m}}{.}40 \pm 0\overset{\mathrm{m}}{.}37 \\ dP &= +0\overset{\mathrm{m}}{.}001 \pm 0\overset{\mathrm{m}}{.}0011 \\ dq &= -0\overset{\circ}{.}06 \pm 0\overset{\circ}{.}036 \end{aligned}$$

and the residuals are given in the last column of Table I. The sum of the weighted squares of residuals was reduced from 104.62 from elements 1, to 43.46 from elements 2, to 36.01 from the final elements. The final elements are

J.D. $2419808.81485 + 3\overset{\mathrm{d}}{.}6871267\ E - 0\overset{\mathrm{d}}{.}0014 \sin 0\overset{\circ}{.}22\ E$ = 1913 Feb. $9^{\mathrm{d}}19^{\mathrm{h}}33\overset{\mathrm{m}}{.}4$ G.H.M.T. $+ 3^{\mathrm{d}}16^{\mathrm{h}}29^{\mathrm{m}}27\overset{\mathrm{s}}{.}75\ E - 2^{\mathrm{m}} \sin 0\overset{\circ}{.}22\ E$. Elements 3.

Elements and observed times appearing in this paper are heliocentric.

TABLE I

Observer	Epoch	J.D.	G.H.M.T. Observed	No. of Obs.	Wt.	$O.-C._1$	$O.-C._2$	$O.-C._3$
Wen.	−994	2416604	$16^{\mathrm{h}}50^{\mathrm{m}}$	5	2	$-1\overset{\mathrm{m}}{.}8$	$0\overset{\mathrm{m}}{.}0$	$+0\overset{\mathrm{m}}{.}5$
Wen.	−984	16641	13 44	3	1	−2.3	−0.6	−0.2
D	−796	17334	18 4.5	13	6	−0.8	−0.5	−0.8
D.	−585	18112	17 44	17	6	+2.7	+1.1	+1.4
Wen.	−578	18138	13 9.5	4	1	+1.7	+0.1	+0.4
D.	−486	18477	18 19.5	23	10	+1.2	−0.8	−0.2
D.	−390	18831	17 26.5	6	1	−0.3	−2.2	−1.3
D.	−281	19233	14 55	4	1	−2.8	−4.2	−3.4
D.	−112	19856	17 59	2	1	+2.2	+2.3	+2.7
D.	0	20269	16 55	17	9	−1.3	−0.2	−0.1

The individual observations were first weighted according to observing conditions. Normals were then formed and plotted, and a curve drawn through them. Residuals were then read off. The observations were corrected for the average residual (usually of the whole night, but sometimes, where there was an abrupt change, for a part of the night). The weights of the observations were then modified according to the magnitude of the average remaining residual. These remaining residuals were found to have the characteristics of accidental errors. Normals, usually of five observations each, were formed and the sum of the weights diminished in cases where all the observations entering into a normal were not made on different nights, the weight of a normal made up of five observations on one night suffering the greatest reduction. The current number of the normal, the current number of the supernormal into which the normals are combined for the final solution, the phase, the weight, the observed difference in magnitude between the variable and comparison star *a*, are given in the first five columns of Table II.

The mean light-curve of RV Ophiuchi, while presenting the relatively simple conditions of a circular orbit and a total eclipse, is decidedly puzzling. The observations, which I have no reason to think are inferior to those of RT Persei and Z Draconis, bring out clearly a strong asymmetry combined with an apparently nonperiodic succession of hollows and humps. The observed secondary minimum is barely deep enough to satisfy the representation of primary minimum on the assumption of uniformly brilliant disks. Any darkening toward the limb requires a still deeper secondary minimum. As the depth of secondary minimum in a light-curve of this form should be strongly affected by darkening, hopes had been entertained of even determining the degree of darkening in this case. Although, as will be seen, the effect of mutual radiation is about the same as in the systems RT Persei and Z Draconis, the ellipticity is much greater than in the two other cases. This was again surprising, since the two stars are so far apart as to warrant no expectation that an ellipticity effect could be detected.

As the depth (d) of secondary minimum is comparable with the ellipticity (c) and effects of mutual radiation (b) it seemed wise to

TABLE II

No.	Super-normal No.	Phase	Wt.	*a*−*v*	O.−C.	O.−C.′
1.........	38	$-6^{h}\,49^{m}5$	14	$+0^{M}.991$	$-0^{M}.005$	$+0^{M}.003$
2.........	38	28.0	14	.973	− .022	− .015
3.........	38	6 5.3	13	.985	− .009	− .002
4.........	38	5 50.6	9	1.001	+ .008	+ .015
5.........	38	28.4	6	.991	− .002	+ .004
6.........	38	5 10.1	6	.969	− .023	− .017
7.........	38	4 50.3	6	1.008	+ .017	+ .023
8.........	38	4 29.0	3	1.003	+ .017	+ .022
9.........	16	3 53.7	5	.866	− .034	− .027
10.........	15	37.0	9	.819	− .013	− .005
11.........	14	21.0	7	.708	− .052	− .043
12.........	13	3 5.2	10	.669	− .001	+ .010
13.........	12	2 54.7	10	.553	− .053	− .040
14.........	11	40.6	8	.467	− .038	− .024
15.........	10	31.6	11	.367	− .065	− .051
16.........	9	18.4	12	.265	− .062	− .048
17.........	9	2 8.3	8	+0.243	+ .013	+ .026
18.........	8	1 58.4	11	+0.135	+ .007	+ .020
19.........	7	44.8	13	−0.031	.000	+ .011
20.........	6	29.5	12	−0.224	.000	+ .010
21.........	5	17.8	13	.414	− .030	− .022
22.........	4	1 5.5	11	.603	− .040	− .034
23.........	3	0 54.7	11	.745	− .030	− .025
24.........	2	39.3	13	.924	− .034	− .031
25.........	1^{1}	26.0	13	1.001	− .021	− .019
26.........	1	14.5	11	.994	+ .004	+ .005
27.........	1	− 0 0.3	11	1.002	− .004	− .004
28.........	1	+ 0 15.8	10	.973	+ .025	+ .024
29.........	1^{1}	28.2	11	.984	− .014	− .016
30.........	....	41.4	12	.979	− .109	− .112
31.........	3	0 53.9	14	.766	− .040	− .045
32.........	4	1 7.4	15	.568	− .030	− .036
33.........	5	17.0	15	.393	+ .032	+ .024
34.........	6	28.4	11	−0.235	− .005	− .015
35.........	7	39.1	9	−0.074	+ .024	+ .013
36.........	7	49.4	12	+0.054	+ .026	+ .014
37.........	8	1 59.3	12	+0.164	+ .027	+ .014
38.........	9	2 10.6	9	.281	+ .031	+ .018
39.........	10	21.2	12	.359	+ .009	− .005
40.........	11	35.3	8	.505	+ .043	+ .029
41.........	12	45.6	14	.553	+ .008	− .006
42.........	13	2 57.3	13	.649	+ .024	+ .012
43.........	14	3 9.9	14	.736	+ .036	+ .025
44.........	15	3 34.0	11	.853	+ .031	+ .023
45.........	17	+ 4 8.1	12	+0.964	+0.020	+0.014

TABLE II—*Continued*

No.	Super-normal No.	Phase	Wt.	$a-v$	O.−C.	O.−C.′
46	19	$+4^{h}35^{m}.6$	9	$+0^{M}.993$	$+0^{M}.003$	$-0^{M}.002$
47	19	5 5.6	8	.990	− .002	− .008
48	19	27.4	8	1.018	+ .025	+ .019
49	19	5 47.5	9	.999	+ .005	− .002
50	19	6 3.7	8	1.005	+ .010	+ .003
51	19	25.3	8	.985	− .010	− .017
52	19	6 52.7	6	1.014	+ .018	+ .010
53	19	7 17.4	3	.970	− .028	− .036
54	19	7 41.2	6	1.008	+ .009	.000
55	19	8 22.2	5	.974	− .027	− .036
56	19	9 0.3	5	.994	− .009	− .019
57	19	9 30.7	5	.987	− .017	− .027
58	20	10 17.2	8	.998	− .008	− .019
59	20	12 27.8	7	.966	− .046	− .059
60	20	13 53.0	10	.984	− .032	− .046
61	20	14 21.5	9	.990	− .027	− .041
62	20	14 38.4	12	1.010	− .008	− .022
63	21	17 40.9	10	1.054	+ .028	+ .012
64	21	18 30.5	12	1.043	+ .015	− .001
65	21	18 47.8	10	1.068	+ .040	+ .024
66	21	19 4.2	10	1.057	+ .028	+ .012
67	21	23.0	7	1.059	+ .029	+ .013
68	21	19 41.1	8	1.046	+ .016	.000
69	21	20 2.7	6	1.050	+ ..020	+ .004
70	21	27.0	9	1.051	+ .020	+ .004
71	22	42.7	8	1.047	+ .015	− .001
72	22	20 58.8	9	1.070	+ .040	+ .023
73	22	21 15.8	8	1.032	.000	− .017
74	22	23 18.9	11	1.074	+ .040	+ .024
75	22	24 20.6	7	1.034	.000	− .016
76	23	27 25.0	5	1.047	+ .017	+ .002
77	23	28 14.7	5	1.069	+ .040	+ .025
78	23	28 57.5	6	1.066	+ .038	+ .023
79	23	30 2.8	6	1.041	+ .015	+ .001
80	23	31 12.4	6	1.047	+ .023	+ .010
81	24	32 30.9	6	1.034	+ .013	+ .001
82	24	33 42.5	4	1.014	− .004	− .015
83	24	34 32.4	6	1.051	+ .034	+ .023
84	24	35 28.4	6	1.035	+ .020	+ .010
85	24	35 58.2	6	1.044	+ .030	+ .021
86	24	36 33.1	6	1.029	+ .016	+ .007
87	25	37 44.6	8	.991	− .020	− .028
88	25	38 13.7	10	1.018	+ .008	+ .001
89	25	38 39.5	8	.994	− .014	− .021
90	25	+39 21.4	7	+1.034	+0.027	+0.021

TABLE II—*Continued*

No.	Super-normal No.	Phase	Wt.	*a*−*v*	O.−C.	O.−C.′
91.........	26	$+39^{h}59^{m}.9$	7	$+0^{M}.978$	$-0^{M}.028$	$-0^{M}.033$
92.........	26	40 30.0	5	+1.013	+ .017	+ .013
93.........	27	41 48.6	6	.973	+ .006	+ .003
94.........	28	42 14.5	8	.942	− .013	− .015
95.........	28	31.7	9	.947	.000	− .002
96.........	29	42 52.1	8	.946	+ .008	+ .006
97.........	29	43 6.2	6	.938	+ .004	+ .003
98.........	30	22.1	8	.917	− .013	− .014
99.........	30	35.9	8	.943	+ .015	+ .014
100.........	31	43 45.8	8	.925	− .002	− .003
101.........	31	43 56.4	8	.885	− .042	− .042
102.........	31′	44 6.5	10	.938	+ .011	+ .011
103.........	31′	18.3	9	.960	+ .033	+ .033
104.........	31′	27.1	6	.922	− .005	− .005
105.........	31	36.3	12	.987	+ .060	+ .060
106.........	30	46.6	9	.947	+ .019	+ .020
107.........	30	44 57.5	8	.941	+ .012	+ .013
108.........	29	45 11.2	9	.973	+ .041	+ .042
109.........	29	25.2	7	.941	+ .006	+ .007
110.........	29	45 42.5	8	.947	+ .005	+ .007
111.........	28	46 20.0	7	.949	− .008	− .005
112.........	27	47 8.9	12	.970	− .008	− .005
113.........	27	28.9	10	.986	− .003	+ .001
114.........	27	47 46.7	10	.993	.000	+ .004
115.........	26	48 16.3	8	1.005	+ .004	+ .009
116.........	32	+48 37.2	9	1.014	+ .009	+ .014
117.........	32	−39 31.1	7	1.005	.000	+ .006
118.........	32	11.0	13	.961	− .047	− .041
119.........	32	39 0.3	12	1.030	+ .022	+ .028
120.........	32	38 51.8	11	1.003	− .005	+ .001
121.........	32	43.0	13	.992	− .017	− .011
122.........	32	32.6	13	1.037	+ .028	+ .035
123.........	32	38 13.1	12	.993	− .016	− .009
124.........	32	37 49.6	13	1.036	+ .026	+ .033
125.........	32	32.9	14	.987	− .023	− .015
126.........	32	20.8	14	1.020	+ .009	+ .017
127.........	32	37 4.7	13	1.018	+ .006	+ .014
128.........	32	36 43.9	10	.990	− .022	− .014
129.........	32	36 22.2	9	1.018	+ .005	+ .014
130.........	32	35 55.3	10	.972	− .042	− .033
131.........	32	42.0	7	1.015	.000	+ .009
132.........	32	35 6.7	2	1.000	− .016	− .006
133.........	33	33 49.8	5	.985	− .034	− .023
134.........	33	32 34.5	5	.982	− .040	− .028
135.........	33	−31 32.4	6	+0.971	−0.052	−0.039

TABLE II—*Continued*

No.	Super-normal No.	Phase	Wt.	*a*−*v*	O.−C.	O.−C.′
136	33	$-30^{\mathrm{h}}48^{\mathrm{m}}.8$	7	$+1^{\mathrm{M}}.005$	$-0^{\mathrm{M}}.020$	$-0^{\mathrm{M}}.006$
137	33	30 19.5	7	.980	− .045	− .031
138	33	29 51.6	8	.989	− .037	− .023
139	34	29 21.6	8	.974	− .052	− .038
140	34	28 56.2	9	1.020	− .008	+ .007
141	34	28 20.1	11	1.018	− .011	+ .004
142	34	27 39.6	8	1.005	− .025	− .010
143	34	27 14.9	8	1.016	− .015	.000
144	34	26 57.0	9	1.000	− .032	− .017
145	34	38.5	8	.988	− .044	− .028
146	34	26 19.3	8	.998	− .034	− .018
147	34	25 47.0	9	1.005	− .027	− .011
148	34	25 11.2	4	1.042	+ .010	+ .026
149	35	24 23.2	6	1.037	+ .005	+ .021
150	35	23 33.7	6	.993	− .039	− .023
151	35	22 48.0	6	1.016	− .016	.000
152	35	22 15.8	6	1.033	.000	+ .017
153	35	21 48.6	6	1.000	− .032	− .015
154	35	27.7	5	.982	− .050	− .033
155	35	21 8.2	6	.996	− .036	− .019
156	35	20 51.4	6	1.054	+ .022	+ .038
157	35	34.2	5	1.050	+ .020	+ .036
158	35	20 15.9	4	1.055	+ .024	+ .040
159	35	19 56.6	3	1.030	.000	+ .016
160	35	31.5	7	1.009	− .020	− .004
161	36	19 8.9	6	1.060	+ .030	+ .046
162	36	18 51.1	10	1.037	+ .009	+ .025
163	36	27.9	9	1.026	.000	+ .016
164	36	18 1.2	8	1.042	+ .015	+ .031
165	36	17 36.0	8	1.001	− .025	− .009
166	36	17 11.6	8	1.044	+ .019	+ .034
167	36	16 40.8	4	1.036	+ .012	+ .027
168	36	15 29.6	4	1.043	+ .023	+ .038
169	37	13 59.0	2	1.020	+ .004	+ .018
170	37	11 31.9	10	1.028	+ .019	+ .031
171	37	10 36.9	15	1.049	+ .043	+ .055
172	38	9 22.4	13	.990	− .013	− .002
173	38	8 58.0	12	.988	− .013	− .002
174	38	36.9	12	1.016	+ .016	+ .025
175	38	18.6	16	1.011	+ .012	+ .021
176	38	8 3.3	14	.988	− .010	− .001
177	38	7 51.1	16	.996	− .002	+ .007
178	38	32.5	11	.999	+ .002	+ .010
179	38	−7 11.6	14	+0.981	−0.015	−0.007

determine these quantities, as well as the light (a) of the system at longitude 90° and the time of secondary mid-eclipse, from the whole curve outside of primary minimum.[1] All these observations were combined into 20 supernormals (this solution was made before the observations were corrected for night errors and the weights modified). The second-order terms were computed with the assumed values of the unknowns. The result of the solution was

$$a=1.021\pm 0.002$$
$$b=0.007\pm 0.004$$
$$c=0.042\pm 0.004$$
$$d=0.052\pm 0.010$$
$$\text{Time of sec.}=\text{time of primary}+\tfrac{1}{2}P+0^{\text{m}}.6\pm 20^{\text{m}},$$

the unit of intensity being the average brightness between eclipses, $a-v=1^{\text{M}}.013$. The probable error of weight unity is ± 0.020, which is, on the average, that of three normals. The probable error of a single normal would be, therefore, ± 0.034. This is not surprisingly large in view of the pronounced asymmetrical difference between the two branches outside of eclipse.

The curve was then rectified to intensity 1.0288, corresponding to $a-v=1^{\text{M}}.045$. From the length of the constant phase during primary minimum we know that this eclipse is total. From the depths of the two minima we find that the radius of the brighter star is ($k=$) 0.54 that of the fainter. In the following, Russell's methods and notation are used.[2] With this value of k no values of $A=\sin^2\theta_2$ and $B'=\dfrac{\sin^2\theta_2-\sin^2\theta_3}{1-2c\cos^2\theta_3}$ could be found which gave a satisfying representation of the mean of the two branches of primary minimum. $k=0.54$, $A=0.0228$, $B'=0.0175$ represents the ascending branch fairly well, but the whole eclipse calls for a value of k in the neighborhood of 0.64. This value of k requires a depth of secondary minimum exceeding the observed by $0^{\text{M}}.012$. Any assumption of darkening calls for a still deeper secondary. The amount of ellipticity, which was well determined by the least-squares solution, is unexpectedly large.[3] Representations of primary minimum were

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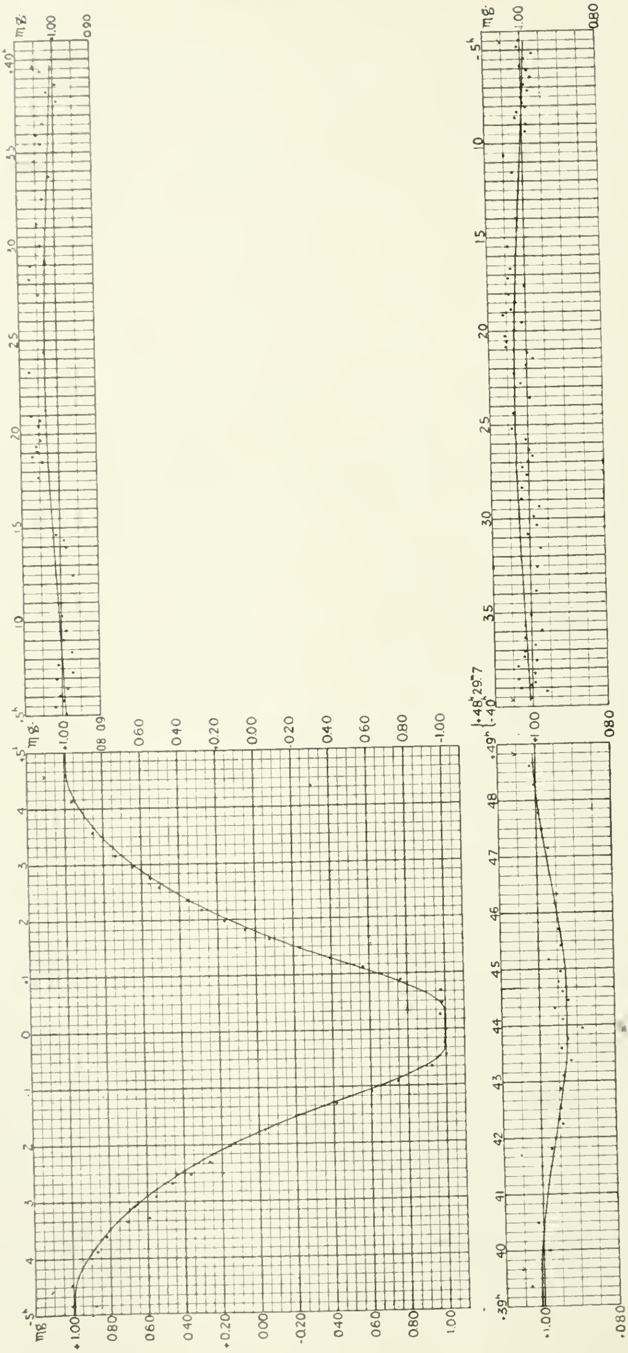
[1] Russell, *Astrophysical Journal*, **39**, 407, 1914.

[2] *Astrophysical Journal*, **35**, **36**, and **39**.

[3] See Shapley's table, *Contributions of Princeton Observatory*, No. 3, p. 115.



Computed Light-Curve of RV Ophiuchi

attempted, assuming the amount of ellipticity anticipated from experience, and assuming no ellipticity at all. In neither case was there much improvement. At this point the discussion of systematic residuals was undertaken, and still the representation was not satisfactory. The thought comes to one that it may be futile to attempt to find elements to represent the mean of the two branches of such an asymmetrical curve as the one under discussion.[1]

Since the value of k is affected by the elements of rectification, it seemed best to attempt a least-squares solution of the entire curve in 7 unknowns, A, B, k, a, b, c, and λ_1 the observed intensity during totality. The intensity during the annular phase of secondary minimum is expressed as a function of k and λ_1. The somewhat complicated equations, based on the assumption of uniformly brilliant disks, are as follows:

$$\text{I.}\qquad \sin^2\theta = A + \psi\cdot B\cdot n\,,$$

where

$$n = \frac{1 - z\cos^2\theta}{1 - z\cos^2\theta_3}\,,\quad z = 2c\,.$$

$$\text{I}'.\qquad dA + \psi\cdot n\cdot dB + \psi\cdot \mathrm{B}\cdot\frac{\delta n}{\delta c}\cdot dc + B\cdot n\cdot\frac{\delta\psi}{\delta k}\cdot dk + B\cdot n\cdot\frac{\delta\psi}{\delta\alpha}\cdot d\alpha = 0\,.$$

In primary eclipse:

$$\text{II.}\quad l = a - b\cdot\cos\theta - \alpha(a + b_1 + b_2 - \lambda_{1r}) - l'\cdot c\cdot\cos^2\theta + bc\cos^3\theta - l'\cdot\frac{c^2}{2}\cos^4\theta$$

where

$$l' = a - \alpha(a + b_1 + b_2 - \lambda_{1r})\;;\quad \lambda_{1r} = l'_1 + b_1 + b_2\;;\quad l'_1 = \frac{\lambda_1}{\sqrt{1 - z}} + b_1\;;$$

also

$$a + b_1 + b_2 - \lambda_{1r} = a - b - \lambda_1 - l'_1\cdot c + bc - l'_1\cdot\frac{c^2}{2}\,.$$

Subscript r means rectified (but expressed in same unit as the observed intensities), and 1 refers to conjunction during total eclipse.

$$\begin{aligned}\text{II}'.\quad dl = (1-\alpha)da - (\cos\theta - \alpha)db + \alpha\cdot d\lambda_1 - (l'\cos^2\theta - l'_1\cdot\alpha)dc \\ - (a + b_1 + b_2 - \lambda_{1r})d\alpha\,.\end{aligned}$$

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[1] *Astrophysical Journal*, 35, 36, and 39.

Eliminating $d\alpha$ between equations II′ and I′, and setting for brevity

$$y = a + b_1 + b_2 - \lambda_{1r}$$

and

$$x = B \cdot n \cdot \frac{\delta\psi}{\delta\alpha}$$

we obtain for primary eclipse

$$dl = \frac{y}{x} \cdot dA + \frac{y}{x}\psi \cdot n \cdot dB + y\,\frac{\frac{\delta\psi}{\delta k}}{\frac{\delta\psi}{\delta\alpha}}\,dk + \alpha \cdot d\lambda_1 + (1-\alpha)da - (\cos\,\theta - \alpha)db$$

$$-\left(l' \cos^2\,\theta - l'_1 \cdot \alpha + \frac{y}{x} \cdot \psi \cdot n \cdot \frac{\delta n}{\delta c}\right)dc\,.$$

During secondary we have

$$\text{III.}\qquad l = a - b\,\cos\,\theta - k^2\alpha\lambda_{1r} - l'c\,\cos^2\,\theta + bc\,\cos^3\,\theta - l'\frac{c^2}{2}\,\cos^4\,\theta$$

where

$$l' = a - k^2\alpha \cdot \lambda_{1r}$$

and

$$\text{III}'.\quad dl = da - (\cos\,\theta + 2k^2\alpha)db - k^2\alpha \cdot d\lambda_1 - (l'\,\cos^2\theta - k^2\alpha l'_1)dc - 2k\alpha\lambda_{1r} \cdot dk$$

$$- k^2\lambda_{1r} \cdot d\alpha$$

and

$$dl = \frac{k^2\lambda_{1r}}{x}dA + \frac{k^2\lambda_{1r}}{x}\psi \cdot n \cdot dB + \left[k^2\lambda_{1r} \cdot \frac{\frac{\delta\psi}{\delta k}}{\frac{\delta\psi}{\delta\alpha}} - 2k\alpha\lambda_{1r}\right]dk - k^2\alpha \cdot d\lambda_1 + da$$

$$-(\cos\,\theta + 2k^2\alpha)db - \left(l'\,\cos^2\,\theta + k^2\alpha l'_1 + \frac{k^2\lambda_{1r}}{x} \cdot \psi \cdot n \cdot \frac{\delta n}{\delta c}\right)dc\,.$$

Between eclipses:

$$\text{IV.}\qquad l = a - b\,\cos\,\theta - ac\,\cos^2\,\theta + bc\,\cos^3\,\theta - a\frac{c^2}{2}\,\cos^4\,\theta$$

and

$$dl = da - \cos\,\theta \cdot db - a \cdot \cos^2\,\theta \cdot dc\,.$$

Intensities may be expressed in any unit. The average intensity between eclipses was taken as the unit. Twenty-two supernormals during primary eclipse, with the twenty already formed from the rest of the curve, gave forty-two equations for the determination of the 7 unknowns. The manner in which the normals are assembled to form the supernormals is indicated in the second column of Table II, where the current number of the supernormal containing the normal is given. Normal No. 30, which can be represented by no set of unknowns, was finally omitted. The equations were divided through by the intensities to equalize the weights. The details of the observations and the least-squares solution will be found in a forthcoming contribution from the Princeton University Observatory. The solution resulted in the following set of values:

$A = 0.02285 \pm 0.00014$
$B = 0.01898 \pm 0.00021$
$k = 0.6244 \pm 0.0139$
$\lambda_1 = 0.1569 \pm 0.0015$
$a = 1.0175 \pm 0.0037$
$b = 0.0081 \pm 0.0027$
$c = 0.0330 \pm 0.0053$

Since 4 of the unknowns, dA, dB, $d\lambda_1$, and dk, do not appear in the fourteen equations outside of the eclipses, the probable errors of these quantities should not be increased, to the full extent, by the very large residuals of asymmetry which prevail in these fourteen equations. The probable errors of these four unknowns as functions of the remaining three were determined from the twenty-five equations in which they appear. In combination with the probable errors of da, db, and dc, found from all thirty-nine equations, the probable errors of dA, dB, $d\lambda_1$, and dk were derived. The probable error of one normal point, usually comprising five observations, is ± 0.023. This is not excessively large, considering the residuals of asymmetry between eclipses. The residuals of the normals, O.−C., are given in the sixth column of Table II. This solution is also the basis of the curve drawn through the plotted observations.

The observations show strong and well-determined asymmetries and irregularities too large to be attributed to errors of observation. There is a well-defined hollow just after primary and another just after secondary. The most conspicuous asymmetry is a greater brightness from middle of primary eclipse through to secondary than on the other side. The secondary minimum gives no evidence of an eccentric orbit and the consequent possible explanation as a periastron effect. The term $0.017 \sin \theta$ takes care of the greater part of this asymmetry between eclipses. The existence of this sine term would probably mean that the advancing side of the bright star is brighter than the following.[1] Stebbins[2] finds a similar asymmetry in the light-curve of δ Orionis, only in this case both components, which do not differ greatly in brightness, are apparently brighter on the advancing sides. An asymmetry in the same sense was found persisting throughout the entire light-curve of RT Persei.[3] To gain an idea of the effect of such an explanation on the residuals of the curve of RV Ophiuchi, I have taken the coefficient as determined between eclipses and assumed that at conjunction the visible disk of the bright star is divided into two equal areas of unequal brightness. The difference in total light of the two halves is 0.015 of the total light of the two stars. The new residuals, O.–C.′, Table II, show what improvement has been made by this partial explanation of the asymmetry. The sum of the weighted squares of residuals has been reduced from 1.143222 to 0.968367, a reduction of 15 per cent. Of the remaining sum 42 per cent is found among the 38 normals during primary eclipse, where, with few changes in sign, there has been a reduction of 21 per cent. The reduction outside of primary minimum is 10 per cent. Evidently the primary minimum requires a larger correction of the same character, while the stretches between eclipses require some explanation of the hollows in the curve. Some improvement during primary minimum could be effected by supposing that the brightness of the disk diminishes steadily from one limb to the other. On the whole, there seems to be some evidence that the advancing side of the brighter component of some eclipsing variables is brighter than the following side.

The elements of the system RV Ophiuchi follow:

Maximum radius of larger star	a_f	0.2069	±0.0010
Minimum radius of larger star	b_f	0.2001	
Maximum radius of smaller star	a_b	0.1291	±0.0006
Minimum radius of smaller star	b_b	0.1249	

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[1] Announced at April 1915 meeting of American Philosophical Society; see *Proceedings*, 54, 54, 1915.

[2] *Astrophysical Journal*, 42, 142, 1915.

[3] *Contributions of Princeton Observatory*, No. 1.

Ratio of the radii of the stars	k	0.624	±0.014
Ratio of the axes of the spheroidal stars	$1+\frac{1}{2}z$	1.033	±0.005
Least apparent distance of centers	$\cos i$	0.0709	±0.007
Inclination of orbit plane	i	85°56′	±24′
Eccentricity of orbit	e	0	
Maximum fraction of light of smaller star obscured during eclipse	α_0	1	
Interradiation difference of light of sides of larger star	$2b_1$	0.0176	±0.0027
Interradiation difference of light of sides of smaller star	$2b_2$	0.0014	
Light of brighter side of smaller star	L_b	0.825	
Light of brighter side of larger star	L_f	0.175	
Ratio of surface brightness			
Of the bright sides of the two stars	$\frac{J_b}{J_f}$	12.2	
Of the sides of the fainter star		1.11	
Difference of light of advancing and following sides of brighter star		0.015	
Density of the brighter star	ρ_b	0.24	
Density of the fainter star	ρ_f	0.06	
(Assuming equal masses)			

SUMMARY

1. New elements for the occurrence of eclipses, containing a sine term of small amplitude, have been derived:

J.D. $2419808.81485+3^{d}.6871267\ E-0^{d}.0014 \sin 0^{\circ}.22\ E=1913$ Feb. $9^{d}19^{h}33^{m}.4$ G.H.M.T.$+3^{d}16^{h}29^{m}27^{s}.75\ E-2^{m} \sin 0^{\circ}.22\ E$.

2. During the total eclipse at primary minimum the star is $2^{M}.03$ fainter than at maximum brightness, while the loss during the annular secondary eclipse amounts to $0^{M}.11$.

3. The observed light-curve shows that RV Ophiuchi, a star of spectral type A, is an eclipsing system. One star emits nearly 5 times as much light as the other but its surface brightness is 12 times as great, the radius of the brighter star being but two-thirds that of the fainter star. The distance between centers is 5 times the radius of the fainter star.

4. The sides of the stars facing each other are found to be brighter than the opposite sides, the ratio being about the same as that found in the systems RT Persei and Z Draconis.

5. The stellar disks are found to be much more elliptical than was to be expected in a system where the radius of the stars is so small a fraction of the radius of the orbit.

6. Darkening toward the limb, if it exists in this case, is disguised by other phenomena. All the results contained in this paper are based on the assumption of uniform luminosity.

7. The light-curve is distinctly asymmetrical, the asymmetry persisting throughout the entire curve. This seems in part to be due to a difference in brightness of the advancing and following sides of the brighter star.

8. The theoretical curve which best represents the observations according to our present knowledge of the causes of light-variation in eclipsing systems still leaves residuals of a locally systematic character.

Princeton University Observatory
December 4, 1915

A VARIATION IN THE SOLAR ROTATION

By H. H. PLASKETT

INTRODUCTION

For a number of years the spectroscopic determination of the solar rotation was in a rather unsatisfactory condition. In order to put matters on a better basis a co-operative investigation was inaugurated at the 1910 meeting of the International Solar Union. The spectrum was divided into a number of regions, one of which was assigned to each observer. While results from all the observers have not yet been published, still those results that have appeared show very large systematic differences. Accordingly, at the Bonn meeting of the Solar Union the following resolution was adopted:

> It is highly desirable to trace to their source the systematic differences that are found in the values of the solar rotation by different observers, and this investigation should take precedence of a continuation of the program adopted in 1910. For this purpose study should be directed to determining the velocity at the solar equator by as many different methods as possible, and it is recommended that the ten lines chosen in 1910 in the region λ 4220 to λ 4280 be used for this purpose. Investigation should also be made into the personal differences that are found in the measures of the same plate by different observers.

This resolution indicates very clearly the present status of the subject. That these "systematic differences" may be ascribed to instrumental or personal errors, or even to a variation in the rate of rotation of the sun, makes the matter most puzzling.

In order to test for the source of these differences, the writer, under favorable instrumental conditions, made, during the summer of 1915, a new series of plates, which were measured in duplicate. The methods used and the results obtained will now be discussed.

SECTION I. METHODS

Since previous work of the writer had pointed in that direction, it was decided that a test should be made for a variation in the solar rotation. In order to afford the best opportunity for detecting such a variation, certain precautions, to be described in full later, were considered in order that instrumental and personal errors might be reduced to a minimum. It was early recognized that this variation might occur in three ways: (1) rapid changes in the rotation rate occurring within a day; (2) slower changes in rate extending over several weeks; (3) a secular change in the rate. In order to cover these three possibilities the following program was drawn up: At least three equatorial plates were to be taken on each fine day for one or two months, and on at least four or five days a large number of plates were to be taken at frequent intervals for the greater part of the day as a test for rapid changes within a day. The means for all the plates from each day should give a good value of the velocity for that day, and these means should make possible a test of slow changes. Finally, a mean of all the plates should give a value of the equatorial velocity comparable with that obtained by the writer in 1913, thus permitting a test for secular change.

INSTRUMENTAL CONDITIONS

The instrumental equipment was practically the same as that hitherto used at Ottawa. A concave mirror of focal length 24.5 m is fed by coelostat and a secondary flat. It forms an image of the sun 23 cm in diameter on the face of a Littrow spectrograph, the beam having first passed through a louvred tube to the basement of the observatory. The optical parts of the spectrograph are a 15 cm Brashear lens of 7 m focus and a plane grating ruled by Anderson, used in the third order. The light from the two limbs of the sun is brought by reflecting prisms to another pair of two-toothed reflecting prisms directly over the slit. Hence on the photographic plate four spectra are obtained in the order east, west, east, west.

The errors to which spectroscopic observations of the solar rotation are subject fall into two main classes: those due to causes operating outside the spectroscope and those due to instrumental conditions within the spectroscope. In the first class are those errors produced by haze and by changes of focus and astigmatism of the mirrors. In the second class the chief source of error probably lies in unequal illumination of the grating by the two limbs. Flexure and change of temperature, which are so important in determinations of stellar radial velocity, play no part in solar rotation, since both limbs are affected equally. In this investigation special precautions were taken to insure that these sources of error were nullified. A brief description of the methods used will be of interest.

Of the sources of error external to the spectroscope, that due to haziness was carefully avoided. Haze, by diffuse reflection, produces blended spectra which give rotation values that are too small. Observations were never made except on days that were free from haze.

The errors due to astigmatism and change of focus were more serious. Within a few days after the mirrors had been freshly silvered it was found that they became unevenly heated. As a consequence, the image became astigmatic while the focus would lengthen abnormally. In order to reduce these difficulties an electric fan was installed which played upon the coelostat mirror. This was found to give very satisfactory results, not only as regards astigmatism, but also in keeping the focus constant.

When the fan was installed and it was seen that the focus remained constant it was decided to determine the value of R, the diameter of the sun in millimeters, by two independent methods: (1) to measure R directly at the proper focal point; (2) to determine the focal length of the mirror for each exposure, and, from the semidiameter of the sun in seconds for that day, to compute R in millimeters. The two values of R thus obtained were found to be in close agreement over the entire series of observations, the mean numerical residual being of the order of half a millimeter, or considerably less than one-fourth of 1 per cent.

As was mentioned before, the only important source of error internal to the spectroscope is that due to uneven illumination of the grating from the two limbs of the sun. Of course, if the emulsion of the photographic plate could be kept exactly at the focus of the camera lens the spectra from the two limbs would have no relative displacement, but it seems almost impossible to do this. Owing probably to uneven heating of the cells holding the prisms it is quite possible for the illumination to change during the course of an exposure and still no change be noticeable when the illumination is tested. It is this source of error, more, probably, than any other, which tends to vitiate the results in spectroscopic determinations of the solar rotation.

The writer had early decided that the best plan was to use a region of the spectrum where there were sufficient telluric lines to act as a check. It is evident that if the spectra from the two limbs are displaced relatively to each other the telluric lines will show a like shift. Thus, by measuring both the solar and telluric lines and applying the proper signs the algebraic sum of their means should give the true radial velocity. In order to carry out this plan it was decided to take the plates in the region of λ 5900, where there are a number of very well defined water-vapor lines.

Trouble was early experienced in securing plates sensitive to the region. Several well-known brands of commercial panchromatic plates were tried, and, though fairly satisfactory results were secured as regards grain and contrast, yet the exposure time was excessive (something over twenty minutes). Accordingly, the writer made some experiments with stained plates. The usual pinachrome solution[1] was tried on Seed Contrast Process plates. The result was in every way more satisfactory, though the exposure was still too long (fifteen minutes). Some Cramer Iso Process plates were then stained. Most satisfactory results as regards both grain and contrast were secured, while at the same time the exposure was cut down to two minutes. Accordingly, with the exception of the first three or four, which are stained Seed Contrast Process, the plates used in this investigation were Cramer Iso Process stained with pinachrome.

Having in this manner secured a check on the chief instrumental sources of error, the program previously outlined was carried out. On every fine day, the sky being free from haze, from three to four plates were taken at the equator. The plates were taken early in the morning, between five and eight o'clock, in order that the telluric lines might be at their greatest strength. By a plate is meant here, and in subsequent writing, one exposure showing four spectra, two from each limb. Thus a number of exposures may be secured on one plate, but for convenience in discussion each one of these exposures will be termed a plate. In this manner 111 plates were secured on 25 days, an average of a little over four plates per day.

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[1] *British Journal and Photographic Almanac*, p. 595, 1915.

MEASUREMENT AND REDUCTION OF PLATES

Owing to the large number of plates to be measured, and to the fact that they were to be measured in duplicate for a reason to be explained later, it was decided to measure the displacements of only five solar and three telluric lines. From earlier work it was evident that a good value of the rotation would be given by five lines. In Table I will be found the wave-lengths, the elements to which the lines are due, and the intensities of these eight lines taken from Rowland's tables. The value of one millimeter in angstrom units was obtained from the mean of five measures of the spectrum of the sun's center. The values were smoothed out by a curve. In the last column of the table will be found the velocity constant. This when multiplied by the displacement in millimeters gives the radial velocity due to the rotation of the sun.

TABLE I

LINES IN THE REGION OF λ 5900

No.	λ	Element	Int.	1 mm in Angstrom Units	Vel. Const.
1.	5857.674	Ca	8	0.69189	17.709
2.	5862.582	Fe	6	.69146	17.683
3.	5866.675	Ti	3	.69112	17.662
4.	5886.193	A(wv)	5	.68944	17.561
5.	5916.475	Fe	3	.68685	17.406
6.	5919.860	A(wv)	7	.68655	17.388
7.	5932.306	A(wv)	5	.68547	17.324
8.	5934.881	Fe	5	0.68526	17.311

The relative displacements of the lines in the spectra of the two limbs were measured. One setting was made in each strip of spectrum, so that two settings were made on the line from each limb. The plate was then reversed and this procedure repeated. Thus eight settings in all were made on each line on each plate.

Telluric displacements were counted positive when they were in the opposite direction to the solar displacements. Hence the algebraic sum of the mean solar and telluric velocities should give the velocity free from instrumental error. The error involved in multiplying the constant telluric displacement, if it exists, by a "velocity constant" varying with the wave-length is of course negligible.

From the writer's previous experience with the possibility of variation[1] in the personal equation, it was decided to measure the plates in duplicate. No plates were measured until all the plates were taken. They were then mixed at random and divided into two bundles. One bundle was measured on the Repsold machine with an eyepiece micrometer and the other on a Toepfer machine with a 300 mm screw. As a plate was measured on one machine it was placed at the bottom of the bundle for the other machine. As a result the maximum time elapsed between two measures of the same plate. As no reduction was made until the measuring was completed, it was impossible for any prepossession to enter the result. Furthermore, since the telluric lines were measured at the same time as the solar, it seems not unlikely that if, for example, the solar displacements were too large, the telluric lines would be measured negatively, and hence an automatic correction would be carried along. The final result thus ought to be free from personal equation as well as instrumental error.

The preliminary reduction was carried out as follows: The mean solar velocity was secured from the five solar lines, and likewise the mean telluric velocity with its proper sign was obtained. Their algebraic sum gave the preliminary v for each plate. As each plate was measured twice, it was the mean of the two measures that was used in the final reduction to the true sidereal rate.

SECTION 2. RESULTS

The unreduced values of the velocity for the 111 plates are given in Table II. In the first column will be found the date G.M.T. at which each plate was taken. In the second column is given the difference between the two measures of the plate, viz., that made on the Repsold machine and that made on the Toepfer. Finally, the means of the two measures are given in the third column.

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[1] "The Solar Rotation in 1913," *Journal R.A.S. of Canada*, **7**, 307, 1914; "The Psychology of Differential Measurements," *ibid.*, **9**, May 1916.

TABLE II

MEASURES

Date G.M.T. 1915	I–II	*v*	Date G.M.T. 1915	I–II	*v*	Date G.M.T. 1915	I–II	*v*
	km	km		km	km		km	km
$171^{\mathrm{d}}994$	−0.015	1.745	$192^{\mathrm{d}}991$	−0.049	1.813	$210^{\mathrm{d}}023$	+0.001	1.820
172.008	+ .116	1.819	.993	+ .023	1.817	.061	− .148	1.760
.022	− .051	1.699	.999	+ .130	1.912	.063	+ .012	1.859
.036	− .052	1.723	193.002	+ .073	1.820	.065	+ .078	1.851
174.128	− .097	1.710	194.994	+ .104	1.879	211.008	− .040	1.718
.146	+ .017	1.725	.999	− .061	1.838	.010	+ .097	1.733
.163	− .062	1.736	195.004	− .029	1.819	.012	− .016	1.715
175.066	− .008	1.658	.008	+ .030	1.846	222.073	+ .062	1.794
.083	+ .051	1.700	200.975	− .126	1.839	.075	+ .044	1.799
176.028	− .005	1.787	.988	+ .027	1.869	.078	− .043	1.800
.060	+ .060	1.702	.993	+ .115	1.838	.080	+ .196	1.755
.079	− .017	1.735	201.045	− .125	1.892	.105	− .056	1.800
177.057	+ .091	1.645	.987	+ .027	1.843	.109	+ .140	1.742
.076	+ .087	1.754	.989	+ .031	1.778	.111	+ .123	1.797
179.044	+ .075	1.671	.997	+ .082	1.799	.132	− .074	1.789
.064	− .007	1.738	202.000	− .039	1.709	.134	+ .084	1.885
.083	+ .039	1.714	204.043	− .103	1.871	.137	+ .022	1.757
182.036	+ .063	1.709	.046	+ .033	1.874	223.156	+ .097	1.805
.042	+ .024	1.735	.049	+ .033	1.774	.157	+ .091	1.852
.051	+ .087	1.734	.051	+ .038	1.842	.160	+ .021	1.846
.058	+ .002	1.660	205.048	− .008	1.749	226.003	+ .005	1.843
184.960	+ .063	1.760	.050	+ .030	1.754	.006	+ .103	1.822
.967	+ .009	1.705	.053	+ .070	1.805	228.070	+ .079	1.824
.974	+ .039	1.765	.055	− .008	1.821	.072	+ .061	1.841
.985	+ .050	1.704	207.966	− .030	1.765	.074	− .126	1.778
190.045	− .024	1.729	.968	− .046	1.884	.106	+ .064	1.824
.049	+ .141	1.775	.971	+ .062	1.880	.108	+ .020	1.789
.052	+ .003	1.814	208.010	+ .009	1.790	.110	− .020	1.833
.056	+ .100	1.699	.013	− .019	1.828	.133	− .067	1.851
190.965	+ .044	1.850	.979	+ .024	1.800	.137	− .032	1.738
.968	− .029	1.851	.983	+ .035	1.788	.139	+ .075	1.782
.973	− .015	1.796	.988	− .013	1.800	.168	+ .011	1.823
.976	+ .036	1.873	209.977	+ .098	1.721	.170	+ .143	1.836
192.001	− .017	1.822	.979	+ .012	1.731	.172	+ .010	1.786
.005	+ .010	1.834	.982	+ .051	1.816	.196	+ .113	1.892
.009	− .102	1.831	210.019	− .146	1.801	.198	− .001	1.773
.013	−0.010	1.782	.021	+0.027	1.861	.201	−0.045	1.762

ERRORS OF MEASUREMENT

It will be of interest to compare the two series of measures. Since the measures are made of the same plates, it is evident that they should agree within the limits of accidental error. Considering the residuals as a whole, it is seen that they are in fair agreement. The algebraic mean of I–II is +0.020 km. That is, there is a constant difference between the two series which must be ascribed to a difference in personal equation of the writer with the two machines. Eliminating this constant difference and taking the numerical mean of the residuals thus left, the mean becomes 0.021 km. This number, 0.021 km, may be taken as a measure of the accidental differences between the two series, and it shows that the two series are in satisfactorily close agreement.

However, it will be noticed that there are several very large residuals (greater than a tenth of a kilometer) in the second column. Allowing for the constant difference, there are 14 residuals out of 111, or 13 per cent of the residuals are greater than a tenth of a kilometer. At first sight it is difficult to see how such large differences could have arisen in measuring the same plate. The writer is of the opinion, however, that it is due very largely to the measures of the small telluric displacements. An examination of the measures shows that in 11 cases out of the 14 it was the telluric velocity that had changed its value by more than a tenth of a kilometer, while the solar velocity had remained the same in the two series. A complete discussion of the whole matter will be found in a recent article[1] by the writer, in which he shows that the habit of measurement is subject to very rapid changes in measuring these smaller displacements.

So much for a comparison of the two series of measures. When the difficulties in the measurement of the telluric displacements are taken into consideration it is readily seen that the two series are in satisfactorily close agreement. Accordingly, in obtaining the final results the mean of the two measures for each plate was used. This mean is given in the third column of Table II.

Again, it is readily seen that while the plates for each day give a fairly homogeneous mean, yet there are some large discrepancies. It is interesting to note that these large discrepancies are accompanied by correspondingly large residuals, showing that they are due to errors of measurement. However, on the whole the results for each day are fairly homogeneous. Taking the mean for each day and then the probable error of that mean, it is found that the mean of the probable errors is +0.012, a satisfactorily small quantity.

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[1] "The Psychology of Differential Measurements," *op. cit.*

VARIATION IN THE SOLAR ROTATION

After this preliminary discussion it is now possible to apply the test for a variation in the rate of rotation. As previously mentioned, this variation may take three forms: (1) rapid changes within a day; (2) slower changes, extending over several weeks; and (3) a secular change since 1913.

Variation within a day.—Unfortunately, owing to the unexpectedly large errors of measurement, it is impossible to give a definite answer to this question. However, large as these errors are, it may be safely said that were there a range of values within the day as great as a tenth of a kilometer it would be readily evident.

Variation over several weeks.—In order the better to test for this variation Table III was formed. Columns one, three, and four

TABLE III

SUMMARY OF RESULTS

Date G.M.T. 1915	No. Plates	I–II	v	V	P.E.	ξ
		km	km	km	km	
172$^{\text{d}}$02	4	−0.001	1.746	1.911	±0.018	13.57
174.15	3	− .047	1.724	1.885	.005	13.38
175.07	2	+ .022	1.679	1.846	.015	13.11
176.06	3	+ .013	1.741	1.914	.016	13.59
177.07	2	+ .089	1.700	1.866	.038	13.25
179.06	3	+ .036	1.708	1.875	.014	13.31
182.05	4	+ .043	1.709	1.871	.012	13.28
184.97	4	+ .041	1.734	1.893	.012	13.44
190.05	4	+ .055	1.754	1.919	.017	13.62
190.97	4	+ .009	1.843	2.009	.011	14.26
192.01	4	− .030	1.817	1.975	.008	14.02
193.00	4	+ .044	1.840	2.000	.016	14.20
195.00	4	+ .011	1.845	2.007	.008	14.25
201.00	4	− .027	1.860	2.026	.008	14.38
201.99	4	+ .025	1.782	1.943	.019	13.80
204.05	4	+ .000	1.840	2.003	.016	14.22
205.05	4	+ .021	1.782	1.951	.013	13.85
207.99	5	− .005	1.829	1.996	.016	14.17
208.98	3	+ .022	1.796	1.954	.003	13.87
210.02	9	− .002	1.802	1.964	.013	13.94
211.01	3	+ .014	1.722	1.885	.004	13.38
222.10	10	+ .050	1.792	1.966	.008	13.96
223.16	3	+ .070	1.834	2.007	.010	14.25
226.00	2	+ .053	1.832	2.006	.007	14.24
228.14	15	+0.019	1.809	1.977	0.007	14.04

are the same as in Table II except that they give the mean for each day. The number of plates used in forming this mean are given in the second column. In the fifth column is given the final mean reduced velocity for each day. The probable error of this mean is given in the next column. For completeness the probable error is given for each day, though, when only two plates are used in forming the mean, it has not much significance. In the last column is given the angular velocity corresponding to *V*.

Using the dates in this table as abcissas and the final reduced values, *V*, as ordinates, Fig. 1 has been constructed. The value of the velocity for each day has been represented by a circle. Owing to rainy weather there are, unfortunately, no observations in the interval 211–222 days. Notwithstanding this, the figure shows quite clearly a cyclic variation with a range of 0.15 km. From a velocity of 1.91 km on the 172d day the rate drops to 1.87 km on the 177th day. It then rises slowly to a maximum of 2.00 km, about which it hovers for nearly two weeks. It then drops quickly to a minimum about the 211th day. At this point rainy weather intervened, and no more observations were secured until the velocity was once more a maximum.

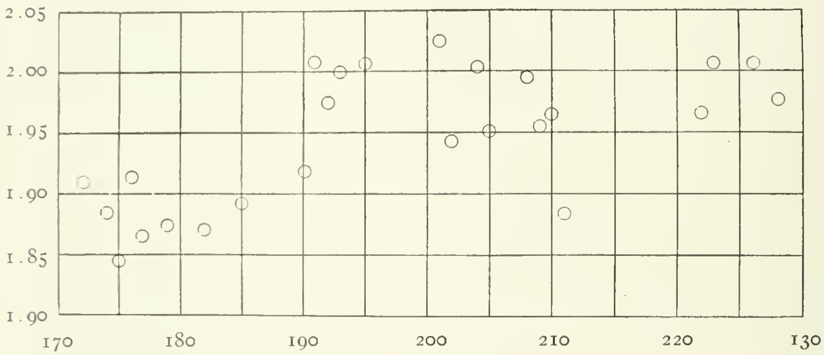

Fig. 1

Owing to the short interval covered by the observations it is impossible to draw any conclusions as to the periodicity of this phenomenon. The most that can be said is that during the summer of 1915 the rate varies cyclically in about a month. In order to show this cyclic variation better the writer has grouped the observations in the interval 172–211 days into 14 normal places. The results are shown graphically in Fig. 2.

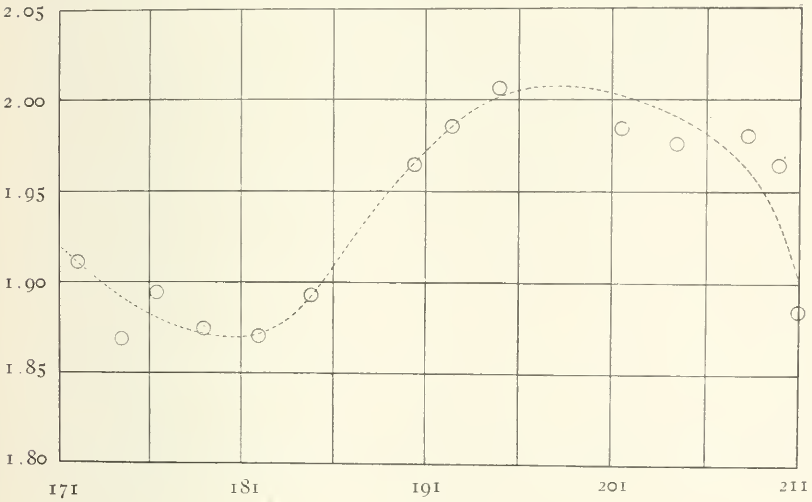

FIG. 2

Secular variation.—The mean equatorial value of the solar rotation during the time the sun was under observation was obtained by taking a simple mean of the values of V in Table III. This gives

$$V = 1.949 \text{ km per sec.}$$
$$\xi = 13\overset{\circ}{.}83 \text{ per day}$$

Comparing these with the values obtained by the writer in 1913, viz.:

$$V = 1.991 \text{ km per sec.}$$
$$\xi = 14\overset{\circ}{.}14 \text{ per day}$$

it is evident that there has been a real change in the interim.

SECTION 3. DISCUSSION OF RESULTS

Before proceeding with this discussion it is of interest, in view of what follows, to know the effective level at which the observations were made. The effective level of the lines, as taken from the results of Mitchell's observation of the flash spectrum, is about 400 km.

CYCLIC VARIATION

In the previous section it was shown that the sun, during the summer of 1915, underwent a cyclic variation in its rotation rate with a range of 0.15 km. This variation was completed in about a month; nothing could be said as to the periodicity of this change.

At first sight it seems strange that a variation of this magnitude, involving a change of nearly 2.4 days in the rotation period, should have so long escaped detection. Second thought shows, however, that it is not so strange as it appears. Until recently observations of sun-spots were the only means of determining the rotation period of the sun. Observations extending over two or three sun-spot cycles have been made by Carrington, Spoerer, and Maunder. In a recent discussion of the Greenwich sun-spot records, Maunder says: "The rotation periods given by different spots in the same zone of latitude differ more widely than do the mean rotation periods for different zones of latitude. It has been forgotten that whatever the cause which produces the variation of rotation rate with latitude, the causes producing differences of rate within any given latitude are more effective still."[1] With such variable standards it would be almost impossible to detect a cyclic change of this magnitude, particularly if the interval required to complete a cycle coincided closely with the rotation period of the sun. The same conclusions may be applied with even more point to determinations of the rotation with either faculae or flocculi.

These objections have no force, however, in connection with spectroscopic determinations of the solar rotation period, and it seems strange that this cyclic variation has not been detected before. Strangely enough, previous observations have shown nearly the same range as that obtained by the writer, but unfortunately the observations have not been properly grouped to show a variation. As a rule, a few plates have been taken spread over long intervals, with rarely more than one plate exposed on a day. As a consequence accidental errors tend to cloak any real variation that might exist. Fortunately the plates taken at Ottawa in 1912[2] form an exception, since they are so grouped as to make a test possible. In

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[1] *Monthly Notices*, **65**, 823, 1905.

[2] *Astrophysical Journal*, **42**, 373, 1915.

Table IV will be found the date G.M.T., the number of plates used in forming the mean, the mean reduced velocity, and its probable error. These results are shown graphically in Fig. 3. Evidence of a variation of the same type as that in Fig. 2 is shown here. The λ 4250 results also show a variation of this character which can readily be seen by consulting Table IV of the original paper. In this case the values fall to a minimum at the end of the series of plates.

TABLE IV

λ 5600, 1912, J. S. PLASKETT

Date G.M.T. 1912	No. Plates	V	P.E.
		km	
158ᵈ20	1	2.055	....
159.20	3	2.002	+0.009
161.15	3	1.992	.014
166.27	4	2.039	.003
167.18	1	1.969	....
170.21	3	1.984	.008
179.29	7	2.011	.005
180.12	3	2.054	.002

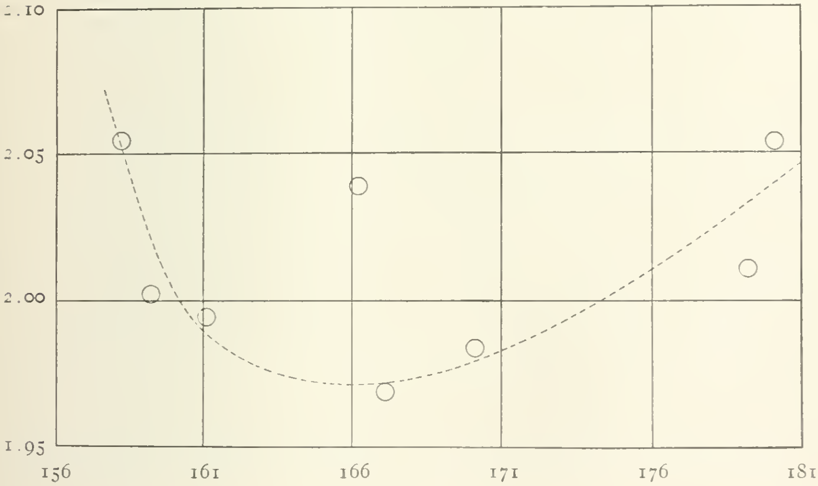

FIG. 3

It is unfortunate that previous work does not throw more light upon this cyclic variation. Observations will have to be extended over several years before data can be gathered sufficient to explain this phenomenon. It is of interest, however, and also important to realize that a physical explanation can be found. Variation in the rotation of the sun, assuming its moment of momentum constant, can be caused by oscillations about its mean form. Moulton[1] has treated these oscillations fully and has shown, in the case of the sun, that the shrinkage of even a second of arc in the diameter will cause a change in diurnal rate of only 0°02. One-tenth of this shrinkage would cause the sun to change in brightness by a whole magnitude, and hence it is evident that the cyclic variation is not caused by oscillations. Emden's solar theory[2] calls for a "velocity of rotation variable in both time and space." The homogeneous layers, each with its own angular momentum and potential temperature, assumed by him to account for the heat exchange between center and surface, explain at the same time the equatorial acceleration. It is evident, if this theory be true, that at any given level the velocity should rise slowly to a maximum and fall abruptly to a minimum. The interval of time covered by this change will depend upon the thickness of the layer. For, consider a warm layer at the given level. As it cools by external radiation it contracts, and, as it contracts, according to Emden, it accelerates. This acceleration will continue until this layer has sunk below the given level (400 km), and a new warm layer with a low angular velocity has appeared. This is the type of variation shown in Fig. 2: a slow rise to maximum and a sudden fall to minimum. This simple process will of course be greatly complicated by exchange of heat between adjacent layers. Sufficient has been said, however, to show, provided Emden's theory be correct, that the phenomenon admits of a physical explanation, and also to indicate that its final elucidation is largely a question of sufficient data.

SECULAR VARIATION

A change in velocity of this nature has long been suspected. Thus Halm thought he had evidence of it in his observations at

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[1] *Astrophysical Journal*, **29**, 257, 1909.

[2] *Ibid.*, **15**, 38, 1902.

Edinburgh. However, as evidence accumulated it began very generally to be felt by astronomers that the evidence in favor of a secular change was inconclusive.

This view is of course greatly strengthened by the fact that observations of the solar rotation from sun-spots show no evidence of secular variation. To quote Maunder again: "There is no clear evidence of anything like a systematic change in the rotation period during the progress of any particular cycle, and very little evidence, if any, of a change from one cycle to another."[1] Evidence such as this may be taken as conclusive as regards a secular change in rate at the sun-spot level. For, though with regard to cyclic changes, sun-spots, because of their proper motions, can furnish no definite evidence, yet the mean over a year should give a good value of the rate for that year.

As a consequence it must be concluded that this secular change is confined to the reversing layer. It is of interest to note that the low value obtained by the writer in this paper is confirmed in a recently published abstract[2] from Mount Wilson. The velocity obtained at that institution during the years 1914 and 1915 is

$$V = 1.949 \text{ km per sec.}$$

Though previous observations are not strictly comparable on account of the possibility of instrumental and personal differences, yet it appears that since 1906 the rotation value has been slowly falling from 2.050 km to 1.950 km.

Here again elucidation of this secular change must await the gathering of more material, as even less is known of the secular change than of the cyclic variation. Observations ought to be continued by all institutions that have been engaged in this work, and measurements, wherever possible, ought to be made by the same observer. In this manner a large quantity of data free from systematic differences could readily be gathered.

CONCLUSIONS

1. A refinement of instrumental conditions led to results practically free from instrumental error.

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[1] *Op. cit.*

[2] *Popular Astronomy*, **23**, 641, 1915.

2. Measures of each plate were made in duplicate. On account of the difficulty of measurement of the small telluric displacements there were a few large accidental errors.

3. The results were inconclusive as regards variation within a day, if the range be less than one-tenth of a kilometer. There was no evidence of a variation with a range greater than this.

4. There was good evidence of a cyclic variation extending over a month with a range of 0.15 km. Too short an interval was covered by the observations to make possible any statement as to periodicity.

5. The mean of all the measures showed that the rotation in 1915 was 0.04 km lower than it was in 1913.

6. These results were in general confirmed by a discussion of previous spectroscopic determinations of the solar rotation.

In conclusion the writer would like to acknowledge the kindly interest shown and the encouragement afforded by the director, Dr. W. F. King, during the progress of this investigation.

Dominion Observatory, Ottawa
December 1915

MINOR CONTRIBUTIONS AND NOTES

ON THE POLE-EFFECT IN A CALCIUM ARC

In the course of an investigation of the pressure-shift of the lines in the arc spectrum of calcium, which is in progress, it seemed desirable to make some observations on the pole-effect of the triplets of the first and second subordinate series.

A horizontal arc, 4 mm long and carrying 4 amperes on a 110-volt circuit was used, with electrodes of metallic calcium, varying in diameter from 7 to 10 mm.

The image of the arc was projected with a magnification of about 3 upon the slit of a 21-ft. concave grating spectrograph, and the spectrum in the first order from $\lambda 3000$ to $\lambda 4200$ was photographed on a single plate. This region includes, according to Kayser's notation, IIN4, IN5, IIN5, and IN6, to which our measures have been confined. The scale of the plates is 2.64 A per mm.

Table I presents the data for the metallic arc between calcium poles:

TABLE I

POLE-EFFECT AT 1 ATMOSPHERE. CALCIUM ELECTRODES

Positive minus Center

	IIN4	IN5	IIN5	IN6
Mean........	+0.008 mm (28)	−0.009 mm (21)	+0.014 mm (7)	−0.011 mm (6)

Negative minus Center

	IIN4	IN5	IIN5	IN6
Mean........	−0.002 mm (26)	+0.004 mm (27)	−0.004 mm (7)	+0.009 mm (7)

The numbers in parentheses indicate the number of measures averaged.

The change of sign with the series is very striking, and apparently a continuous change in wave-length is indicated from one pole to the other. From a qualitative standpoint, the conclusions, we think, are quite definite, but on account of the unsteady conditions in the arc between calcium electrodes no great weight can be attached to the numerical values.

It is noteworthy that in the case of the calcium arc the shift in going from the center to the positive pole is opposite to that obtained in going from the center to the negative pole. As has been observed by Goos[1] and by St. John and Babcock,[2] this is not the case with the lines of the *d* and *e* groups in the spectrum of iron, where the shift in going from the center of the arc to the positive pole is in the same direction, though of a smaller amount, as that found in going from the center of the arc to the negative pole. St. John and Babcock state that a longer exposure time is required

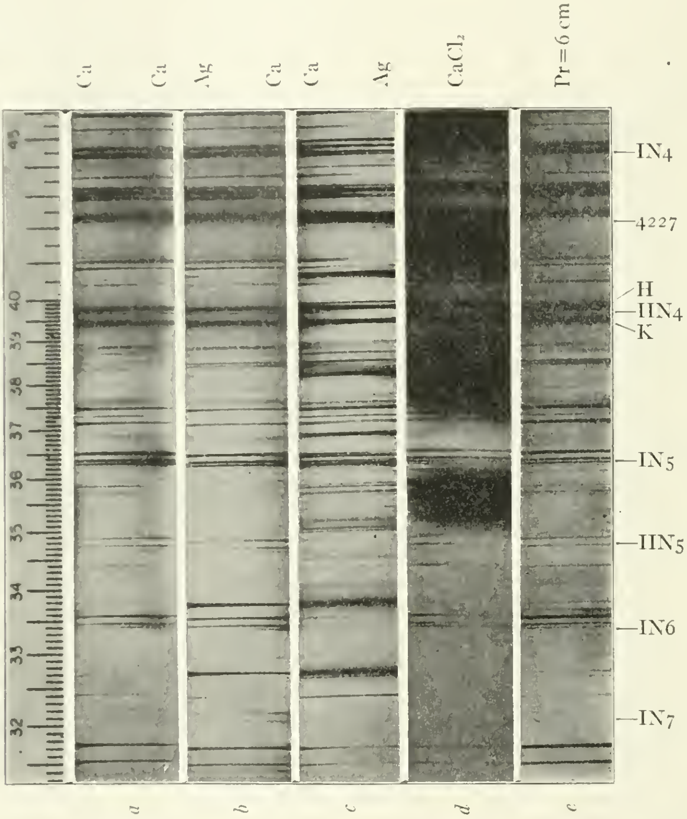

FIG. 1

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[1] *Astrophysical Journal*, 38, 141, 1913.

[2] *Ibid.*, 42, 231, 1915.

at the center of the arc than at either pole in order to secure lines of the same strength. We had noted that a longer exposure was required at the negative pole than at the center of the arc, and a longer exposure at the center than at the positive pole, in the case of the subordinate series lines of calcium. We therefore projected the image of our calcium arc on the slit of a quartz spectrograph. When both poles were calcium, we found that the lines of the first and second subordinate series were strongest at the positive pole and diminished continuously in intensity from the positive pole to the negative, as shown in Fig. 1*a*.

Since the wave-length appeared to depend upon the intensity of the part of the line from which the light is taken, it seemed desirable to vary, if possible, the intensity-gradients across the arc and examine the corresponding pole-effects. With the positive pole calcium, and the negative pole silver, Fig. 1*b*, the intensity-gradient for the lines under investigation was the same as when both poles were calcium; but with clean Ag+ and Ca−, the gradient was reversed (Fig. 1*c*), and the pole-effect was also reversed, as shown in Table II.

TABLE II

POLE-EFFECT AND REVERSED INTENSITY-GRADIENT

Silver+ and Calcium−

Negative minus Center

IIN4	IN5	IIN5	IN6
+0.008 mm	−0.011 mm	+0.017 mm	− 0.015 mm
+0.006	−0.012	....	....

When ordinary carbon rods were charged with calcium by immersing them in solutions of $CaCl_2$, the intensity-gradients were as shown in Fig. 1*d*, i.e., strongest at the negative pole and diminishing to the positive pole. This is opposite to the intensity-gradient when both poles are metallic calcium; and the pole-effect is also opposite, as may be seen in Table III.

St. John and Babcock found that the pole-effect disappeared *in vacuo*. We have also found that with calcium electrodes at a pressure of about 6 cm, the intensity-gradients are as shown in Fig. 1*e*. It is not easy to secure consistent measurements of the pole-effect with electrodes of metallic calcium either *in vacuo* or at atmospheric pressure, since the arc does not burn steadily, but the general conclusion seems to be that the pole-effect in this case is zero or very small, as shown in Table IV.

TABLE III

REVERSAL OF POLE-EFFECT WITH INTENSITY-GRADIENT

Vapor-Density and Pole-Effect. Carbon Poles Charged with $CaCl_2$. Density-Ratio = 20 to 1

Negative minus Center

	IIN4	IN5	IN6
Denser vapor	+0.005 mm	−0.007 mm	−0.019 mm
	+0.004	−0.008	−0.021
	+0.003	−0.007	....
	+0.004	−0.005	....
Mean	+0.004	−0.007	−0.020
Rarer vapor	+0.004	−0.006	....
	+0.003	−0.005	....
Mean	+0.004	−0.006	....

The employment of carbon electrodes charged with $CaCl_2$ in solutions of different concentrations presents an opportunity of demonstrating the effect of vapor-density on the pole-effect, and also upon the absolute wave-length of these lines. The measures of Table III taken between negative and center with arcs whose relative vapor-densities, as judged from exposure-times, were

TABLE IV

POLE-EFFECT *in vacuo*. CALCIUM ELECTRODES

Positive minus Center

IIN4	IN5	IIN5	IN6
0.000 mm	−0.002 mm	−0.001 mm	0.000 mm
+0.002	+0.005	+0.001	+0.001
+0.001	−0.002	....	−0.002
−0.002	−0.004	....	−0.006
+0.005	+0.001	....	−0.003
+0.001	−0.002	....	0.000
0.000	....	....	....
Mean.. +0.0008	−0.0003	0.000	−0.0016

about 20 to 1 would seem to indicate that the magnitude of the pole-effect is independent of the density of the radiating ions. The effect of vapor-density on absolute wave-length was investigated by comparing the same part of carbon arcs charged with different amounts of $CaCl_2$, and from Table V it would seem that the absolute wave-length is unchanged by vapor-densities which differ by as much as 10 to 1. A similar conclusion was reached by St. John and Babcock regarding wave-lengths in the furnace spectrum of iron. The series lines of calcium here considered are well suited to this investigation since they are extremely sensitive to pole-effect and pressure-shift, and in these phenomena the lines of the first and second subordinate series show displacements of opposite sign.

TABLE V

WAVE-LENGTH AND VAPOR-DENSITY

Carbon Poles Charged with $CaCl_2$.

Density-Ratio = 10 to 1

High Density minus Low Density

IIN4	IN5
+0.001 mm	+0.002 mm
−0.001	+0.002
+0.001	−0.002
−0.001	−0.001
0.000	0.000
Mean.. 0.000	0.000

TABLE VI

POLE-EFFECT AND PRESSURE-SHIFT

	IIN4	IN5	IIN5	IN6
Positive minus negative....	+0.010 mm	−0.013 mm	+0.018 mm	−0.020 mm
Pressure-shift×1.7........	+0.010	−0.012	+0.017	−0.022

We find, by a comparison with a preliminary determination of the pressure-shifts for these lines (Table VI) that there probably exists a parallel relationship between the two phenomena, as St. John and Babcock also observed for the iron arc. The displacements in both cases are in the direction of the unsymmetrical widening with increased intensity and pressure.

The conclusion indicated by the measures cited is that the pole-effect for the lines of the subordinate series depends upon the intensity-gradient from pole to pole across the arc, i.e., when light taken from any part of a line is compared with that taken from a stronger part of the same line, there is a difference in wave-length similar to that caused by an increase in pressure: the wave-length at the stronger part of the line corresponds to the wave-length at the greater pressure.

The fact that we obtained the same value for the wave-length with a small or a much larger amount of $CaCl_2$ present, the ratio of exposure-times being about 1 to 10, whereas the exposure-ratios between center and positive pole were about 1 to 2 or 3, indicates that mere density of the radiating ions is probably not the underlying cause of the pole-effect.

It therefore occurs to us that possibly the amplitude of vibration of the electrons is the factor which disturbs the wave-length. On this hypothesis, lines which are shifted to the red by pressure are increased in wave-length and lines that are shifted to the violet are decreased in wave-length with increasing amplitude of vibration. The well-known facts that increase of current causes changes in wave-length similar to pole-effect changes, and that the pole-effect is greater at the negative than at the positive pole in the iron arc, in which the negative pole is brighter although the positive pole is hotter, are consistent with this hypothesis. There may also be significance in the fact that the arc increases in intrinsic brightness with increase of pressure, which suggests the possibility that pressure-shift may also be associated with the amplitude of vibration of the electrons, since pressure-shift, for symmetrical lines at least, is independent of vapor-density. It will be recalled also, in this connection, that the pressure-shift of Ti lines has been found[1] to be 40 or 50 per cent higher in the Ti spark than in the arc.

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RYERSON PHYSICAL LABORATORY
January 1916

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[1] *Astrophysical Journal*, **35**, 10, 1912.

CORRECTIONS TO THE MEASURED VELOCITY OF A SPECTROSCOPIC BINARY, DEPENDING UPON THE LENGTHS OF EXPOSURE OF THE SPECTROGRAMS

If a spectroscopic binary is faint and its period of oscillation is short, it may happen that the length of exposure necessary to secure a measurable spectrogram will amount to a considerable fraction of the period. This case has arisen a number of times and is likely to occur more frequently in the future. Thus the binary 12 Lacertae has recently been shown by Dr. Young of Ottawa to have a period of less than five hours, and some of the spectrograms from which this result was deduced were exposed for more than half the period. This star is also being observed at Allegheny by Mr. Daniel, who confirms the short period. Some of the Allegheny spectrograms were exposed over two hours.

It is obvious that velocities derived under such circumstances need a correction depending upon the length of exposure. Let u_1 be the phase-angle at the beginning of the exposure and u_2 that at the end. Then in a circular orbit the true velocity at the mid-time of exposure is

$$K \cos \frac{u_2+u_1}{2}. \tag{1}$$

Here and in what follows we count the velocity from that of the center of mass of the system, and we count u, as usual, from the ascending node. The measured velocity from this same spectrogram, aside from accidental errors, will be the mean velocity during the interval u_1 to u_2; that is

$$\frac{K}{u_2-u_1}\int_{u_1}^{u_2} \cos u \, du ;$$

or

$$K \cos \frac{u_2+u_1}{2} \cdot \sin \frac{u_2-u_1}{2} \cdot \frac{2}{u_2-u_1}. \tag{2}$$

Comparing this with (1) we see that the measured velocity needs the correction

$$K \cos \frac{u_2+u_1}{2}\left(1-\frac{2}{u_2-u_1} \sin \frac{u_2-u_1}{2}\right). \tag{3}$$

The numerical values for this coefficient of K are given in Table I. The vertical argument is the mean phase, and the horizontal argument is the length of exposure expressed in fractions of the period of oscillation, and also in degrees of arc.

TABLE I

VALUES OF $\cos\frac{u_2+u_1}{2}\left(1-\frac{2\sin\frac{u_2-u_1}{2}}{u_2-u_1}\right)$

$u_2-u_1=$	0° 0.00	18° 0.05	36° 0.10	54° 0.15	72° 0.20	90° 0.25	108° 0.30	126° 0.35	144° 0.40	162° 0.45	180° $=u_2-u_1$ 0.50	
$\frac{u_2+u_1}{2}$												$\frac{u_2+u_1}{2}$
0°.....	0.00	0.00	+0.02	+0.04	+0.06	+0.10	+0.14	+0.19	+0.24	+0.30	+0.36	360°
10.....	.00	.00	+ .02	+ .04	+ .06	+ .10	+ .14	+ .19	+ .24	+ .30	+ .36	350
20.....	.00	.00	+ .02	+ .03	+ .06	+ .09	+ .13	+ .18	+ .23	+ .28	+ .34	340
30.....	.00	.00	+ .01	+ .03	+ .06	+ .09	+ .12	+ .16	+ .21	+ .26	+ .31	330
40.....	.00	.00	+ .01	+ .03	+ .05	+ .08	+ .11	+ .15	+ .19	+ .23	+ .28	320
50.....	.00	.00	+ .01	+ .02	+ .04	+ .06	+ .09	+ .12	+ .16	+ .19	+ .23	310
60.....	.00	.00	+ .01	+ .02	+ .03	+ .05	+ .07	+ .09	+ .12	+ .15	+ .18	300
70.....	.00	.00	+ .01	+ .01	+ .02	+ .03	+ .05	+ .06	+ .08	+ .10	+ .12	290
80.....	.00	.00	.00	+ .01	+ .01	+ .02	+ .02	+ .03	+ .04	+ .05	+ .06	280
90.....	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	270
100.....	.00	.00	.00	− .01	− .01	− .02	− .02	− .03	− .04	− .05	− .06	260
110.....	.00	.00	− .01	− .01	− .02	− .03	− .05	− .06	− .08	− .10	− .12	250
120.....	.00	.00	− .01	− .02	− .03	− .05	− .07	− .09	− .12	− .15	− .18	240
130.....	.00	.00	− .01	− .02	− .04	− .06	− .09	− .12	− .16	− .19	− .23	230
140.....	.00	.00	− .01	− .03	− .05	− .08	− .11	− .15	− .19	− .23	− .28	220
150.....	.00	.00	− .01	− .03	− .06	− .09	− .12	− .16	− .21	− .26	− .31	210
160.....	.00	.00	− .02	− .03	− .06	− .09	− .13	− .18	− .23	− .28	− .34	200
170.....	.00	.00	− .02	− .04	− .06	− .10	− .14	− .19	− .24	− .30	− .36	190
180.....	0.00	0.00	−0.02	−0.04	−0.06	−0.10	−0.14	−0.19	−0.24	−0.30	−0.36	180

We see from (3) that if the spectrograms have all been exposed for the same length of time the correction is proportionate to the true velocity at the corresponding mean phase; consequently the effect of the error that we are discussing would be to diminish the semi-amplitude of oscillation without changing in any other way the shape of the velocity-curve. While this is strictly true only if the lengths of exposures are the same, it will be approximately true in any case that will arise in practice, so long as the average length of exposure is about the same for various parts of the velocity-curve.

A convenient procedure for applying this correction is as follows: The observations are plotted in the usual way and from this is derived a first approximation, K', to the semi-amplitude. This will be too small by an amount that we may ascertain from the first line in the table. Thus, if the average length of exposure is four-tenths of the period, we find 0.24 below this in the first line, and therefore a better approximation, K'', to the semi-amplitude may be found thus:

$$K''=K'\frac{1}{1-0.24}=1.32\,K'.$$

We now apply to each measured velocity the product of K'' and the corresponding number in the table.

If we develop (3) in ascending powers of u_2-u_1 we obtain

$$K\cos\frac{u_2+u_1}{2}\left\{\frac{(u_2-u_1)^2}{24}-\frac{(u_2-u_1)^4}{1920}+\text{etc.}\right\}. \tag{4}$$

From this we see that whatever the mean phase may be, the correction depending upon the length of exposure is very nearly proportionate to the square of the length of exposure. Incidentally we see that

$$K\left(\frac{u_2-u_1}{5}\right)^2\cos\frac{u_2+u_1}{2}$$

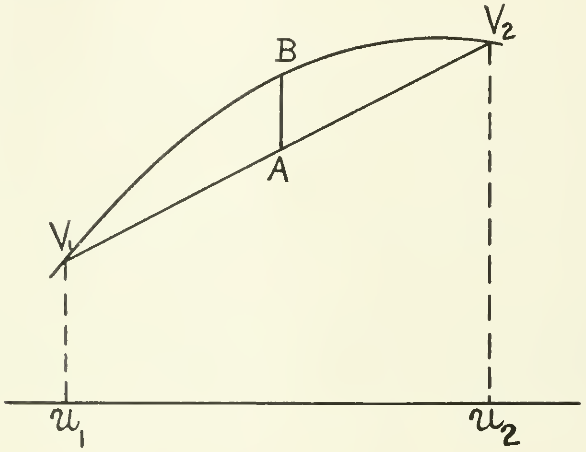
is a good approximation to the correction even for very large values of u_2-u_1.

An even better approximation is obtained by subtracting from the true velocity the mean of the velocities at u_1 and u_2; this difference is very closely equal to three times the correction, the error being always less than

$$K\frac{(u_2-u_1)^4}{2880}\cos\frac{u_2+u_1}{2}.$$

This approximation lends itself readily to graphical evaluation. In the diagram let V_1V_2 be a portion of the true velocity-curve, V_1 corresponding to u_1, and V_2 to u_2. From the middle point of the chord V_1V_2 erect the vertical AB; the correction is equal to one-third of AB.

We have confined our attention to circular orbits. There would be no particular difficulty in extending the present results to include elliptic orbits, but the necessity for this has not yet arisen, and there is reason to believe that it cannot arise, the orbits of all short-period binaries being very nearly circular. It is worthy of remark that the graphical correction described in the preceding paragraph would apply nearly as well to orbits of moderate eccentricity as to circular orbits.



Long exposures not only cause the systematic error that we have discussed, but they also tend to increase accidental error in a number of ways: (1) changes of temperature in the spectrograph are likely to be greater; (2) differential flexure is greater; (3) the effective mean time of exposure may be uncertain by reason of changes in the transparency of the atmosphere and in the seeing; (4) if the spectrogram is exposed near the times of maximum or minimum velocity, the lines may be made asymmetric, and the observer may have considerable difficulty in deciding upon what portion of the line to make the micrometer settings; this error would probably vary to considerable extent with the strength of the spectrogram and with the manner in which it was developed.

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January 21, 1916

NOTE ON "TERRESTRIAL" RADIATION

The radiation from the surface of the earth which does not escape immediately to space is absorbed, mainly by the aqueous vapor in an air layer close to the surface, and is thereafter passed on from layer to layer as air radiation in a vertical progressive propagation outward to space, but a slower one than that of the immediately transmitted part.

We cannot expose an instrument having a horizontal surface of indefinitely great extent, and thus resembling the earth's surface, and allow such an instrument to radiate out to space through the atmosphere; but by using a small instrument whose radiating surface constitutes virtually an element of the desired surface, we can let it radiate to a correspondingly small element of sky close to the zenith with identically the same relative radiating power that the sum of all the elements in the widely extended surface of the earth exerts to the similarly extensive sum of the sky elements above that surface. Or what is the same thing, the rate of transmission of radiant energy per square centimeter of surface between each square kilometer of the earth's surface and a parallel layer of atmosphere a few hundred meters thick immediately above the surface (which is practically the absorbent layer) is identically that of an instrument of 1 sq. cm in a vertical direction through an air column of this section.

As a convenient mode of measuring radiation to the sky, the instrument may have a wider angular aperture than the ideal one, but the principle remains the same. A curve of sky radiation at different altitudes above the horizon, obtained with an instrument whose angular aperture covered 24 sq. degrees of sky, has been published by me in the *American Journal of Science* for April, 1913 (Fig. 2, **35**, 383). A less accurate curve, obtained with an instrument of 32° angular aperture, or one which has more than thirty times the angular area of my instrument, and is therefore rather too coarse for such work, has been given by Mr. Anders Ångström in the *Astrophysical Journal* for January, 1914 (Fig. 2, **39**, 97). These curves may be summed hemispherically by zones, as Mr. Ångström has done, and if the experiment is performed by night the sum should be equal to the radiation of the same instrument exposed horizontally to the entire hemisphere of the sky, and may indeed be labeled "nocturnal radiation"; but this is a purely instrumental or laboratory fact, and represents no fact of nature. The *terrestrial* radiation to space, which is correctly given by the process described above, is more than twice as great; and therefore where Ångström gets transmissions of 5 to 25 per cent for "nocturnal radiation," I get an average of 42 per cent for the transmission of terrestrial radiation in middle latitudes. Dr. Nils Ekholm[1] gets results in Scandinavia in entire agreement with mine, obtaining transmissions of 55 to 70 per cent, appropriate to a more northern clime.

Finally, I have found quite similar transmission (48 per cent) for radiation somewhat resembling that of the earth in quality, but coming in the opposite direction, namely, for that emitted by the heated surface of the moon, whence it follows that Ångström's values are far too low.

FRANK W. VERY

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[1] "Om Naturens Värmehushållning," *K. Vetenskapsakademiens Årsbok*, 1914, p. 314.

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ON THE ALBEDO OF THE PLANETS AND THEIR SATELLITES

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In a preceding paper, the photometric observations of the sun, moon, and planets were reviewed, and their stellar magnitudes derived. In the present communication, values of the albedo of the various bodies are deduced from these data, and the material bearing on the albedo of the earth is considered.

I. THE DEFINITIONS OF ALBEDO, AND THE THEORY OF ITS DETERMINATION

According to Lambert's original definition[1] of the albedo of a diffusely reflecting surface, it is the ratio of the amount of light diffusely reflected in all directions by an element of this surface to the incident amount of light which falls on this element. With the law of diffuse reflection proposed by Lambert, this ratio is the same for all angles of incidence; but Seeliger[2] has shown that for other laws of reflection, whether derived from theoretical assumptions or from observation, this is not generally the case. Following his notation, suppose that an element $d\sigma$ of the surface is placed in a beam of parallel light of intensity L (such that a unit of area under normal incidence would receive the quantity of light L). Under any angle of incidence i the element will receive the amount $L \cos i\, d\sigma$. The intensity of the diffusely reflected light, at unit distance, will in general depend both on the angle of emanation ϵ and on the azimuth of the plane of reflection. The mean intensity for all azimuths, and a given value of ϵ, will be proportional to L and $d\sigma$, and may be expressed in the form

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[1] See Müller, *Photometrie der Gestirne* (Leipzig, 1897), pp. 52–55.

[2] *Abhandlungen der K. bayer. Akad. d. Wissenschaften*, **16**, 430, 1888.

$$\gamma L\, d\sigma f(i,\ \epsilon),$$

where γ is a constant. Integrating over a hemisphere of unit radius described about the element, to find the whole amount of reflected light, and dividing by the amount of the incident light, the albedo, according to Lambert's definition, is given by the equation

$$\mu' = 2\pi\gamma \int_0^{\frac{\pi}{2}} \frac{f(i,\ \epsilon)}{\cos i} \sin \epsilon\, d\epsilon\,. \tag{1}$$

This will be a function of i unless the law of reflection is such that

$$f(i,\ \epsilon) = \cos i\, f(\epsilon)\,.$$

If then we are to speak of *the* albedo of the surface, we must choose in some more or less arbitrary way,[1] either the value of μ' for some particular angle of incidence, as a mean value for all such angles. Seeliger decides to weight the different values of μ' according to the probability of their occurrence, which is proportional to sin i, and so obtains, as the definition of the albedo,

$$\mu = \int_0^{\frac{\pi}{2}} \mu' \sin i\, di = 2\pi\gamma \int_0^{\frac{\pi}{2}} \tan i\, di \int_0^{\frac{\pi}{2}} f(i,\ \epsilon) \sin \epsilon\, d\epsilon\,, \tag{2}$$

which may be expressed verbally as follows: "An element of surface is successively illuminated by beams of parallel light coming at random from all directions. The *mean of the ratios* of the diffusely reflected light to the incident light is the albedo."

Other definitions might equally well be given; for example: "An element of surface is exposed to rays of parallel light coming from all directions at random, or from a uniformly bright sky. The

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[1] "Einige Willkür ist natürlich hierbei nicht zu umgehen."—Seeliger, *loc. cit.*

ratio of the whole amount of light diffusely reflected to the *whole amount* incident on the element is its albedo." In this case, if the intensity of the light coming from unit solid angle of the sky is L, the amount of light which reaches the element under angles of incidence between i and $i+di$ will be

$$2\pi L \cos i \sin i \, di \, d\sigma \, ,$$

of which the fraction μ' is reflected. Integrating over the hemisphere, it is found that the whole amount of incident light is $\pi L d\sigma$, and that the albedo on this definition is given by the equation

$$A = 2\int_0^{\frac{\pi}{2}} \mu' \sin i \cos i \, di = 4\pi\gamma \int_0^{\frac{\pi}{2}} \sin i \, di \int_0^{\frac{\pi}{2}} f(i, \epsilon) \sin \epsilon \, d\epsilon \, . \qquad (3)$$

Still another definition, and from a more definitely astronomical viewpoint, was proposed long ago by Bond,[1] as follows: "Let a sphere S be exposed to parallel light. Then its albedo A is the ratio of the whole amount of light reflected from S to the whole amount incident on it." This definition leads again to the equation (3). If r is the radius of the sphere, and L the intensity of the incident light, the whole amount incident on the sphere is $\pi r^2 L$. The zone of the sphere upon which the angle of incidence is between i and $i+di$, as seen from the direction of the light, has the projected area $2\pi r^2 \cos i \sin i \, di$, and reflects the fraction μ' of the incident light. Hence it is easily found that

$$A = 2\int_0^{\frac{\pi}{2}} \mu' \sin i \cos i \, di \, ,$$

as above.

Unless μ' is independent of i, Bond's and Seeliger's definitions will lead to different values for the albedo of the same body, the difference arising because Seeliger's gives much greater weight to the small angles of incidence, in spite of the fact that the amounts of both incident and reflected light under these conditions are small. From the astronomical standpoint, several reasons can be urged in favor of Bond's definition; for example: (1) It corresponds more closely to the actual circumstances of observation. (2) In dealing with such questions as the effect of solar radiation on the temperature of a planet, where the amount of heat absorbed by its surface is the determining factor, Bond's definition of the albedo will always lead to correct results, while Seeliger's may not. (3) The law of diffuse reflection $f(i, \epsilon)$, and the law connecting μ' with i are unknown for the planetary surfaces, so that computations by Seeliger's formulae must rest on more or less arbitrary assumptions. (4) For a number of bodies, however (the moon, Mercury, Venus, and in part for Mars), the relation between the amounts of light reflected in different directions—that is, at different phase-angles—is known from observation, so that the albedo on Bond's definition can be computed independently of all assumptions. The last point is important. It is well known that the manner in which the brightness varies with the phase-angle is very different for different bodies, and that not one of them agrees with any of the laws derived from theoretical assumptions, so that an empirical law must be derived from the observations in each case.

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[1] *Proceedings of the American Academy of Arts and Sciences*, N.S., **8**, 232, 1861.

If the brightness of the planet (reduced to standard distances from the earth and sun) at the full phase be taken as unit, that at any other phase-angle α may be called $\phi(\alpha)$. The whole amount of light reflected by the planet to the celestial sphere will be proportional to

$$\int_0^{\pi} \phi(\alpha) \sin \alpha \, d\alpha \, .$$

If it shone in all directions with the brightness of the full phase, the emitted light would be 2.0 on the same scale. Now let r be the mean radius of the planet's disk, and R its distance from the sun, and M_0 be the ratio of the apparent brightness of the planet at the full phase, and at distance Δ from the earth, to that of the sun at unit distance. The fraction of the sun's whole radiation which the planet intercepts is $\frac{r^2}{4R^2}$. If it shone in all directions with its full brightness, the whole amount of emitted light would be $M_0\Delta^2$ times that emitted by the sun. The albedo of the planet, according to Bond's definition, is therefore given by the equation

$$A=\frac{R^2M_0\Delta^2}{r^2}\cdot 2\int_0^\pi \phi(a)\sin a\,da\,. \tag{4}$$

The albedo here appears as the product of two factors, of which one depends only on the geometrical and photometric relations of the planet as observed at the full phase, and the other entirely upon the law of variation of its brightness with phase. The first factor, which may be called p, may be expressed in a variety of forms. If σ is the angular semi-diameter of the planet as seen from the earth, s that of the sun as seen from the planet, and S that of the sun seen from the earth, then

$$p=\frac{M_0R^2\Delta^2}{r^2}=\frac{M_0\sin^2 S}{\sin^2\sigma\sin^2 s}=\frac{M_0 R^2}{\sin^2\sigma}\,, \tag{5}$$

which are well-known formulae.

If σ_1 denotes the semi-diameter of the planet in seconds of arc, and g its stellar magnitude at the full phase, reduced in both cases to unit distance from the earth and sun, and if G is the sun's stellar magnitude at unit distance, it is easily shown that

$$\log_{10}p=\frac{2}{5}(G-g)-2\log_{10}\sin\sigma_1\,.$$

Introducing the value of G (-26.72 on the Harvard scale)[1] and remembering that

$$\log_{10}\sin\sigma_1=\log_{10}\sigma_1-5.3144\,,$$

this becomes

$$\log_{10}p=-\frac{2}{5}(g+0.15)-2\log_{10}\sigma_1\,. \tag{6}$$

The factor p may also be defined as the ratio of the actual brightness of the planet at the full phase to that of a self-luminous body of the same size and position, which radiates as much light from each unit of its surface as the planet receives from the sun under normal illumination.

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[1] *Astrophysical Journal*, 43, 105, 1916.

If now

$$q = 2\int_0^\pi \phi(\alpha)\sin\alpha\, d\alpha\,, \qquad A = pq\,. \tag{7}$$

The factor q is different for the various laws of diffuse reflection, and still more so for the planets as they actually are. For the principal theoretical laws it is found that, for a sphere,[1] on Lambert's law:

$$\phi(\alpha) = \frac{1}{\pi}(\sin\alpha + (\pi-\alpha)\cos\alpha),\ \text{and}\ q = \frac{3}{2}\ ;$$

on the Lommel-Seeliger law:

$$\phi(\alpha) = 1 - \sin\frac{\alpha}{2}\tan\frac{\alpha}{2}\log\cot\frac{\alpha}{4}\,,\quad q = (1-\log 2)\frac{16}{3} = 1.6366$$

and on Euler's law:

$$\phi(\alpha) = \cos^2\frac{\alpha}{2}\,,\quad q = 2\,.$$

For the Lommel-Seeliger law,

$$f(i,\epsilon) = \frac{\cos i\cos\epsilon}{\cos i + \cos\epsilon}\,,$$

and it is found that

$$\mu' = 2\cdot\pi\gamma[1 - \cos i\log(1+\sec i)]\,,\quad A = \frac{8}{3}\pi\gamma(1-\log 2) = 0.8183\ \pi\gamma\,.$$

The factor in brackets in the first of these expressions ranges from 0.308 when $i = 0°$ to unity when $i = 90°$. Since μ' can never exceed unity, it follows that $\pi\gamma$ cannot be greater than 0.5, nor A than 0.409. Hence a planet for which p exceeds 0.25 cannot reflect light in strict accordance with the Lommel-Seeliger law.

For those heavenly bodies whose brightness can be observed over a sufficient range of phase-angles, the value of q can be determined by mechanical quadrature.[2] Since in all cases the falling off in brightness with advancing phase is more rapid than is predicted by the simple theories, the values of q are smaller. Müller's observations of Venus (treated in this way) lead to the value 1.194 for q, while the light-curve derived in the previous paper for the moon gives $q=0.694$. For Mercury, Müller gives two empirical formulae. One gives practically the same value as for the moon; while the other, which agrees better with an observation made near full phase during a total solar eclipse, makes $q=0.420$.

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[1] For a flat disk, normally illuminated and viewed, the value of q is 1.0 if it reflects according to Lambert's law, and the same as for a sphere on the other two laws.

[2] There is of course a gap in the observed curves near each conjunction with the sun, widest near inferior conjunction. But since the factor $\sin\alpha$ in the integrand is here small, a free-hand extension of the observed curves suffices to obtain reliable values of q.

It is therefore clear that, in the absence of knowledge of the law of variation with phase, little can be told about the albedo of a planet by observations of the brightness at full phase alone. Indeed, observations made when the phase is decidedly gibbous are actually better for this purpose. If $p(\alpha)$ denotes the value of p computed from the brightness at phase-angle α instead of at the full phase, the albedo is $p\ (\alpha)\ \frac{q}{\phi(\alpha)}$. The values of the second factor, for the six different laws of variation with phase which are considered above, are given in Table I.

TABLE I

α	Euler	Lambert	Seeliger	Venus	Moon	Mercury
0°.........	2.00	1.50	1.64	1.19	0.72	0.42
20°.........	2.06	1.60	1.77	1.54	1.06	0.83
40°.........	2.26	1.88	2.08	2.00	1.64	1.63
50°.........	2.44	2.12	2.32	2.32	2.07	2.28
60°.........	2.67	2.46	2.64	2.72	2.68	3.21
80°.........	3.41	3.66	3.60	3.88	4.77	6.41

In the neighborhood of phase-angle 50° the values resulting from all the six laws are nearly the same. If Euler's so-called "law," which has no sound physical basis and entirely fails to represent any observed facts, is ignored, the constant 2.20 represents the other five values of $\frac{q}{\phi(\alpha)}$ with an average deviation of only 4.6 per cent, and a maximum error of 6.8 per cent. If, therefore, M_{50} represents the ratio of the brightness of the planet at phase-angle 50° to that of the sun, the equation

$$A=2.20\ M_{50}\frac{R^2\Delta^2}{r^2} \tag{8}$$

will give the albedo, according to Bond's definition, within a few per cent, whatever may be the law of variation with phase, within the range of all those laws which have so far been derived from observation or seriously suggested theoretically.

Mars can be observed up to a phase-angle of 47°, and its albedo can therefore be determined with almost as much certainty as that of Venus or the moon. In the case of the brighter asteroids, which have been followed up to phase-angles between 20° and 30°, considerable extrapolation is necessary, and the results must be correspondingly uncertain. For Jupiter and the outer planets, the phase-angle can never be more than a few degrees, and attempts to determine their albedo must involve assumptions regarding the unknown laws of variation of their brightness in the unobservable phases. Since, however, the falling off in brightness with increasing phase-angle is certainly less for Jupiter and Saturn than for Venus, it is very probable that for these planets q is greater than 1.20. The assumption that q is 1.5, as would follow from Lambert's law, is probably within 15 per cent of the truth.

II. THE BRIGHTNESS OF THE EARTH-SHINE, AND THE STELLAR MAGNITUDE OF THE EARTH

The brightness of the earth, as seen from another planet, can be determined observationally only by measures of the illumination of the moon by the light reflected from the earth. The only recent series of observations of this, and doubtless the best, are these of Very,[1] who measured, with a photometer of his own design, the relative surface brightness of the sunlit portion of the moon, the portion lit by earth-shine, and the neighboring sky, the true brightness of the earth-shine being the difference of the last two quantities. His observations are summarized in Table II. The first four columns are taken from his paper, and give (1) the date of observation, (2) the elongation of the moon, which is the phase-angle of the earth at the moment, and is taken as negative for the waning moon, (3) the observed ratio of the surface brightness of the moon to that of the sky, and (4) that of the sunlit part of the moon to the earth-shine, after correction for the light of the sky. In the following columns, which are added by the writer, and contain a new reduction of Very's observations, appear (5) the ratio of the brightness of the sky to that of the earth-shine, (6) the difference in surface brightness between the sunlit and earth-lit portions of the moon, expressed in stellar magnitudes, (7) the ratio of the mean surface brightness of the moon at this phase to that of the full moon, expressed in magnitudes and taken from Table III of the preceding paper,[1] (8) the correction to the observed intensity of the earth-shine to reduce to mean distance, (9) the deduced ratio of the surface brightness of the earth-shine at mean distance and the observed phase to that of the full moon, which is obviously the difference of magnitude between the sun and the earth at mean distance and the given phase, as seen from the moon. The resulting

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[1] *Astronomische Nachrichten*, **196**, 269–290, 1912.

TABLE II

1	2	3	4	5	6	7	8	9
Sept. 28, 1911	70°	52	549	10.6	6.85	+2.01	−0.10	8.76
30	96	3095	4596	1.5	9.15	+1.48	− .11	10.52
Oct. 2	118	1149	3313	2.9	8.80	+1.10	− .10	9.80
26	54	3033	2889	0.9	8.65	+2.42	− .11	10.96
29	87	3626	14540	4.0	10.40	+1.67	− .10	11.97
Nov. 16	− 52	1871	2290	1.2	8.40	+2.39	+ .02	10.81
17	− 40	8579	2735	0.3	8.59	+2.65	.00	11.24
27	78	1358	1932	1.4	8.22	+1.86	− .08	10.02
Dec. 14	− 69	9380	2224	0.2	8.42	+2.08	.00	10.52
Feb. 14, 1912	− 42	10164	5769	0.6	9.40	+2.61	− .11	11.90
20	31.5	2476	698	0.4	7.11	+3.10	−0.02	10.19

values of the difference of magnitude between the earth and sun are too much influenced by the errors of observation, which are unavoidably large, to permit of an independent deduction of the law of variation of the earth's brightness with phase. To determine what influence various assumptions regarding this would have on the results, they have here been reduced on four different hypotheses, assuming for the law of the earth's variation in brightness with phase the Lommel-Seeliger law, Lambert's law, and the empirical laws found by Müller for Venus and in the preceding paper for the moon. The results are given in Table III, which gives, for the observations in order as above, and for each phase law, (1) the correction to "full earth," (2) the deduced difference in magnitude between the mean full earth and the sun, and (3) the same quantity after correction for the systematic error described below. Upon grouping the results according to the relative brightness of the sky and earth-shine at the time of observation, as is done in the lower part of the table, it becomes evident that this greatly influences the measures, the computed brightness of the earth increasing with the brightness of the sky.

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[1] *Astrophysical Journal*, **43**, 114, 1916.

The observations of September 28, a hazy night, when the sky averaged ten times as bright as the earth-shine, and varied in brightness during the measures by four times the amount of the latter, should be entirely rejected. The other three groups can be brought into fair agreement on the assumption that the brightness of the sky-illumination at the point where the combined effect of it and the earth-shine was measured was, on the average, greater by 20 per cent than at the points outside the limb where it alone was measured. Although care was taken by Very to measure the light of the sky "at distances from the bright part of the moon comparable to those at which the earth-shine was measured," this assumption does not appear unreasonable when it is noticed that on October 26 ("sky clear") it was found that the sky immediately adjacent to the bright limb was ten times brighter than at the points where measures were made outside the dark limb. A very small difference in the distances from the bright part of the moon of the points at which the brightness of the sky and the earth-shine were measured would therefore explain the observed variations. On this hypothesis, the tabular results for the brightness of the earth-shine should be multiplied by the factor $1-0.20x$, where x is the measured ratio of the brightness of the sky to that of the earth-shine. The last column under each heading in Table III gives the difference of magnitude of the sun and full earth resulting from each night's measures, after application of this correction. The group-means given in the table show that the systematic difference depending on the brightness of the sky has disappeared, though the mutual accordance of the individual determinations is not improved. In forming final means, the observations made when the sky was more than twice as bright as the earth-shine have been given half-weight. The average deviation of a single observation reduced to unit weight is given in the table. The magnitude of these quantities is due to the extreme difficulty of the observations. As a basis for further discussion, the mean of the values obtained with and without the systematic correction here suggested has been adopted. Taking the mean of the results of the first three assumptions concerning the variations with phase, which represent the observations equally well, and better than the fourth, it may be

TABLE III

Sky: Earth-Shine	Lommel-Seeliger			Lambert			Venus			Moon		
	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
10.6...	−0.8	(8.0)	.....	−0.9	(7.9)	.....	−1.2	(7.5)	.....	−1.9	(6.9)	.....
1.5...	−1.2	9.3	9.7	−1.4	9.1	9.5	−1.7	8.9	9.3	−2.7	7.9	8.3
2.9...	−1.8	8.0	8.9	−1.9	7.9	8.8	−2.3	7.5	8.6	−3.7	6.1	7.2
0.9...	−0.4	10.5	10.7	−0.4	10.5	10.7	−0.8	10.2	10.5	−1.2	9.8	10.0
4.0...	−1.0	11.0	12.7	−1.1	10.9	12.6	−1.4	10.6	12.3	−2.3	9.7	11.4
1.2...	−0.4	10.4	10.7	−0.4	10.4	10.7	−0.7	10.1	10.4	−1.2	9.6	9.9
0.3...	−0.2	11.0	11.1	−0.2	11.0	11.1	−0.5	10.7	10.8	−0.9	10.3	10.4
1.4...	−0.8	9.2	9.5	−0.9	9.1	9.4	−1.2	8.8	9.1	−2.0	8.0	8.3
0.2...	−0.6	9.9	9.9	−0.7	9.8	9.8	−1.0	9.5	9.5	−1.7	8.8	8.8
0.6...	−0.3	11.6	11.7	−0.3	11.6	11.7	−0.6	11.3	11.4	−0.9	11.0	11.1
0.4...	−0.2	10.0	10.1	−0.2	10.0	10.1	−0.4	9.8	9.9	−0.7	9.5	9.6
Means	Obs.											
3.4...	2	9.5	10.8	.....	9.4	10.7	.....	9.0	10.4	.....	7.9	9.3
1.2...	4	9.9	10.2	.....	9.8	10.1	.....	9.5	9.8	.....	8.8	9.1
0.4...	4	10.6	10.7	.....	10.6	10.7	.....	10.3	10.4	.....	9.9	10.0
General mean		10.15	10.47	.....	10.10	10.41	.....	9.81	10.13	.....	9.20	9.54
Average deviation..		± 0.72	0.75	.....	± 0.74	0.80	.....	± 0.75	0.79	.....	± 0.95	0.95
Adopted.....		10.31		10.25			9.97			9.37		

estimated that the mean full earth, as seen from the moon, is $10\overset{\mathrm{M}}{.}2$ fainter than the sun, and of magnitude −16.5, or 40 times brighter than the full moon appears to us. The magnitude of the earth as seen from the sun would then be −3.5, and as seen from Venus when nearest, −6.3. The probable error of these values must be at least $\pm 0\overset{\mathrm{M}}{.}25$, or over 20 per cent, and arises more from the errors of the observations than from any uncertainty in the assumptions made during the reductions.

Professor Very's own reduction of his observations leads to substantially the same value for the brightness of the full earth. From his data, combined with certain results of Arago, he concludes that, when the moon is at quadrature, the sunlit half of its disk is 5100 times brighter than that illuminated by the earth-shine. But at this time the mean surface brightness of the moon is 0.16 times that of the full moon, according to Very's reduction-curve, while, assuming the earth to behave like Venus, its light is 0.28 that of the full earth. Hence the ratio of sunlight to full earth-light on the moon is $5100\times\frac{0.28}{0.16}$, or 9000, corresponding to a difference of magnitude of 9.89, which agrees within $0^{\mathrm{M}}1$ with the value here derived on the assumption of the same phase law.

It will, however, be shown later that the value of the earth's albedo which Professor Very deduces from his observations is about twice too great.

In a later paper[1] Very gives the results derived by him from measures of spectrograms of the earth-lit and sunlit portions of the moon, obtained by Slipher at the Lowell Observatory. His conclusions regarding the relative intensity of the light of these two sources depend on assumptions regarding the photographic action of exposures of very different durations to light of different brightness. For example, on January 3, 1911, one spectrogram was obtained of the earth-shine, with exposure 2520 seconds, and two of the sunlit moon, with exposures of 16 and 4 seconds. According to Very's measures, the mean ratio of the intensity of the lunar spectrogram of 16^{s} exposure to that of the earth-shine (corrected for diffuse light from the sky) is 7.109, while the corresponding ratio for the 4^{s} exposure is 3.913. On the assumption that the photographic intensity was proportional to the product of the intensity i of the light by the time t of exposure, the first of these comparisons would make the sunlit moon 1120 times as bright as the earth-shine, and the second 2460. Very concludes from these and similar figures that "the Cramer instantaneous isochromatic plates are more sensitive than the product $i\times t$, when the exposure is to a light as feeble as that of the earth-shine spectrum, enduring for upward of an hour, and less sensitive than $i \times t$, when the illumination approaches that of the lunar spectrum, with a duration of a few seconds only," and hence that, if the values of the relative brightness of the moon and earth-shine obtained as above are plotted against the exposures on the moon and a curve drawn to represent them, and extended to give the ratio which would have been obtained for an indefinitely short exposure on the moon, this will be the correct value.

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[1] *Astronomische Nachrichten*, **201**, 353–399, 1915.

On this hypothesis, the shortest exposures on the moon should give the best results, but even they will make the brightness of the earth-shine come out too great. Acting on this principle, Very's results from four nights' spectrograms have been reduced anew in the same manner as his visual observations. The details are given in Tables IV A and IV B. Table IV A contains data derived from Very's paper, and gives, in successive columns, the date of observation; the exposure on the earth-shine; the shortest exposure on the moon; the ratio of the brightness of the sky to that of the earth-shine (taken from Very's tables [*op. cit.*] on pp. 374, 375, and

TABLE IV A

Date	Exposures		Ratio of Sky to Earth-Shine	Ratio of Photographic Intensities	*M*:*E*
	Earth-Shine	Moon			
1911					
Jan. 3.......	2520^s	4^s	0.9	3.91	2460
4.......	4800	2	3.3	1.39	3340
	....	2	....	1.18	2830
1912					
Aug. 8.......	4800	1	0.2	1.36	3529
9.......	4260	1.5	0.1	1.06	2674

TABLE IV B

Elongation	Difference Magnitude	Reduction to			*S*:*E*
		Full Moon	Full Earth	Mean Distance	
+39°.3......	8.48	+2.83	−0.55	−0.01	10.75
+51.2......	8.81	+2.49	−0.74	+0.01	10.57
	8.63				10.39
−47.8......	8.87	+2.48	−0.69	+0.08	10.74
−34.0......	8.57	+2.83	−0.47	+0.10	11.03

370); the ratio of the photographic intensity of the lunar spectrum to that of the earth-shine (after correction for the light of the sky); and finally the ratio $M:E$ of the surface brightness of the sunlit part of the moon to that of the earth-lit portion, computed as described above. Table IV B gives, for the observations in order, the elongation of the moon; the difference in brightness between the sunlit and earth-lit portions, reduced to stellar magnitudes; the correction for the difference in surface brightness between the full moon and the moon at the given phase, taken from Table III of the preceding paper; the reduction to full earth, on the assumption that its phase-variations follow the same law as in the case of Venus; the reduction to mean distance of the earth and moon; and finally the ratio $S:E$ of the light of the sun and mean full earth, expressed in stellar magnitudes.

The mean of these last quantities, giving each night equal weight, is 10.75, as against 9.97 derived from the visual observations on the assumption of the same phase law. The photographic observations therefore make the earth-shine only half as bright as do the visual observations. This is just what might be expected if the plates had followed the ordinary law for faint illumination and long exposure, and been "less sensitive than $i\times t$." This was certainly the case for the exposures to the lunar spectrum. As only one exposure on the earth-shine was made on each night, the measures afford no direct means of determining whether it was so in this case too. It seems reasonable, however, to assign very little if any weight to these observations, for the present purpose, in comparison with the visual measures.

Professor Very concludes that the photographic measures are in excellent agreement with the visual observations, because they give values of the ratio $M:E$ which agree well with a curve previously derived from his visual measures.[1] But this curve is not consistent with his assumptions with regard to the phase-variations of the earth and moon. For elongations of 30°, 50°, and 90° it gives the values of $M:E$ as, respectively, 2450, 3150, and 5100. According to Very's curve for the surface brightness M of the moon, its values at these elongations are 0.08, 0.092, and 0.16.

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[1] *Astronomische Nachrichten*, **196**, 274, 1912 (Fig. 3).

This demands that the brightness E of the earth-shine at these phases should be in the ratios 1.04, 0.92, and 1.00; that is, that the earth's light should be as bright at quadrature as when it was nearly full. If the earth follows the same law as Venus, these ratios should be 2.74, 2.05, and 1.00. Hence Very's curve makes the earth-shine more than twice as faint, in proportion to the sunlit region, at the elongations at which observations were made, as his theoretical assumptions indicate.

Very's visual color-estimates, Slipher's spectrograms, and Tikhoff's photographs[1] agree in showing that the earth is bluer than ordinary moonlight. All three observers explain this on the assumption that a considerable part of the light reflected from the earth is scattered by its atmosphere, and is blue like the sky. Tikhoff has derived provisional numerical results from his photographs, and concludes that the ratio of the light scattered by the molecules of the air, which varies inversely as the fourth power of the wave-length, to that reflected by clouds, dust, and the earth's surface is 0.18 at λ 6400, 0.32 at λ 5650, and 1.28 at λ 3750. Taking the mean effective visual wave-length at 5200, for this very faint light, and the photographic wave-length as 4200, it would follow that the earth's photographic albedo should be about 40 per cent greater than its visual albedo. The reflecting power of the earth for the sun's radiation as a whole, according to a summary calculation from Abbot's energy-curve, should be the same as for λ 5400, and therefore practically equal to the visual albedo.

III. CONCLUSIONS AND SUMMARY

It may be well at this point to summarize the results of the discussion, including both the present paper and its predecessor.

1. Bond's definition of albedo, as the ratio of the whole amount of light reflected in all directions from a sphere illuminated by parallel rays to the amount of light incident on the sphere, appears to be the most suitable for astronomical purposes.

2. The albedo A of any planet, according to this definition, may be expressed as the product of two factors, of which one, q, depends only on the law of variation of its light at different phases, while

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[1] Pulkowa, *Mittheilungen*, 6, 22, 1914.

the other, p, depends on the brightness at full phase and the geometrical relations. The factor p may be defined verbally as the ratio of the observed brightness of the planet at full phase to that of a flat disk of the same size and in the same position, illuminated and viewed normally, and reflecting all the incident light in accordance with Lambert's law, or algebraically by equations (5) or (6). The factor q is defined by equation (7) and may be found by mechanical quadrature when the law of variation of brightness with phase is known. Its value varies from 1.635 for the theoretical law of Lommel and Seeliger to 0.420 in the case of an empirical law found from the observations of Mercury. In all the cases, theoretical or observed, which demand practical consideration, it may be found within a few per cent by multiplying the ratio of the brightness at phase-angle 50° to that at full phase by 2.20.

3. The visual stellar magnitude of the sun, on the Harvard scale, is -26.72 ± 0.04, according to the mean of the observations of Zöllner, Fabry, Ceraski, and W. H. Pickering, which were made by different methods and are in excellent agreement. The photographic magnitude, according to the observations of King and Birck, is -25.93, and its color-index $+0^{\mathrm{M}}.79$, agreeing within the errors of observation with the average for stars of Class G.

4. Müller's observations of the brighter planets require a correction of approximately $-0^{\mathrm{M}}.06$ to reduce them to the Harvard scale, while that for Uranus and Neptune is insensible. King's photographic observations, combined with Müller's visual observations at the same phases, give for the color-indices on the scale of King's system: Venus, $+0^{\mathrm{M}}.78$; Mars, $+1^{\mathrm{M}}.38$; Jupiter, $+0^{\mathrm{M}}.50$; Saturn, $+1^{\mathrm{M}}.12$.

5. The results of the seven observers who have determined the variations of the moon's brightness with phase are all in satisfactory agreement with the mean light-curve, which is given in Table III of the preceding paper,[1] and may be regarded as very well determined. The full moon is 8.7 times brighter than the first quarter, and 10.0 times brighter than the last quarter.

6. The results of different observers for the stellar magnitude of the mean full moon are discordant. Herschel's observations

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[1] *Astrophysical Journal*, 43, 114, 1916.

have been reduced with modern photometric magnitudes of his comparison stars, and an error which had crept into the earlier reduction corrected. The visual magnitude adopted as the weighted mean of those observations not obviously affected by systematic error is -12.55, with a probable error of $\pm 0^{M}_{.}07$, as derived from the observations, but very likely greater in reality. The photographic magnitude, -11.37, has been well determined by King. The resulting color-index, $+1^{M}_{.}18$, is in accordance with the spectro-photometric work of Wilsing and Scheiner, which shows that moonlight is decidedly redder than sunlight.

7. The intensity of sunlight from the zenith is 103,000 meter-candles, and that of mean full moonlight 0.24 meter-candle. A standard candle, if of approximately the same color as the stars, would appear of magnitude -14.18 at a distance of 1 m, and $+0.82$ at 1 km.

8. Very's observations of the intensity of the earth-shine indicate that the mean full earth, as seen from the moon, is forty times brighter than the full moon as seen from the earth, and that the stellar magnitude of the earth as seen from the sun is -3.5, with an uncertainty of at least 25 per cent, or $0^{M}_{.}20$. Owing to the blue light reflected by the sky, the photographic albedo of the earth is probably about 40 per cent greater.

Collecting now the numerical data, the results are exhibited in Table V (for the objects named in the first column) as follows: (1) the stellar magnitude at mean opposition, m_0 (for Mercury and Venus, the magnitude at full phase, mean distance from the sun, and unit distance from the earth); (2) the magnitude g which it would have at full phase and unit distance from the earth and sun; (3) the assumed mean semi-diameter σ_1 at unit distance; (4) the quantity p, computed by the equation (6) p. 177; (5) the factor q defined above; (6) the albedo A, according to Bond's definition; (7) the color-index (when known); and (8) the photographic albedo, according to Bond's definition.

Two sets of values are given for Mercury, the first corresponding to Müller's linear empirical formula for the phase-variations and the second to his quadratic formula.[1] The data for Saturn refer to the ball of the planet, with ring invisible. The results obtained for the earth from Very's visual observations are reduced on all four assumptions regarding the law of variation with phase. The values of q for Mercury, Venus, and the moon have been derived by mechanical integration from the observed laws of variation with phase, and are accurate. Those for Mars and the asteroids have been determined from the loss of brightness at phase-angle 50°. In the former case this should give a nearly correct value. For the asteroids considerable extrapolation is necessary, and the tabular values, which have been computed with the mean law of variation for all the asteroids which have been observed, are considerably uncertain. The value $q=1.5$ which has been assumed for the outer planets, in the impossibility of observing them except very near the full phase, may be in error by 15 per cent. The same

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[1] *Astrophysical Journal*, 43, 108, 1916.

TABLE V

Object	m	g	σ_1	p	q	Visual Albedo A	Color-Index	Photographic Albedo
Moon	−12.55	+0.40	2″.40	0.105	0.694	0.073	+1.18	0.051
Mercury	− 2.94	−0.88	3.45	.164	0.42	.069	....	......
	− 2.12	−0.06	3.45	.077	0.72	.055	....	......
Venus	− 4.77	−4.06	8.55	.492	1.20	.59	+0.78	.60
Mars	− 1.85	−1.36	4.67	.139	1.11	.154	+1.38	.090
Jupiter	− 2.29	−8.99	95.23	.375	1.5:	.56:	+0.50	.73:
Saturn	+ 0.89	−8.67	77.95	.420	1.5:	.63:	+1.12	0.47:
Uranus	+ 5.74	−6.98	36.0	.42	1.5:	.63:	....	......
Neptune	+ 7.65	−7.06	34.5	.49	1.5:	.73:	....	......
Ceres	+ 7.15	+3.70	0″.53	.10	0.55:	.06:	....	......
Pallas	+ 7.84	+4.38	0.34	.13	0.55:	.07:	....	......
Juno	+ 8.95	+5.74	0.14	.22	0.55:	.12:	....	......
Vesta	+ 6.04	+3.50	0.27	.48	0.55:	.26:	....	......
Jupiter I	+ 5.54	−1.16	2.38	.46	1.5:	.69:	....	......
II	+ 5.69	−1.01	2.08	.51	1.5:	.76:	....	......
III	+ 5.08	−1.62	3.62	.30	1.5:	.45:	....	......
IV	+ 6.26	−0.44	3.49	.11	1.5:	.16:	....	......
Titan	+ 8.30	−1.26	2.9	0.33	1.5:	0.50:	....	......
THE EARTH (FROM VERY'S VISUAL OBSERVATIONS)								
Lommel-Seeliger	....	−3.46	8.79	0.27	1.64	0.45	+0.45:	0.6:
Lambert	....	−3.52	8.79	.29	1.50	.43	....	......
Like Venus	....	−3.80	8.79	.37	1.20	.45	....	......
Like Moon	....	−4.40	8.79	0.65	0.70	0.45	....	......
THE EARTH (FROM VERY'S REDUCTION OF SLIPHER'S SPECTROGRAMS)								
Like Venus	....	−3.02	8.79	0.18	1.20	0.22	....	......

value has been adopted for the satellites of Jupiter and Saturn, in the total absence of observational information. If these small and presumably atmosphereless bodies behave like Mercury, Mars, or the moon, the actual values of q for them would be much smaller, and the tabular values of A may in these cases be regarded as upper limits.

Concerning the diameters of the various bodies which are employed in the reductions, it may be briefly stated that they have been taken from the sources which appeared most likely to be free from systematic error, especially that arising from irradiation. Thus Sampson's values[1] for Jupiter and its satellites, derived from the Harvard eclipse observations, and Abetti's small value for Neptune,[2] corrected for irradiation have been adopted, while the adopted diameters of Venus and Mercury are greater than those given by the measures during transits. Barnard's diameters[3] for the four asteroids are used. The most remarkable value, $0''.54$ for Vesta at unit distance, is confirmed by the interferential measures of Hamy,[4] who finds $0''.53$.

In conclusion, a few remarks may be made upon the physical interpretation of these results.

For such purposes as the computation of the probable temperature of a planet, the value of the albedo given in the column headed A should unquestionably be used, as it represents the amount of solar light which is actually reflected from the planet. When attempting to estimate the diameters of the fainter asteroids and satellites, on the basis of their observed magnitudes at opposition, the quantities p, which are independent of the peculiarities of the phase-variation of the planets, should obviously be employed. It is easy to see that the values p should also be used for comparison with the observed reflecting powers of terrestrial substances. It has been recognized since Zöllner's time that the rapid decrease in brightness of the moon with phase arises mainly from the rough character of its surface, the shadows of the irregularities being invisible to us at the full, but covering more and more of the surface with advancing phase. Zöllner concluded that the observed variation could be accounted for by supposing that the whole surface of the moon was covered with mountains having slopes that were inclined at an angle of 52° to the horizontal, but his analysis suffers from an error pointed out by Searle.[1] The effect is too great to be explained by the shadows of the visible lunar mountains, but the very plausible assumption that a great part of the moon's surface is covered by broken fragments of rock, in whose interstices innumerable shadows are formed, appears sufficient, recalling Seeliger's theory of Saturn's rings, in which the variation at small phase-angles is still more pronounced. Very little of the solar radiation which penetrated into a crevice between two such rocks would get back again into space, for most of that which was diffusely reflected from the region where the sunbeam struck would be caught on the walls of the crevice. Hence a rough surface of this sort would be a much better absorber of incident radiation than a smooth, diffusely reflecting surface of the same material. At the full phase, however, when our line of sight reaches right down the path of the sun's rays, we shall see the sunlit patches at the bottom of the crevices, and the surface will appear just about as bright as a smooth surface of the same material. Hence there is good reason to believe that if the moon's surface could be smoothed out, without otherwise altering its properties, the brightness of the full moon, and the value of p deduced therefrom, would be but little changed, while the brightness at large phase-angles, and hence the values of q and A, would be considerably increased. For the outer planets, however, which appear considerably brighter at the center than at the limb, the values of A may be nearer the true reflecting power of the surface than those of p.

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[1] *Harvard Annals*, **52**, 333.

[2] *Memorie della Società degli Spettroscopisti Italiani* (2), **1**, 113, 1912.

[3] *Astronomische Nachrichten*, **157**, 262, 1902.

[4] *Comptes Rendus*, **128**, 851, 1899.

Wilsing and Scheiner[2] have determined the reflecting power of many ordinary rocks, using an approximately flat, rough, natural surface normal to the incident and reflected rays. Their formula of reduction gives exactly the quantity which has here been designated by p, and the conditions of observation appear very similar to those which obtain in the case of the full moon. Their results for the albedo of a number of typical rocks are: "Liparitbimsstein," 0.56; yellow sandstone, 0.38; red granite, 0.36; volcanic ash, 0.18; syenite, 0.13; trachyte-lava, 0.10; clay-slate, 0.07; basalt, 0.06. In almost all cases the reflecting power was greater for the red end of the spectrum than for the blue, the average difference being about 20 per cent.

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[1] See Müller, *Photometrie der Gestirne*, p. 77.

[2] *Potsdam Publications*, **20**, Part IV, 1909.

Abbot,[1] from energy measurements, concludes that the albedo of a cloud surface is about 0.65, and, after allowance for atmospheric absorption, computes that the albedo of the earth, in substantially the sense of Bond's definition, would be 0.60 if it were completely covered with high clouds, 0.14 if it were cloudless, and 0.37 as matters actually are. These values are provisional, but no later ones have been published.

The results of the present discussion therefore entirely confirm the familiar views that the reflecting power of Venus and the outer planets is comparable with that which might be expected from cloud surfaces, while that of Mercury, Mars, and the moon, which have little or no atmospheres, is similar to that of ordinary rocks. The high albedo found for Neptune is somewhat uncertain, on account of the great difficulty of getting precise measures of the planet's diameter. The most remarkable features are the high values for Vesta and the first and second satellites of Jupiter—none of which can have much if any atmosphere—but even these can be matched by some terrestrial rocks.

It is also worthy of remark that the values of p for Venus, the earth, Vesta, Jupiter, Saturn, Uranus, and Neptune are higher than the theoretical limit, 0.25, for a body reflecting according to the Lommel-Seeliger law, and that, for all the bodies which have values of p below the limit, the known variations with phase are inconsistent with this law, except in the sole case of the fourth satellite of Jupiter, where there are no data.

The value of the earth's albedo found from Very's visual observations comes out almost the same on all the assumptions regarding the variations with phase, as it ought to do for observations made at phase-angles averaging not far from 50°. It is intermediate between the values for the cloudless and cloudy planets, and agrees with Abbot's estimate within the error of the observations.

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[1] *Annals of the Smithsonian Astrophysical Observatory*, **2,** 161–163.

Professor Very, from the same observations, has derived the value 0.89 for the albedo of the earth. His reasoning is substantially as follows: The light of the full earth, as seen from the moon, is 1/9000 that of the sun (according to his own reduction, a result which differs little from that of the present paper). But, according to Zöllner, the light of the full moon is 1/618,000 that of the sun; and the angular area of the earth as seen from the moon is 13.4 times that of the moon as seen from the earth. Hence, if the earth were of the same albedo as the moon, its light at the full would be 1/46,120 of sunlight, and its actual albedo must be 5.12 times that of the moon. Zöllner's value for the albedo of the moon is 0.174; therefore that of the earth is 0.89.

As Guthnick has pointed out,[1] the trouble here lies with the value adopted for the albedo of the moon. Zöllner, in his original discussion,[2] distinguishes between the "apparent albedo" of the moon, which is that derived from the observations in the ordinary way by application of Lambert's law for a smooth, diffusely reflecting sphere, and the "true albedo," which depends on his assumption that the whole surface of the moon is covered with declivities of an average slope of 52°, the sides of which reflect diffusely according to Lambert's law. Under these highly artificial conditions, the slopes, receiving sunlight more obliquely than would a level surface, would appear less luminous at full moon, and a corresponding correction is necessary to find the "true albedo" which the moon would exhibit if these hypothetical irregularities were smoothed flat. Zöllner's "true albedo," 0.174, does not therefore represent the actual intensity of the light of the full moon. His "apparent albedo," 0.120, as can easily be verified, corresponds to his observed ratio of moonlight to sunlight, and to Lambert's law. The corresponding value of p, in the notation of the present paper, is 0.080. Hence, what Very's argument really shows is that the value of p for the earth is 0.080×5.12, or 0.41. On his assumption that the law of variation with phase is the same as for Venus, this must be multiplied by $q = 1.20$ to get the actual albedo, which thus comes out 0.49. This is about 10 per cent higher than the value found above, the difference arising from the difference between the values deduced by Very and by the writer for the brightness of the full earth, but being less than the probable error of either.

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[1] *Astronomische Nachrichten*, **198**, 253, 1914.

[2] *Photometrische Untersuchungen*, p. 162.

It appears, therefore, that the value of the earth's albedo resulting from Professor Very's measures of the earth-shine, far from being inconsistent with Abbot's value of the solar constant (1.93 calories), is actually in agreement with it, within the unavoidable uncertainty of the photometric measurements.

In conclusion, the attention of photometric observers may be called to the uncertainty that still exists concerning some of the most fundamental data. Further and more accurate observations, by visual and photo-visual methods, of the relative brightness of the moon and stars, and of the standard candle and the stars, and observations of the earth-shine, both visual and photographic, are especially desirable.

ADDENDUM

Mr. Evershed, in a recent letter to *Nature*,[1] calls attention to the fact that the disk of the moon, when rising or setting behind distant snow-clad mountains, appears very similar in color and brightness to the snow, and queries whether this observation is consistent with the low albedo ordinarily attributed to the moon, especially when it is considered that the rays reaching the observer from the snow and the moon have traversed approximately equal thicknesses of the atmosphere. His observations regarding the moon's appearance under these circumstances are confirmed by the writer's very vivid recollections of sunrise at Taormina, with the moon setting behind the snowy slope of Etna. On that occasion, however, the moon, though as white as the snow, was hardly as bright, seeming in comparison almost unsubstantial, like a bubble.

Yet the photometric observations summarized above show beyond a doubt that the albedo of the moon is actually very low—probably less than one-fifth of the value for a snow-clad satellite—and add to the difficulty by indicating that the moon's light is decidedly yellowish.

The solution of the problem is probably to be found in two facts: (1) In comparing the moon, seen in daylight, with distant snows, the comparison is instinctively made between the high lights in both areas, that is, between the brightest parts of the disk and the fully illuminated snow, while the dark

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[1] *Nature*, **96**, 369, 1915.

maria are compared with the shadows on the snow. As the albedo of these bright areas is much greater than the average for the moon's disk as a whole, this diminishes the difficulty, though it does not by itself remove it. (2) When the moon is seen by daylight, most of the light which appears to come from it comes in reality from the illuminated air between the observer and the moon, and relatively little from the moon itself. A glance at the half-moon by daylight shows that the *maria* are conspicuously blue, while the phase limb is hardly visible, the light of the sky overpowering the faint illumination of this part of the lunar surface. With even a low telescopic power these effects are greatly exaggerated. According to Kimball's measurements, the diffused light of the sky at noon, even on a clear day, illuminates a horizontal surface about one-tenth as strongly as the direct rays of the sun. It follows that the mean surface brightness of the noonday sky is nearly equal to that of the full moon, and therefore several times greater than that of the half-moon. When the moon and sun are both low in the sky, their light is much diminished by atmospheric absorption. That of the sky is also less than with a high sun, but not so greatly weakened, as is obvious without instrumental aid to anyone who compares the illumination of a vertical surface by the rays of the setting sun with that of a neighboring horizontal surface by the sky.

When the moon is rising or setting, with the sun above the horizon, the light which comes from the sky in front of it must therefore be considerably greater than that which really comes from the moon, and it must appear at least two or three times brighter, and also much whiter, than it would if rising, under otherwise identical circumstances, during the night.

The combination of these two factors appears to be sufficient to explain Mr. Evershed's observations.

Princeton University Observatory
January 20, 1916

ON THE TEMPERATURE AND RADIATION OF THE SUN[1]

By FELIX BISCOE

I. OBSERVATIONS WITH THE SPECTRO-BOLOMETER

The purpose of this part of the paper is the determination of the temperature of the sun from the intensity of radiation for individual wave-lengths in its spectrum. The observations at the Smithsonian Astrophysical Observatory[2] at Washington will be employed here.

1. *The intensities of radiation of the sun outside the earth's atmosphere.*—The observations mentioned above were principally made at Mount Wilson and Mount Whitney in 1909–10, the rays of the whole visible solar surface being employed. The coefficients of transmission of the earth's atmosphere were determined by the secant law. In the mean from the whole observational material the intensities I of the solar radiation shown in Table I were obtained for the different wave-lengths λ in μ in the normal spectrum, outside the earth's atmosphere, expressed in arbitrary units.

TABLE I

λ	I	λ	I	λ	I
0.300........	539	0.430......	5321	0.800......	2665
0.325........	1271	0.450......	6027	1.000......	1657
0.350........	2684	0.470......	6240	1.300......	898
0.375........	3459	0.500......	6062	1.600......	532
0.390........	3614	0.550......	5623	2.000......	247
0.400........	4338	0.600......	5042	2.500......	43
0.420........	5251	0.700......	3644	3.000......	14

For the extreme values of the wave-lengths in this table the intensities I are affected with some uncertainty. The value of the solar constant, that is, the quantity of heat which outside the earth's atmosphere the sun radiates upon the square centimeter of the earth with vertical incidence per minute, reduced to the mean distance of the earth, is determined to be

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[1] Extract from the author's paper in the *Warsaw University News*, 1915 (in Russian). See also the author's paper in *Astronomische Nachrichten*, **183**, 241, 1910, entitled "Die Temperatur der Sonne."

[2] C. G. Abbot, F. E. Fowle, and L. B. Aldrich, *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **3**, 196–201, 134, and 157, 1913.

$$S = 1.932 \text{ gm cal.}$$

2. *Determination of the coefficients of transmission of the solar atmosphere as well as the intensities of radiation of the solar photosphere.*—That part of the observations mentioned which concerns itself particularly with the decrease in intensity on the sun's disk toward the edge was chiefly made in 1907 at Washington. From the mean of many observations which were made at different degrees of transparency of the earth's atmosphere, as well as at different zenith distances of the sun, there were derived for the selected wave-lengths λ at definite fractions ρ of the solar radius from the center, the accompanying ratios of intensity (Table II) expressed in units of the intensities at the center of the sun's disk.

TABLE II

λ \ ρ	0.00	0.20	0.40	0.55	0.65	0.75	0.825	0.875	0.92	0.95
0.323.....	1.0000	0.960	0.897	0.835	0.775	0.690	0.660	0.530	0.452	0.382
0.386.....	1.0000	.9801	.9258	.8556	.7920	.7097	.6326	.5543	.4826	.4177
0.433.....	1.0000	.9780	.9271	.8663	.8060	.7290	.6471	.5827	.5098	.4499
0.456.....	1.0000	.9857	.9416	.8848	.8314	.7564	.6810	.6160	.5377	.4706
0.481.....	1.0000	.9871	.9438	.8914	.8401	.7706	.7007	.6378	.5658	.4988
0.501.....	1.0000	.9850	.9454	.8945	.8449	.7773	.7106	.6501	.5826	.5171
0.534.....	1.0000	.9870	.9499	.9018	.8561	.7919	.7282	.6720	.6053	.5478
0.604.....	1.0000	.9894	.9568	.9129	.8722	.8160	.7606	.7102	.6485	.5935
0.670.....	1.0000	.9906	.9612	.9241	.8870	.8376	.7860	.7404	.6803	.6292
0.699.....	1.0000	.9902	.9629	.9261	.8903	.8412	.7925	.7476	.6909	.6371
0.866.....	1.0000	.9922	.9690	.9388	.9108	.8711	.8302	.7918	.7440	.6987
1.031.....	1.0000	.9978	.9774	.9508	.9251	.8886	.8507	.8159	.7724	.7298
1.225.....	1.0000	.9948	.9756	.9530	.9316	.9007	.8654	.8336	.7944	.7565
1.655.....	1.0000	.9965	.9820	.9657	.9504	.9275	.9012	.8770	.8471	.8155
2.097.....	1.0000	0.9965	0.9858	0.9709	0.9563	0.9361	0.9149	0.8924	0.8664	0.8377

The values given in Table I furnish us with the mean intensities of the radiation of the whole visible solar surface. For our further investigations we need, however, the values for radiation of the whole visible solar surface, but of such intensity as is found at the center of the sun's disk, the same to be expressed for each wave-length in the same units as in Table I. In order to accomplish this, we draw on the axis of abscissas, at the fraction ρ of the solar radius R, the ordinates y of the magnitude of the relative intensities of Table II, which we assume to be valid up to the middle points of the adjacent intervals of the abscissas. Then the entire intensity of radition of the visible solar surface in units of the intensity at the center will be expressed for each wave-length by the sum of the volumes of cylinders which result from rotating about the axis of ordinates of the separate rectangles. The formula for the volumes is

$$V_0 = 2\pi R\frac{0.0+0.1}{2}R(0.1-0.0)y_0,\ V_1 = 2\pi R\frac{0.1+0.3}{2}R(0.3-0.1)y_1,\ \ldots\ .$$

$$V_{10} = 2\pi R\left(0.975+\frac{1-0.975}{3}\right)R\frac{1-0.975}{2}\frac{y_9}{2},$$

where in the last volumes the rotation of a triangle with the height equal to the half of the last ordinate is assumed. If we now divide the sum of these volumes by the base πR^2, we shall obtain for each wave-length, with the exception of the limiting values of Table II, the mean intensities m of the radiation on the visible solar surface in units of the intensity at the center of its disk given in Table III.

If we now divide the intensities I of Table I by the corresponding fraction m, we obtain for each wave-length the value of the radiation for the whole visible solar surface reduced to the intensity at the center of the disk.

On the assumption that the decrease in brightness on the solar surface is produced in the inappreciably radiating solar atmosphere itself, the coefficients of transmission of the solar atmosphere can be derived as follows from Table II:

If I is the intensity of radiation of the solar photosphere; I_0, the intensity outside the earth's atmosphere at the center of the visible solar disk; I_z, that at any point whatever in the apparent distance ρ from the center expressed in units of the solar radius; further, if z be the angle which the ray of light proceeding from this point and reaching the observer forms with the solar radius prolonged through this point; and ξ is the refraction of this ray in the solar atmosphere; then Laplace's theory of extinction yields the following equation:

$$\log\frac{I_z}{I} = -K\frac{\xi}{\sin z} = -K\alpha\sec z\ ,$$

where K denotes a constant, and the refraction on the sun is expressed by $\xi=\alpha \tan z$. From this expression it follows that

$$\frac{I_z}{I_0}=p^{\sec z-1} \tag{1}$$

if the transmission coefficient $\frac{I_0}{I}$ is set equal to p.

From the fundamental equation of the theory of refraction applied to the points of a light-ray where it leaves the solar surface or the point of observation, there follows the relation:

$$\sin z=\frac{D\cdot\sin\sigma}{R\cdot\mu}=\frac{\sin\sigma}{\sin\sigma_0\cdot\mu}=\frac{\rho}{\mu}, \tag{2}$$

if R is the radius of the sun, μ the coefficient of refraction at its surface, D the distance of the place of observation from the center of the sun, σ the apparent distance of the point of the solar surface under consideration from its center, while σ_0 indicates the apparent radius of the sun, as it must appear without its atmosphere. For the quotient $\frac{\sin\sigma}{\sin\sigma_0}$ can therefore be substituted the apparent distance ρ of the point from the sun's center, expressed in units of the sun's radius. Thus (1) contains the two unknowns p and μ when we consider (2). If we denote by p_0 and μ_0 their approximate values and if we let the true values be $p=p_0+\delta p$ and $\mu=\mu_0+\delta\mu$, then we obtain for the solution, after differentiating (1),

TABLE III

λ	m	I	f	T	λ	m	I	f	T
0.386	0.705235	3573	12437	7200	0.670	0.812568	4040	7420	7200
0.433	0.719581	5450	17077	7700	0.699	0.816577	3657	6560	7100
0.456	0.741503	6120	16538	7700	0.866	0.842530	2300	3702	6800
0.481	0.754034	6210	15605	7700	1.031	0.858651	1580	2370	6700
0.501	0.760813	6060	14573	7700	1.225	0.867743	1060	1538	6800
0.534	0.773816	5790	13058	7600	1.655	0.892505	480	632	7000
0.604	0.795179	5000	9820	7400	......	....	......	......	......

$$\sigma=-\frac{I_z}{I_0}+p_0^{\sec z_0-1}+(\sec z_0-1)p_0^{\sec z_0-2}\cdot\delta p-p_0^{\sec z_0-1}\frac{\log p_0}{0.4343}\cdot\frac{\sec^3 z_0\cdot\sin^2 z_0}{\mu_0}\cdot\delta\mu\,.$$

Nine such equations were then formed according to Table II for each of the wave-lengths of Table III. The solution of these equations gave the most probable values for p and μ as they are represented in Table IV. After introducing these values in (2) and (1), a certain theoretical value $\frac{I_z}{I_0}$ is obtained, which differs from the empirical values of Table II in the sense O−C by small amounts which are further given in Table IV.

If we finally divide the intensities I of Table I, after they have been divided by the appropriate fractions from Table III, still further by the corresponding fractions p of Table IV, we obtain for each wave-length the intensity of radiation J of the whole solar photosphere directed toward us, which would appear equally bright all over except for the absorption in the solar atmosphere. The two operations mentioned were performed at once because at first the intensities I were interpolated from Table I, for which the values m and p are given in Tables III and IV, respectively. The results are contained in the fourth and ninth columns of Table III.

3. *A determination of the temperature of the solar photosphere from an application of the radiation formula for a perfect radiator.*—If S' is the solar constant corrected for absorption in the solar atmosphere; if $u=15'59''.6$, being the mean apparent radius of the sun, and if i is the specific intensity of radiation of an element of surface, then we obtain from the fundamental photometric laws for self-luminous bodies, as well as from the definition of the solar constant,

$$S'=60\, i\, \pi \sin^2 u\,.$$

From analogous values for the monochromatic solar constant S'_λ we obtain, for the intensity of radiation i_λ at the wave-length λ, the formula

$$i_\lambda=\frac{S'_\lambda}{60\, \pi \sin^2 u}\,.$$

The quantity S'_λ is here determined as follows. In Fig. 1 the energy-curves A and B are so constructed that to 1 mm along the axis of abscissas corresponds a range of wave-lengths

$$\delta\lambda=0.000001 \text{ cm}$$

TABLE IV

λ	p	μ	0.20	0.40	0.5	0.65	0.75	0.825	0.875	0.92	0.95
0.380	0.40737	1.07292	−0.0040	−0.0068	−0.0069	−0.0020	+0.0105	+0.0300	+0.0342	+0.0540	+0.0621
0.433	0.44353	1.06814	−0.0075	−0.0111	−0.0071	−0.0033	+0.0092	+0.0202	+0.0364	+0.0547	+0.0690
0.456	0.49906	1.05919	−0.0017	−0.0044	−0.0037	+0.0005	+0.0076	+0.0194	+0.0323	+0.0448	+0.0544
0.481	0.52776	1.05698	−0.0012	−0.0061	−0.0051	−0.0024	+0.0056	+0.0191	+0.0314	+0.0486	+0.0578
0.501	0.54657	1.05502	−0.0039	−0.0070	−0.0069	−0.0048	+0.0023	+0.0164	+0.0296	+0.0501	+0.0608
0.534	0.57299	1.05388	−0.0027	−0.0061	−0.0067	−0.0041	+0.0019	+0.0151	+0.0294	+0.0480	+0.0656
0.604	0.64035	1.04796	−0.0023	−0.0073	−0.0121	−0.0125	−0.0089	+0.0026	+0.0155	+0.0329	+0.0502
0.670	0.67003	1.04497	−0.0019	−0.0063	−0.0078	−0.0080	−0.0022	+0.0087	+0.0230	+0.0415	+0.0584
0.699	0.68265	1.04374	−0.0026	−0.0060	−0.0087	−0.0090	−0.0049	+0.0070	+0.0204	+0.0382	+0.0543
0.866	0.73741	1.04207	−0.0021	−0.0061	−0.0086	−0.0076	−0.0034	+0.0065	+0.0179	+0.0351	+0.0524
1.031	0.77639	1.03890	+0.0026	−0.0017	−0.0050	−0.0060	−0.0048	+0.0014	+0.0105	+0.0254	+0.0406
1.225	0.79488	1.03900	−0.0008	−0.0054	−0.0068	−0.0057	−0.0022	+0.0030	+0.0117	+0.0268	+0.0428
1.655	0.85077	1.03515	−0.0004	−0.0045	−0.0056	−0.0046	−0.0022	+0.0017	+0.0084	+0.0210	+0.0330

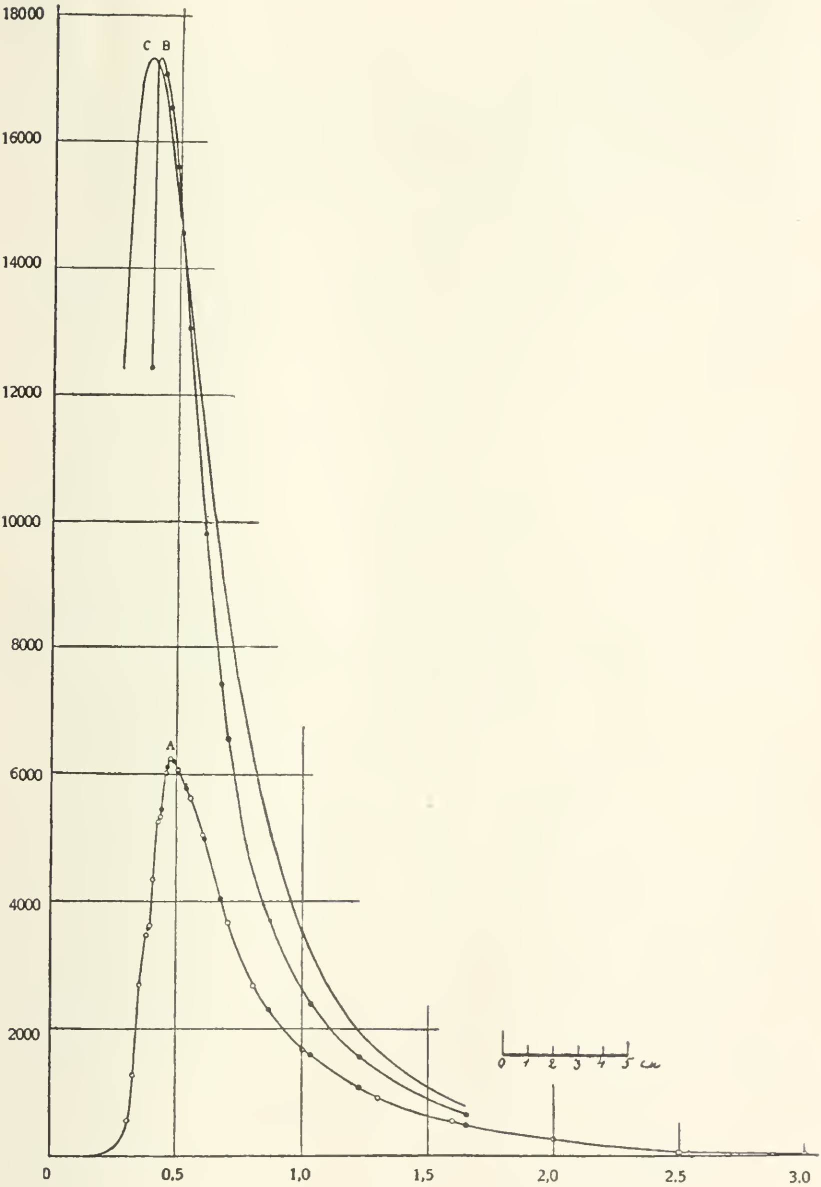
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0,5
1,0
1,5
2,0
2,5
3.0
C
B
A
0 1 2 3 4 5 cm

FIG. 1

and in the axis of ordinates the arbitrarily assigned unit of the quantity I is that in Table I and of f in Table III. Then to the correct solar constant S' corresponds the total surface F' sq. mm, between the energy-curve B and the axis of abscissas, while to the quantity S for the interval of wave-length $d\lambda$ for a definite wave-length λ there corresponds an element of surface f sq. mm. Thence follows

$$S'_\lambda = \frac{S'}{F'} \cdot f ,$$

where the ratio $\frac{S'}{F'}$ is also equal to that for the original solar constant $S = 1.932$ gm cal., and the corresponding surface $F = 36{,}000$ sq. mm of the corresponding energy-curve A.

If we apply the formula for i_λ to an interval of 1 cm instead of the interval of wave-length $\delta\lambda = 0.000001$, we obtain

$$i_\lambda = \frac{f \cdot S}{d\lambda \cdot F \cdot 60\pi \sin^2 u} . \tag{1}$$

For the determination of the absolute temperature T for a perfect radiator, from its intensities of radiation between wave-lengths λ and $\lambda + d\lambda$, Planck has given the following formula

$$i_\lambda = \frac{c_1 \lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1} , \tag{2}$$

where the constants are

$$c_1 = 0.282 \cdot 10^{-12} \quad \text{gm cal. per sec per sq. cm}$$

and

$$c_2 = 1.4597 \text{ cm degrees.}$$

Thus (2) can be employed by comparison with (1) for the determination of the temperature of the solar photosphere. After substitution of the appropriate values there results the following numerical equation:

$$\frac{1.4597}{e^{\frac{c_2}{\lambda T}} - 1} = \frac{[0.699150 - 18]}{\lambda^5 \cdot f} ,$$

where for λ we substitute the wave-lengths in centimeters and for f the values from Table III. These absolute temperatures T of the solar photosphere, determined from the separate wave-lengths λ, are collected in the fifth and tenth columns of Table III. The energy-curve of a perfect radiator of the absolute temperature 7700° C is represented in Fig. 1 by *C*.

4. *Discussion.*—It was assumed in this paper as a fact of observation that the sun is surrounded by a limiting stratum, the photosphere, which possesses not only an apparent but a material reality; also that the density of the photosphere from the limb to the deeper strata increases so rapidly that a comparatively small thickness of it is quite opaque; and that the surface of the photosphere radiates like a perfect black body. The gaseous strata lying above the photosphere, on account of their relatively small density and hence also on account of small temperature, can yield only a very nappreciable radiation in comparison with the photosphere; they reveal themselves, by a selective and general absorption of the photospheric radiation, as the solar atmosphere. On this assumption the coefficients of transmission of the solar atmosphere were derived, which appear entirely probable values, in respect to their absolute magnitude and their increasing progress with increasing wave-lengths. The indices of refraction derived therefrom have also a natural progression, decreasing with increasing wave-length; as to their absolute magnitudes, it may be remarked that in these is contained, not only the effect of the density on the absorption, which is taken into account, but also the effect of the height of the solar atmosphere, which operates similarly but which has not been taken into account; hence, the indices of refraction could only be smaller. The values of the unknown distribution of intensity on the solar surface derived from these values so far differ very little from that actually observed.

From the final intensity of radiation in the solar photosphere we obtain by the formulae for a perfect radiator the absolute temperature of the solar surface to be, on the average, $T_m = 7300° \pm 100°$ C. If we consider the great effect of the numerous corrections on the final intensities of radiation, as well as the uncertainties in the latter, particularly for longer wave-lengths, on the derived temperatures, we may assert that the true radiation of the solar surface may be represented by the known laws for purely thermal radiation of perfectly black bodies of definite temperature. There are other topics which concern themselves with the same thing. In certain theories there is assumed a continuous transition of conditions in the sun as we pass from inside outward, and the observed limitation of the solar body is regarded as an optical appearance. The lack of validity of this assumption has been shown from various sides.[1] On the assumption of a continuous transition of condition in the sun, the distribution of brightness on the sun's surface has been derived by Schwarzschild[2] from an assumed equilibrium of radiation in the solar atmosphere. Independently of the fact that the equilibrium of radiation leads to an incomprehensible atmosphere which reaches into infinity with a very high constant temperature at its extreme strata, the distribution of the brightness of the white light on the solar surface derived from it gives deviations from those actually observed, which are appreciably larger than those resulting from our assumption. The distribution of intensity similarly derived by E. Oepik[3] for different wave-lengths also gives greater residuals than does ours. The solar temperature was also derived by D. A. Goldhammer[4] from the intensities of radiation in its spectrum, but without taking into account the absorption in the solar atmosphere, whence no agreement of the separate values could be expected. G. A. Shook[5] determined the solar temperature from the intensities of radiation directly observed for several wave-lengths at different points of the solar disk; and as was to be expected, the separate values vary in a manner corresponding to the progression of absorption for different wave-lengths on the sun's disk. C. G Abbot[6] remarks that the intensities for different wave-lengths of the radiation coming to us from the whole visible solar surface, outside the earth's atmosphere, do not correspond to the distribution of energy in the spectrum of a perfect radiator of a definite high temperature. If these total intensities are not reduced to those at the center of the sun's disk, and then subsequently corrected for the absorption in the solar atmosphere, we have no reason to expect an agreement of the corresponding curves.

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[1] R. Emden, *Gaskugeln*, "Schmidt'sche Theorie."

[2] "Über das Gleichgewicht der Sonnenatmosphäre," *Nachrichten von der kgl. Ges. der Wiss. zu Göttingen*, 1, 41, 1906.

[3] "Zur Theorie der Sonnenstrahlung," *Astronomische Nachrichten*, **198**, 49, 1914.

[4] "Die Temperatur der Sonne," *Annalen der Physik*, **25**, 905, 1908.

[5] "A Determination of the Sun's Temperature," *Astrophysical Journal*, **39**, 277, 1914.

[6] C. G. Abbot, *op. cit.*, 198–201.

II. OBSERVATIONS WITH FILTERS

In this part of the paper we shall examine my own observations of the radiation of the sun and its variation. The observations were made with filters in connection with the compensation pyrheliometer of Knut Ångström.

1. *The filter for radiation.*—I employed as monochromatic filters colored glasses from Jena, which transmitted, respectively, only limited regions in the red, green, and violet. The polished surfaces were 3×3 cm, and the thicknesses were: red, 4.898; green, 4.958; violet, 4.954 mm. The coefficients of transmission of the filters, i.e., the intensities of the transmitted radiation expressed in percentage of the incident radiation, were determined for different wave-lengths as follows. The radiation, in the one case of the sunlight, and in the other of a black body (that of O. Lummer and F. Kurlbaum) raised to a temperature of 1573° Abs., was dispersed in a spectrometer, and the intensities were then measured, both with and without the filters, by the galvanometer deflections from a linear thermopile. In the mean from such experiments, the values of the coefficients of transmission for the separate filters shown in Table VII were obtained.

TABLE VII

λ...	0.405	0.433	0.456	0.481	0.501	0.534	0.604	0.670	0.699	0.866	1.031	1.225	1.655	2.097
r...	0	1	2	4	3	3	9	86	87	91	88	84	87	86
g...	0	0	1	2	5	8	2	0	0	0	0	2	7	17
v...	10	15	12	6	3	1	0	0	0	0	0	0	1	1

2. *Determination of the solar constant, both complete and monochromatic, by means of the compensation pyrheliometer, with filters.*—The observations in question were made by me in part in the year 1913, at Sardar-Bulag, longitude 44°23′, latitude 39°42′, altitude 2350 m, in the Pass between the large and small Ararat in the Caucasus; but principally from the year 1912 at the University of

Warsaw. The method of observation with the pyrheliometer is well known.[1] The filters were placed in a cylindrical cap of the instrument. Table VIII represents a sample page of the records.

TABLE VIII

SATURDAY, JULY 12, 1913

Therm.=17°0–20°0. Barom. 578.9–579.7 mm. Abs. humidity=9.6 mm.
Rel. humidity=55 per cent

O WITHOUT FILTER		*r*		*g*		*v*	
Time	Angle	Time	Angle	Time	Angle	Time	Angle
5^h36^m	44°	5^h46^m	38°	5^h58^m	80°	6^h15^m	66°
	41		35		70		78
	45		38		82		67
	44		37		78		55
5 42	43	5 50	37	6 6	79	6 45	95
6 54	58	7 5	43	7 14	83	7 30	72
	54		42		95		75
	55		44		86		82
	54		43		105		78
6 58	58	7 8	44	7 26	61	7 42	73
7 46	61	7 52	46	8 5	86	8 18	85
	57		44		95		79
	64		44		93		84
	59		45		94		83
7 49	58	7 58	44	8 14	92	8 32	78
8 40	65	8 46	46	8 54	85	9 11	80
	62		45		95		75
	64		46		95		85
	59		44		95		80
8 43	64	8 49	46	9 5	95	9 23	87
10 44	64	9 37	46	9 51	88	10 7	93
	58		44		107		90
	68		49		92		85
	60		46		105		78
10 52	65	9 43	45	10 3	102	10 19	88
10 56	65	10 31	46	11 7	95	11 30	82
	61		48		90		85
	61		45		118		92
	61		46		91		63
11 00	67	10 37	47	11 22	85	11 44	120

The observations were usually made as follows: first, without filter; then with filter—red, green, violet, successively; and in each case the heating current was commuted five times, and the time at the beginning and end of each group was noted. The theories of extinction are strictly applicable for monochromatic light. They should theoretically yield for the earth's atmosphere larger coefficients of transmission in proportion as a greater path is covered, when white light is used,[1] but quantitatively, by experiment from photometric observations of stars, no such difference has been noted;[2] therefore, in what follows Laplace's extinction theory is employed, not only for the monochromatic observations, with filters, but also without them. According to this theory, if I_z represents the observed intensity of radiation of the sun at a zenith distance z, and if I denotes the same outside the earth's atmosphere, ξ the refraction, and p the coefficient of transmission of the latter, then we shall have

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[1] K. Ångström, "Kompensationspyrheliometer," *Wied. Ann.*, **67**, 633, 1899.

$$\log \frac{I_z}{I} = -K \frac{\xi}{\sin z} \tag{1}$$

and

$$\log \frac{I_0}{I} = \log p = -K\alpha_0 \, .$$

Since the refraction at $z=45°$ is represented by the series

$$\xi_{45} = \alpha_0 (1 - 0.001 \ . \ . \ . \ .) \, ,$$

we may employ, in the determination of p, the refraction at 45° for that incident in place of α_0. Further, there exists between the intensity of radiation sought, I_z, and the intensity i_z actually measured, the relationship

$$I_z = c \cdot i_z^2$$

or

$$I = c \cdot i^2 \, ,$$

where c is the constant of the pyrheliometer. Thus we get

$$\log i_z^2 = \log i^2 - K \cdot \frac{\xi}{\sin z} \, ,$$

or

$$Y = Y_1 - \tan \alpha \cdot X \tag{2}$$

that is, the equation of a straight line. Therefore, if we draw from the abscissas $X = \frac{\xi}{\sin z}$ and ordinates $Y = \log i_z^2$, according to the observations with one filter, the corresponding points ought to lie in a straight line, the intersection of which with the Y-axis furnishes us with $Y_1 = \log i^2$, and its acute angle with the X-axis, moreover, furnishes $\tan \alpha = K$.

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[1] C. G. Abbot, *op. cit.*, 2, 13. [2] G. Müller, *Die Photometrie der Gestirne*, p. 144.

The abscissas were determined by computing for the middle of the time of observation for each group the apparent zenith distance of the sun, and the true refractions from the meteorological data. The ordinates were determined by taking the mean of each five observed angles and multiplying it for the observations zero and r by 0.005A, and for g and v by 0.0005A, which furnishes us the intensities of the current. As the constant of the instrument the value $c = 13.72$ was taken. The most probable values for $\log i^2$ and K are computed by the method of least squares from equations of condition of the following form,

$$-\log i_z^2 + \left(\log i_0^2 - K_0 \frac{\xi}{\sin z}\right) + \delta(\log i_0^2) - \frac{\xi}{\sin z}\delta(K_0) = 0\,,$$

where the initial values were obtained from a straight line drawn approximately.

The results of the observations thus reduced are given in Tables IX A and IX B where the following abbreviations were used: t = the time; S_0 = the full solar constant; S_r, S_g, and S_v, respectively, the portions transmitted by the filter; ϵ = its mean error in per cent; p = the coefficients of transmission; n = the number of groups of observations; T = the mean temperature; B = the barometric pressure; e = the absolute humidity; f = the relative humidity; and r = the "relative number" of sun-spots.

3. *Investigations of the short-period variations of the solar constant and their causes.*—In the first place, attention should be called to the effect noted in our observations of the outbreak of the volcano Katmai in Alaska on the general transparency of the earth's atmosphere; this was reported from many places, in the second half of 1912, during which period our observations began. Thus the coefficients of transmission p for white and colored light are especially small in Table IX B for August 1912 and the deviations of the solar constant S_0 are striking.

TABLE IX A—SARDAR-BULAG

t	S_o	ϵ	p	n	S_r	ϵ	p	n	S_g	ϵ	p	n	S_v	ϵ	p	n	T	B	e	f	r
1913 July 11	1.644	3	0.863	6	0.820	2	0.912	6	0.039	8	0.872	7	0.033	6	0.733	7	18.2	577.5	7.7	36	10
12	1.742	3	0.842	6	0.851	0	0.896	6	0.038	5	0.870	6	0.032	6	0.830	6	18.5	579.3	9.6	55	13
13	1.535	1	0.886	5	0.811	1	0.909	5	0.037	5	0.877	4	0.029	10	0.802	3	17.6	578.0	10.9	62	8
14	1.563	2	0.890	7	0.830	1	0.900	7	0.037	5	0.864	7	0.041	10	0.672	7	19.6	577.5	10.2	43	7

TABLE IX B—WARSAW

t	S_o	ϵ	p	n	S_r	ϵ	p	n	S_g	ϵ	p	n	S_v	ϵ	p	n	T	B	e	f	r
1912 Aug. 5	1.354	15	0.601	6	0.790	5	0.638	6	0.032	19	0.719	6	......	...	......	...	21.5	749.8	11.6	65	0
13	1.461	7	0.512	16	0.935	5	0.524	15	......	...	......	...	......	...	......	...	14.8	745.9	7.5	63	0
31	1.415	5	0.770	7	......	...	......	...	0.034	4	0.809	6	0.038	34	0.488	6	18.0	749.0	9.7	67	0
Sept. 14	1.366	11	0.707	7	......	...	......	...	0.031	4	0.795	7	......	...	......	...	11.2	753.5	7.2	77	19
Dec. 22	1.820	12	0.728	9	0.780	9	0.813	9	......	...	......	...	......	...	......	...	4.0	751.8	4.3	79	7
1913 Jan 4	1.549	12	0.784	5	1.096	11	0.786	5	0.036	17	0.838	5	......	...	......	...	− 0.7	764.6	4.3	99	7
16	1.384	6	0.837	5	0.847	1	0.858	5	0.029	10	0.888	4	......	...	......	...	− 6.3	757.3	1.6	56	7
1914 Apr. 29	1.528	5	0.812	7	0.820	4	0.851	7	0.036	6	0.833	4	......	...	......	...	16.5	752.4	6.8	49	(60)
May 4	1.750	3	0.793	8	1.069	2	0.770	7	......	...	......	...	......	...	......	...	11.0	758.8	4.0	44	(0)
15	1.603	3	0.789	5	0.982	1	0.795	4	......	...	......	...	......	...	......	...	11.3	758.8	5.2	55	(c)
19	1.734	3	0.803	7	0.807	2	0.912	7	......	...	......	...	......	...	......	...	18.2	754.5	7.1	48	(0)
23	2.023	5	0.655	8	1.114	2	0.691	7	0.063	5	0.711	4	......	...	......	...	16.9	759.8	7.0	52	(20)
25	1.535	4	0.780	10	0.895	5	0.780	10	......	...	......	...	......	...	......	...	20.5	749.7	8.2	50	(0)
June 13	1.816	5	0.765	8	0.817	4	0.886	8	......	...	......	...	......	...	......	...	20.8	752.9	7.2	42	(45)
14	1.888	3	0.753	7	0.757	4	0.941	7	0.045	3	0.671	6	0.032	4	0.690	6	20.6	752.0	7.2	42	(55)
15	1.549	5	0.827	6	0.914	3	0.788	6	0.041	5	0.749	5	0.040	5	0.600	5	21.9	751.8	9.8	53	(32)
July 2	2.070	5	0.607	5	0.942	7	0.789	5	......	...	......	...	......	...	......	...	21.8	754.0	11.1	58	(0)
23	1.507	2	0.789	8	0.968	3	0.723	8	......	...	......	...	......	...	......	...	26.0	740.1	14.1	59	(0)

But independently of this, there are noted further conspicuous oscillations of the values for the solar constant which follow each other at short intervals. On closer investigation, we may establish therefrom the fact that these oscillations of the solar constant stand in a relation with the variations of the corresponding coefficients of transmission of the earth's atmosphere: large values of the solar constant correspond to small values for the coefficients of transmission and vice versa.

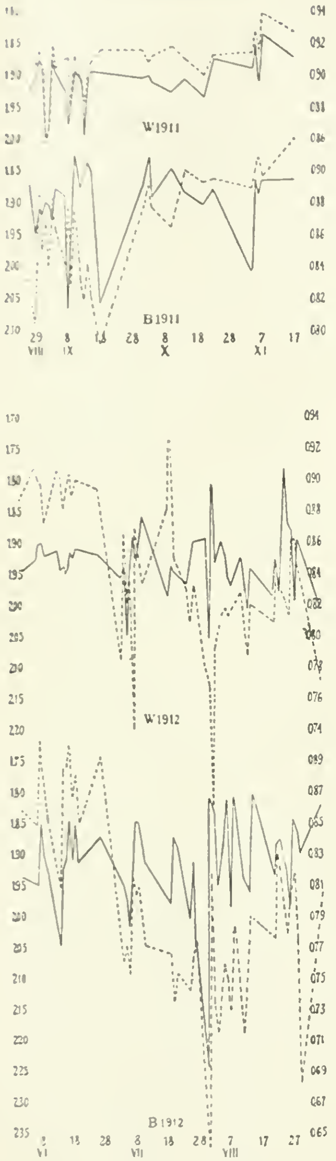

Fig. 2

To investigate this in a more general manner the corresponding results for white light for other observers are compared. The same relation came out in case of the observers between 1902 and 1907 at Washington; also between 1905 and 1912 at Mount Wilson, California, and from 1911 to 1912 at Bassour in Algeria.[1]

Fig. 2 represents graphically the observations which were made for the purpose of clearing up the nature of the short-period variations of the solar constant, which were obtained simultaneously in 1911 and 1912 at Mount Wilson (W), and in Bassour (B). The abscissas here denote the time, and the ordinates at the left denote the final values of the solar constant and at the right the apparent coefficients of transmission; here the solar constant is denoted by the solid line, and the coefficients of transmission by the dotted lines. Since the ordinates increase at the left and right in the opposite sense, the result is that the two phenomena are certainly demonstrated to have a parallel progression.

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[1] C. G. Abbot, *op. cit.*, 2, 96–98; 3, 102–24.

We may furthermore note that the above-mentioned effect of the eruption of Katmai in the latter half of 1912 showed itself both at Mount Wilson and at Bassour in a simultaneous decrease of the coefficients of transmission at the end of June and still more at the end of July. But since a parallel progression of the coefficients of transmission with the solar constants is demonstrable at both stations, we may indirectly expect an approximately simultaneous occurrence of the extreme values for the solar constant at both stations. C. G. Abbot called attention to such close coincidence of the extreme values of the solar constant from these last observations at two widely separated stations; but he interpreted this as evidence for an actual, short-period variation of the sun's own radiation.[1]

Our observations, however, lead to the opinion that the cause of the variations is to be sought, not in cosmical, but in terrestrial meteorological phenomena and in the methods of reduction. It is interesting to make a comparison of the observations on two days with extreme values of the coefficients of transmission and of the solar constant yielded therefrom, as well as of the absolute humidities observed, because we thus get an indication of the effect of the transparency of the layers of our atmosphere on the phenomena in question.[2] The suspicion that the contrast between the portions of the sun at the center and the portions near the limb stands in some relation to the solar constant was therefore also brought into the question of such a connection with the meteorological phenomena.[3]

TABLE X

t	S	p	e
1907 February 15	1.872	0.797	1.45
May 14	2.034	0.660	14.60

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[1] *Ibid.*, **3**, 2, 14, 117, 128, 168.

[2] *Ibid.*, **2**, 98; **3**, 101, 135.

[3] *Ibid.*, **3**, 163–165.

We now have to consider how to interpret the fact that small values for the coefficients of transmission of the terrestrial atmosphere yield large values of the solar constant and vice versa. According to formulae (1) and (2) there will correspond to the small values of the coefficients of transmission p large angles of inclination α with the X-axis for the straight line representing the observations; similarly, large ordinates at the intersection of this line with the Y-axis correspond to large values of the solar constant. According to these equations we also have

$$I_z = I \cdot p^{\frac{X}{a_0}},$$

so that $\frac{X}{a_0} = F(z)$ denotes the path traversed by the light in units of the length of path for the zenith distance zero. Now we believe that on the days of feebly transparent atmosphere which have yielded small values for the coefficients of transmission, the strongly absorbing layers of the atmosphere lie relatively deep, at the surface of the earth, so that we may take for the paths traversed $F(z) = \sec z$. This would, however, yield larger abscissas X, particularly at large zenith distances, and in contrast with the values of the mean refraction as we have computed them. The consequence of this would be that the diminished angle of inclination α also depresses the crossing point of the Y-axis so that the value of the solar constant would be diminished. Consider similarly the days of very transparent atmosphere, with only feeble absorption in strata at high levels, when observations long before or after noon yield almost the same large value of radiation as at noon. Then the paths $F(z)$, increasing less rapidly with the zenith distance than those computed with the refraction, would furnish shorter abscissas X and consequently larger values for the solar constant. It therefore appears that it would be better in reducing the observations to introduce an expression of the form

$$I_z = I \cdot p^{F(z)^n} \tag{3}$$

where $n \gtreqless 1$ makes possible a larger variability for the length of path traversed. This can also be obtained if the lengths of path are determined in units of the varying height of the absorbing stratum of the atmosphere. Let the height of the absorbing atmosphere, in units of the radius of the earth, be h, then we get for the zenith distance z from the triangle: observing station-earth's center-limiting point of the atmosphere, for the angle at the latter point

$$\sin \alpha=\frac{\sin z}{1+h};$$

hence for the angle at the center of the earth

$$\beta=z-\alpha$$

and therefore for the path in the atmosphere, in units of the height of the atmosphere h,

$$F(z)=\frac{\sin \beta}{h \sin \alpha}. \tag{4}$$

It is pleasing to find that the expressions (3) and (4) yield a better representation of the observations, even on a single day.[1] It would therefore be possible to solve the observations of a few successive days so that they can furnish the same most probable value of the solar constant I at different values of the unknowns p, n, or h; or to derive the most probable values of I from combining in a mean the results for a few successive days.

Inasmuch as the observations at elevated stations give smaller variations of the final values, we may conclude that the mean values employed in the first part of this paper, derived from very many observations of the intensities of the solar radiation as well as for the solar constant, are not affected by appreciable errors.

III. CONCLUSION

The very detailed and valuable spectro-bolometric investigations of the distribution of energy in the solar spectrum demand numerous corrections for the reduction of their final values and also require a very expensive arrangement of the observatory for making the observations. For investigating the variations over short intervals of time of the sun's own radiation in different colors, the observations with the quasi-monochromatic filters and the corresponding reductions are sufficiently exact, and at the same time are very convenient and simple.

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[1] A. Bemporad, *Meteorologische Zeitschrift*, **24**, 306, 1907; and author's paper, *Astronomische Nachrichten*, **183**, 241, 1910.

It is also possible to determine the temperature of the sun by the same process of observation with the filters, and this either for the effective temperature outside the earth's atmosphere or for the absolute temperature of the solar photosphere. This can be done by the method of isochromatic curves.[1] If the radiation of a black body at different temperatures is measured through the same filter which was used for the solar radiation, then the logarithms of the intensities of radiation, corrected to the same apparent size of the radiating objects, taken as ordinates, with the corresponding reciprocals of the absolute temperatures as abscissas, give a set of points which lie on a straight line. For the determination of the effective solar temperature, we take the part of the solar constant transmitted by the filter as the sun's intensity of radiation. But if we also measure[2] the intensities of the radiation through the same filter at different points of the sun's disk, and determine by the process given in I, 2 (this paper) the intensity of the radiation of the solar photosphere transmitted by the filter, we may derive from this same isochromatic curve the absolute temperature of the photosphere. Preliminary calculations necessary for this, as well as the preparatory experiments with the black body, have been carried out and are described in my original paper (*loc. cit.*).

On account of the great theoretical and practical importance of the investigations as to the possible variations in the solar radiation, either of short or long periods, it would be very desirable that observations of the sun similar to those described here with filters should be carried on simultaneously at numerous, widely separated stations over a long period of time.

WARSAW, PHYSICAL LABORATORY OF THE
IMPERIAL UNIVERSITY
May 1, 1915

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[1] G. K. Burgess and H. la Chatelier, *The Measurement of High Temperatures*, 1913, pp. 321, 323.

[2] G. A. Shook, *op. cit.*

ON THE CHANGES IN THE SPECTRUM, PERIOD, AND LIGHT-CURVE OF THE CEPHEID VARIABLE RR LYRAE[1]

By HARLOW SHAPLEY

I. INTRODUCTION

The seventh-magnitude star RR Lyrae is the brightest representative of the cluster-type subdivision of the Cepheid variables.[2] Because of its brightness and its period of 13.5 hours the star has appropriately received more consideration of various kinds than other members of the class. Its study, therefore, may contribute more than that of the ordinary cluster-type star to our knowledge of the nature of Cepheid variation.

The variability of the light was first noted at the Harvard College Observatory by Mrs. Fleming prior to July 1899, and was announced in 1901.[3] The discovery was effected in a unique manner. An exposure of 6½ hours on the field of the variable was interrupted for a few seconds every half-hour by means of an automatic device that stopped the driving clock and thus permitted the stars to drift to new places on the plate. The resulting rows of thirteen images furnished for each star a means of detecting any conspicuous short-period variation.[4] The new variable was immediately placed upon the observing-list of the polarizing photometer at Harvard, and the long series of measures by Professor Wendell are of highest value in studies of the star's peculiarities.[1]

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 112.

[2] We must here except β Cephei, for which the amplitude of light-variation is too small for ordinary photometric investigation. It is very likely that in the course of time many other naked-eye stars will be included among the cluster-type variables of small light-variation; for instance, β Canis Majoris for which the spectrum and the spectroscopic orbit are nearly identical with those of β Cephei (*Lick Observatory Bulletins*, **6**, 22, 1910), and 12 Lacertae (*Journal of the Royal Astronomical Society of Canada*, **9**, 423, 1915). Similarly other bright stars may be found to be Cepheids of longer periods and of small variation, such as is suggested very recently by Guthnick and Prager for α Cygni (*Astronomische Nachrichten*, **201**, 443, 1915). For 12 Lacertae, observations of the spectrum by the writer, using the 10-inch portrait lens and objective prism, have shown no conspicuous change throughout the four-hour period either in stellar magnitude or in spectrum lines.

[3] *Harvard Circulars*, No. 54, 1901; *Astronomische Nachrichten*, **154**, 423, 1901.

[4] *Harvard Circulars*, No. 29, 1898; *Astronomische Nachrichten*, **147**, 93, 1898.

That RR Lyrae is a spectroscopic variable of the typical Cepheid variety was discovered at the Lick Observatory by Kiess, who also published photometric observations of the star and some considerations of changes in the intensity of the continuous spectrum.[2] As is usual for Cepheid variables the spectroscopic study discloses an abnormally small mass function and a very small apparent orbit, while no trace of a secondary spectrum is found. The periodic shift of the spectral lines suggests the explanation that the variable moves in an orbit about an obscure companion, but the improbability of the double-star interpretation has been pointed out in a previous article.[3] It was found, in fact, from Wendell's photometric observations of RR Lyrae that the time of the rise to maximum light varied greatly from that required by a uniform period,[4] and that to a certain extent the variation was erratic. More recently Martin and Plummer, in a photographic study of the light-curve, have considered the possible interpretation of this type of variation, concluding that the evidence is strongly against the double-star interpretation, and that a hypothesis involving pulsations is much more reasonable.[5]

The present communication contains a further discussion of the anomalies of the light-variation, as far as they are shown in the several available series of measures, as well as a report on some observations of the star's spectrum. It is hoped that through investigations of this kind the true explanation of Cepheid variation can be definitely determined, and this in turn should throw light on the method of stellar evolution.

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[1] *Harvard Annals*, **69**, Part I, 45, 1909; Part II, 124, 1914.

[2] *Lick Observatory Bulletins*, **7**, 140, 1913.

[3] "On the Nature and Cause of Cepheid Variation," *Mt. Wilson Contr.*, No. 92; *Astrophysical Journal*, **40**, 448, 1914.

[4] *Publications of the American Astronomical Society*, Report of the 16th Meeting, p. 16, 1914; *Popular Astronomy*, **22**, 144, 1914.

[5] *Monthly Notices*, **75**, 566, 1915.

II. VARIATION OF THE SPECTRUM

Harvard classifies the spectrum of RR Lyrae as F.[1] Kiess observes that the hydrogen lines $H\beta$, $H\gamma$, and $H\delta$ are of equal intensity and that the calcium line K is about 0.8 as intense as $H+H\epsilon$.[2] On a photograph of the spectrum, made by the writer with a 15-degree prism mounted in connection with the 10-inch Cooke photographic triplet, the line K was seen to be less than 0.1 the intensity of $H+H\epsilon$. A series of observations was accordingly undertaken for the purpose of determining the class of spectrum at different phases of the light-variation. The results are collected in Table I.

Most of the plates listed in the first column contain multiple images, and the corresponding exposures are designated by postscript letters. The phases in the fifth column will be discussed in a later section of the paper. The brightness in the seventh column increases with the number designating it.

TABLE I

THE SPECTRUM OF RR LYRAE

No. Plate	Date	G.M.T.	Length of Exposure	Phase	Spectrum	Brightness	Remarks
	1915						
28*a*	Nov. 27	16^h 55	10^m	-0^d031	A0	5	
28*b*		17 22	15	− .013	A0	5	Low
32*a*	Dec. 8	14 22	10	+ .093	A6	2	
32*b*		14 33	10	+ .100	A8	2	
33		15 23	15	+ .135	A5	2	
34*a*		16 9	11	+ .167	A8	1	Diffuse
34*b*		16 23	11	+ .177	A8	1	
42*a*	Dec. 9	14 51	10	− .021	B9	5	
42*b*		15 4	10	− .012	A0	4	
42*c*		15 19	15	− .001	A2	4	
44		17 20	10	+ .083	A4:	3	Haze
52*a*	Dec. 10	14 29	16	+ .397	A8:	0	Faint
52*b*		14 49	24	+ .411	F2	0	
53		16 8	16	+ .466	A7:	1	Clouds
55		17 16	11	+ .513	A2:	3	Drift
56*a*	Dec. 11	14 45	10	+ .275	>A2	2	Clouds
56*b*		15 4	26	+0.288	>A5	2	Clouds

A conspicuous change of spectrum with magnitude is at once apparent. The determinations of the class, except in the cases marked as uncertain, are fairly definite, though high accuracy is not to be expected because of the small dispersion. The classification was made without knowledge of the phase of variation or the relative brightness of the star, and similarly the estimate of the integrated brightness of the spectrum, relative to that of neighboring stars, was made without knowledge of the phase or the spectral type. The relation of the variation of spectrum to variation in light is shown in Fig. 1, the spectral classification being plotted along the mean light-curve derived from Wendell's observations.

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[1] *Harvard Annals*, **55**, 25, 1907; **56**, 194, 1912.

[2] *Op. cit.*, p. 144.

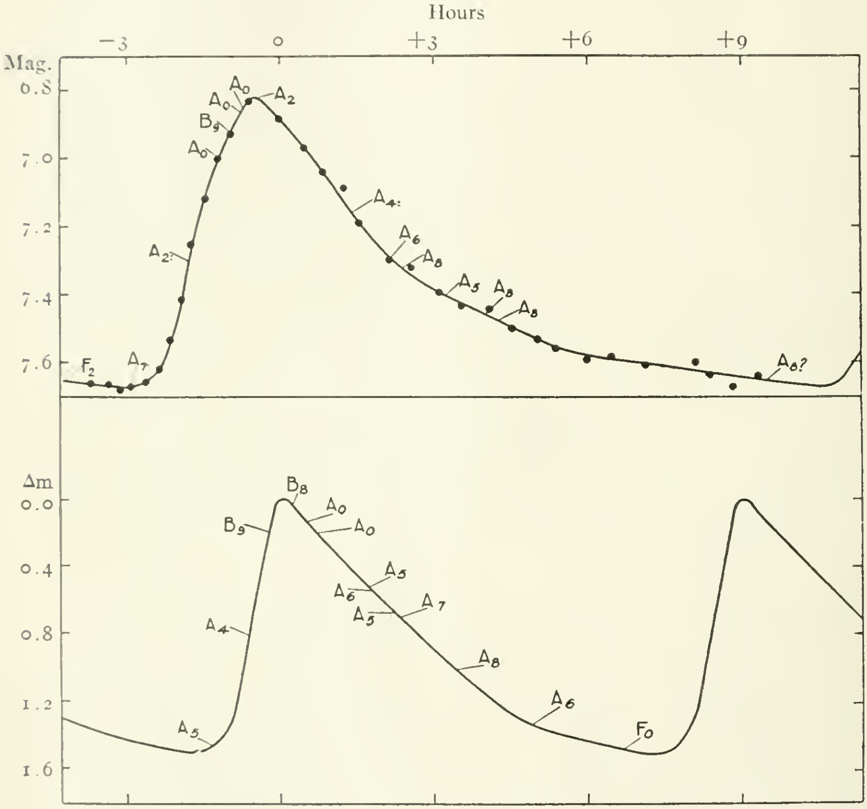

Fig. 1.—*Top:* Mean visual light-curve of RR Lyrae (observations by Wendell) and variations of spectrum (Shapley).

Bottom: Photographic light-curve of RS Boötis (Seares and Shapley) and variation of spectrum (Pease).

A notable feature of this change of spectrum with changing light is that the variation is from one recognizable spectral type to another. Moreover, it is a change from one spectral type to another that adjoins it in the usual conception of the evolutionary sequence of spectral classes. We might say that in this star we have visible proof of the evolution of stars along a spectral series—a phenomenon that is universally believed to exist, but for which our evidence heretofore (if we except the phenomena of novae) is altogether the indirect evidence of the continuous gradations from one star to the next of various constant characters (such as spectrum, motion, intrinsic luminosity), and not the direct fact of variation.[1]

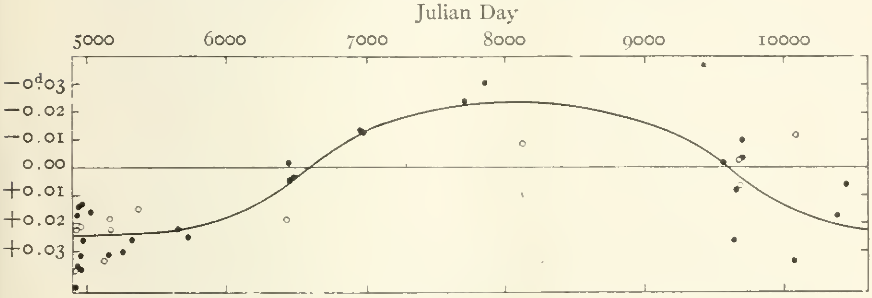

FIG. 2.—Long-period variation in light-elements of RR Lyrae. Open circles indicate observations of weight less than 3.

The data of Table I could be increased without trouble, but beyond establishing the qualitative result there is little more to be done to advantage with low dispersion. The change of spectrum with brightness is considered proved. This is in complete agreement with the results for SW Andromedae[2] and RS Boötis.[3] For all variables of the cluster type we should expect a similar variation of the spectrum, at least for those for which the photographic range exceeds the visual.

The photographic range of RR Lyrae, according to Martin and Plummer, does not exceed the visual, a result hardly to be expected

[1] See "A Short Period Cepheid with Variable Spectrum," *Proceedings of the National Academy of Sciences*, **2**, 132, March, 1916.

[2] *Mt. Wilson Contr.*, No. 92, p. 10; *Astrophysical Journal*, **40**, 457, 1914.

[3] *Mt. Wilson Contr.*, No. 92, p. 10; *Astrophysical Journal*, **40**, 457, 1914; *Publications of the Astronomical Society of the Pacific*, **26**, 256, 1914.

if the change in spectrum is from a normal A type at maximum to the redder F type at minimum.[1] Their result, however, is not necessarily conclusive, for none of the photographic series of measures completely covers the maximum. Moreover, Fig. 3 indicates that the individual maxima may vary greatly in height. It may be, then, that simultaneous observations of the spectrum and of the photographic and visual light-curves, referring the light-curves to standard magnitude scales, would show either a visual range less than the photographic, or, when equal, no change in the spectrum.

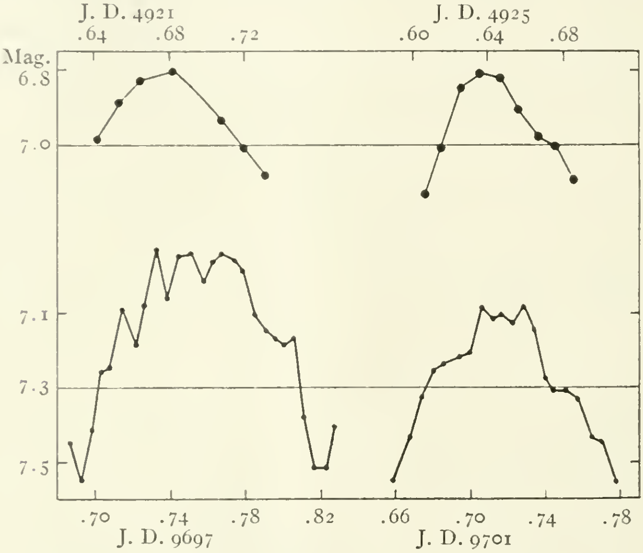

FIG. 3.—Variations in the shape of maxima of RR Lyrae.

III. THE MEAN PERIOD AND ITS SLOW VARIATION

From the first year's observation of RR Lyrae, Wendell found the approximate period $0^d.5668$, and chose as a convenient epoch of reference the round numbers J.D. 2414856.5000, G.M.T.[2] In this case, as in many others where light-elements have been determined at Harvard, the initial epoch is not intended to be coincident with a principal phase of the light-variation—maximum or minimum.[1] In the present case the chosen epoch comes nearly an hour after the observed maximum, but the two have been used as coincident in various discussions of the period. This has led to some error in determinations of the light-elements and particularly in comparisons of photographic and visual variations; but the chief criticisms of the two or three recent discussions must be that the long and important series of Harvard observations have been almost completely neglected, and that the periodic irregularities in the form of the light-curve have been ignored.

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[1] The variation in photographic magnitude is 0.80, and visually (Wendell) is 0.84.

[2] *Harvard Annals*, 69, Part I, 95, 1907; Part II, 165, 1914.

No attempt to improve the light-elements was made at Harvard after 1900, and the phases of all the observations by Wendell are computed from the foregoing approximate elements. Hertzsprung found that the mean period should be lengthened,[2] deriving the value $0^{d}.56682$. Fontana[3] suggested, on the basis of data he considered weak, that the period should be still longer, $0^{d}.5668267$; and Kiess derived at the same time the value $0^{d}.566826$, finding difficulty in harmonizing photographic and visual observations. In the most recent discussion Martin and Plummer conclude that the period is increasing secularly from the value given by Wendell in 1899 to $0^{d}.566863$ in 1914, giving the formula for maximum:

$$\text{J.D. } 2414856.470+0.566798E+(10^{-4}E)^2\div 3. \qquad (1)$$

That the period of RR Lyrae is not constant is shown clearly in the two Harvard series.[4] Plotting the observations, it is also immediately evident that Wendell's approximate period soon fails to represent his measures. A better uniform period for the observations of the first three years is the same one that, within the known errors and irregularities, represents the observations by Martin twelve years later. The period, then, is probably not longer now than it was fifteen years ago, but in the interval it has not remained constant.

A closer examination of this question, which may have an important bearing in the interpretation of Cepheid variation, is afforded by the material collected in the following tables. The various sources of light-observations, so far as they are known to the writer, are enumerated in Table II.

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[1] See a case cited in *Popular Astronomy*, **20**, 656, December 1912.

[2] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **46**, 284, 1911.

[3] Fontana was the first to suspect a variation in the mean period, *Memorie della Società degli Spettroscopisti Italiani*, Serie II, 2, 188, 1913.

[4] We are not yet considering the short-period oscillations.

In all these series of measures very few continuous sets cover light-maximum, and but little could be determined of the variations in the period if we depended only on actual measures at the brightest phase. On the other hand, a great many determinations are possible of some selected point on the steep rising branch of the curve, for much of Wendell's work was evidently planned with an investigation such as the present in view. His series of nightly measures usually cover only the steepest part of the ascending branch, and, though they serve but little in showing directly the changes in the shape of the curve, they are of prime importance in studying the period, and especially in demonstrating the oscillations in the time of the rise to maximum light.

TABLE II

SOURCES OF OBSERVATIONS OF RR LYRAE

Observer	Place	Publication	Observing Method	Interval	Number
Wendell I.	Harvard	*H.A.*, **69**, 45	Polarizing	1899.6–1902.1	241
Wendell II.	Harvard	*H.A.*, **69**, 124	Polarizing	1903.8–1907.7	61
Von Zeipel.	Upsala	*A.N.*, **177**, 372	Zöllner	1906.9–1908.0	38
Haynes.	Laws Obs.	Unpublished	Zöllner	1908.4–1908.5	24
Hertzsprung.	Potsdam	Unpublished*	Photogr.	. . . .	160
Fontana.	Catania	*Spett. It.*, **2**, 183	Wedge	1912.4–1912.5	44
Townley.	Lick Obs.	*L.O.B.*, **7**, 141	Wedge	1912.6–1912.7	40
Kiess.	Lick Obs.	*L.O.B.*, **7**, 141	Wedge	1912.6–1912.9	130
Shapley.	Princeton	This paper	Polarizing	1913.8–1913.9	28
Martin.	Dunsink	*M.N.*, **75**, 566	Photogr.	1913.8–1914.9	108

* *Vierteljahrsschrift der Astronomischen Gesellschaft*, **49**, 192, 1914. For Hertzsprung's notes on the proper motion, see *Astronomische Nachrichten*, **196**, 205, 208, 1913.

The time on the ascending branch when the magnitude is halfway between the minimum and maximum values has been chosen as the epoch of reference, and for convenience will be referred to as the time of *median magnitude*. The method of determining these times is as follows: Neglecting the computed and published phases, the observations from all sources are reduced to Greenwich heliocentric mean time and plotted on a uniform scale, furnishing fragmentary light-curves for each Julian Day on which the star was observed.[1] A mean light-curve, plotted to the same scale on tracing paper, is superposed on the plot of each night's measures and adjusted in the time co-ordinate so as to satisfy all the observations as closely as possible. The time of the median magnitude is then read off, with an error of not more than two or three minutes in a good series of observations.

For the photographic observations the mean curve by Plummer and Martin was used. To determine the time of median magnitude for the visual observations a mean curve was derived from Wendell's first series. Allowance was made in each observer's work for the difference in range of variation, a factor depending on the choice of comparison stars as well as on the scale of the photometer. To form the mean visual light-curve the phases in *Harvard Annals*, **69**, Part I, are corrected to conform to the period $0^{d}.566826$, and the initial epoch adopted by Wendell is retained. The resulting normal magnitudes are given in Table III and the curve is plotted in Fig. 1.[2]

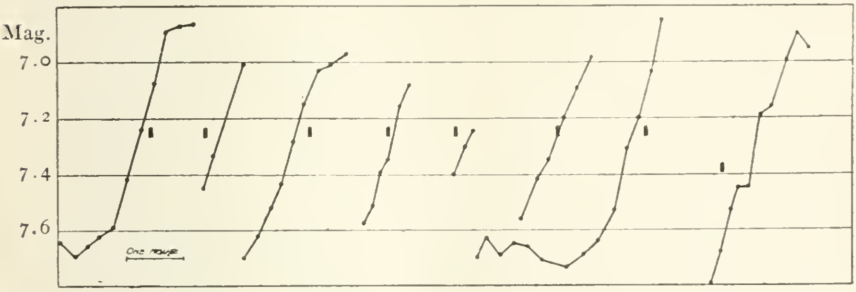

FIG. 4.—Observed ascending branches and computed times of median magnitude for epochs 144, 195, 225, 765, 2755, 3766, 5271, 9210. The existence of the short-period oscillation is illustrated.

It should be noted that the determination of mean curves for stars of this type probably has no value beyond occasional use, such as the present. The great deviation in the form at different epochs[3] makes a mean curve representative of only those possibly widely unlike maxima which chance to compose it, and at most but a rough model of the curve at an unobserved maximum.[1] With this idea in mind no attempt is made to derive a definitive mean curve from all the observations, or even from the homogeneous Harvard series alone.

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[1] Such as are plotted in Fig. 4.

[2] It was found after the completion of this work that the correction for the equation of light applied at Harvard to the phases in this series of measures was made with the wrong sign. As the correction is not large, the error has little effect on the mean curve. The times of observation recorded at Harvard are geocentric.

[3] See Fig. 3.

TABLE III

NORMAL MAGNITUDE CURVE OF RR LYRAE

No.	Phase	No. Observations	Magnitude	No.	Phase	No. Observations	Magnitude
1......	$0^{d}018$	8	6.97	18......	$0^{d}369$	4	7.67
2......	.034	5	7.04	19......	.388	7	7.64
3......	.051	7	7.09	20......	.412	3	7.66
4......	.065	3	7.19	21......	.426	7	7.66
5......	.087	3	7.30	22......	.436	10	7.68
6......	.108	4	7.32	23......	.444	6	7.67
7......	.130	3	7.39	24......	.456	9	7.66
8......	.148	3	7.43	25......	.466	8	7.62
9......	.169	7	7.44	26......	.475	9	7.53
10......	.190	5	7.50	27......	.486	15	7.41
11......	.209	6	7.53	28......	.495	17	7.25
12......	.230	8	7.56	29......	.505	22	7.12
13......	.248	4	7.59	30......	.515	18	7.00
14......	.268	6	7.58	31......	.525	12	6.93
15......	.296	4	7.61	32......	.540	5	6.83
16......	.336	4	7.60	33......	0.564	5	6.88
17......	0.348	4	7.64				

The short series of observations by von Zeipel, Haynes, and Fontana contain no sets of successive observations sufficient to give the time of the rise to single maxima, but in each case a mean curve of more or less weight can be obtained which gives a determination of the time of median magnitude for the mean date. These observations cannot be used, however, to discuss short-period oscillations.

The observations by the writer, made with the polarizing photometer and 23-inch refractor of the Princeton Observatory, have been reported upon briefly in an earlier note.[2] The individual observations are now given in Table IV. The comparison star is

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[1] Harmonic analyses of the light-variations of cluster-type stars, if they are to have any meaning, must make allowance for these wide oscillations. See, for example, the diagrams of maxima of XX Cygni, *Astrophysical Journal*, **42**, 159, 160, 1915.

[2] *Popular Astronomy*, **22**, 144, 1914.

B.D.$+42^\circ 3328$, for which the Harvard[1] visual magnitude is 8.96. My observations show the same range as Wendell's, but the whole curve is $0\overset{M}{.}15$ below his, evidently because the comparison stars are different and their adopted magnitudes may have errors of that amount. Only one measure of the time of the median magnitude is obtainable from this series, but that determination is of considerable weight (Fig. 4).

TABLE IV

PRINCETON OBSERVATIONS OF RR LYRAE

No.	E.S.T.	G.H.M.T.	Julian Day	Mag. Diff.	Mag.	Remarks
	1913		2420000+			
1............	Oct. 23 $6^h 16^m$	$11^h 16^m$	64.469	1.15	7.81	
2............	27.5	28	478	1.15	7.81	
3............	38	38	485	1.17	7.79	
4............	49	49	499	1.18	7.78	Clouds
5............	7 4.5	12 5	503	1.26	7.70	Thick
6............	15	15	510	1.24	7.72	
7............	30	30	521	1.22	7.74	Clouds
8............	45	45	531	1.23	7.73	
9............	Nov. 5 5 56	10 56	77.456	1.24	7.72	
10............	6 5.5	11 5	462	1.11	7.85	
11............	16.5	16	469	1.10	7.86	
12............	26	26	476	1.11	7.85	Haze
13............	43	43	488	1.14	7.82	
14............	54.5	54	496	1.28	7.68	
15............	7 4.5	12 4	503	1.43	7.53	
16............	13	13	509	1.51	7.45	
17............	23.5	23	516	1.51	7.45	
18............	35	35	524	1.77	7.19	
19............	45	45	531	1.80	7.16	
20............	8 0	13 0	542	1.97	6.99	
21............	12	12	550	2.07	6.89	
22............	23	23	558	2.01	6.95	
23............	Nov. 17 5 35	10 34	89.440	1.98	6.98	
24............	46	45	448	1.97	6.99	
25............	57	56	456	1.96	7.00	
26............	6 7.5	11 6	462	1.98	6.98	
27............	20	19	472	1.90	7.06	
28............	30	29	478	1.85	7.11	

The epoch of the one maximum (photographic) available from Hertzsprung's observations is taken from the statement by Kiess.[2] It is corrected to the time of the median magnitude by means of the Dunsink mean photographic curve—a doubtful procedure, hence the low weight assigned it in Table V.[3]

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[1] *Harvard Annals*, **63**, 178, 1913.

[2] *Op. cit.*, p. 140.

[3] The observation is not plotted in Fig. 2..

The data for the discussion of the period are summarized in Table V. The heliocentric times in the second column refer to the median magnitude. The number of the epoch in the third column is counted from the initial epoch used by Wendell. Weights depend on the number and arrangement of the observations and on the general reliability of the photometric method involved.

TABLE V

OBSERVATIONS OF MEDIAN MAGNITUDE

No.	J.D. and G.H.M.T.	Epoch	Observer	Weight	$O-C_1$	$O-C_2$	$O-C_3$
	2400000+						
1.....	14913.690	100	Wendell I	2	$+0^{\rm d}.015$	$+0^{\rm d}.012$	$+0^{\rm d}.014$
2.....	14921.624	114	"	7	+ .013	+ .010	+ .012
3.....	14925.599	121	"	6	+ .021	+ .018	+ .020
4.....	14933.514	135	"	2	.000	− .004	− .002
5.....	14938.611	144	"	9	− .005	− .009	− .007
6.....	14947.684	160	"	3	− .001	− .005	− .003
7.....	14967.538	195	"	4	+ .014	+ .010	+ .012
8.....	14968.667	197	"	4	+ .009	+ .005	+ .007
9.....	14980.554	218	"	9	− .007	− .011	− .009
10.....	14984.521	225	"	8	− .008	− .012	− .010
11.....	14993.602	241	"	6	+ .004	.000	+ .002
12.....	15018.533	285	"	8	− .006	− .010	− .008
13.....	15150.621	518	"	2	+ .011	+ .007	+ .009
14.....	15171.592	555	"	4	+ .009	+ .005	+ .007
15.....	15175.550	562	"	2	.000	− .004	− .002
16.....	15184.616	578	"	3	− .004	− .008	− .006
17.....	15290.625	765	"	7	+ .008	+ .004	+ .005
18.....	15315.561	809	"	7	+ .004	+ .001	+ .002
19.....	15387.538	936	"	3	− .007	− .010	− .009
20.....	15663.591	1423	"	5	.000	− .001	.000
21.....	15743.517	1564	"	6	+ .003	+ .002	+ .003
22.....	16418.608	2755	Wendell II	3	− .001	+ .012	+ .012
23.....	16443.528	2799	"	8	− .021	− .008	− .008
24.....	16456.571	2822	"	5	− .015	− .001	− .001
25.....	16477.543	2859	"	6	− .016	− .001	− .001
26.....	16991.641	3766	"	8	− .033	.000	− .001
27.....	17000.711	3782	"	9	− .032	+ .001	.000
28.....	17703.002	5021	Von Zeipel	9	− .043	− .001	− .003
29.....	17844.704	5271	Wendell II	10	− .049	− .006	− .008
30.....	18122.474	5761	Haynes	1	− .026	+ .018	+ .016
31.....	18919.393	7167	Hertzsprung	2	− .070	− .032	− .035
32.....	19570.165	8315	Fontana	9	− .018	+ .004	.000
33.....	19635.946	8431	Townley	7	+ .010	+ .029	+ .025
34.....	19659.735	8473	"	6	− .008	+ .010	+ .006
35.....	19692.600	8531	Kiess	2	− .019	− .003	− .007
36.....	19693.743	8533	"	3	− .009	+ .007	+ .003
37.....	19697.694	8540	"	6	− .026	− .010	− .014
38.....	19701.668	8547	"	6	− .020	− .004	− .008
39.....	20077.515	9210	Shapley	10	+ .019	+ .024	+ .019
40.....	20093.341	9238	Martin	2	− .027	− .022	− .027
41.....	20390.388	9762	"	6	+ .002	+ .002	− .003
42.....	20453.296	9873	"	4	−0.009	−0.009	−0.014

The residuals in the sixth column, $O-C_1$, show the representation of the observations by the linear formula:

$$\text{Med. Mag.} = \text{J.D. } 2414856.992 + 0^{d}566830\,E. \qquad (2)$$

The agreement is fair at the beginning and end of the series, but for an intervening interval of ten years all residuals are negative, and the probability of a long-period change is at once apparent. Plotting $O-C_1$ against the epochs, it is found by graphical methods that the following harmonic expression represents these residuals with considerable accuracy:

$$-0^{d}020 - 0^{d}024 \sin (0^{\circ}0340\,E - 104^{\circ}5). \qquad (3)$$

The period of this variation is 10600 E, or about 16.5 years, and the amplitude is more than an hour. The necessity of the correction is exhibited in Fig. 2; the available data are somewhat fragmentary, however, and it cannot be claimed that the formula will satisfy fully the future deviations from a uniform period. The recent rough estimates of magnitude, listed in Table I, were made a year later than any observations involved in the derivation of the revised light-elements, and they appear to be in very good agreement with prediction. The phases in that table are computed by means of formula (5) below.

The residuals in the seventh column of Table V, $O-C_2$, result from correcting those of the preceding column by means of (3). The new residuals were then discussed by least squares to determine corrections to the initial epoch, T, and the mean period, P. Forty-two equations of condition were involved in the solution, and the corrections in days are as follows:[1]

$$\left.\begin{aligned} \Delta T &= -0.0020 \pm 0.0011 \\ \Delta P &= +0.0000074 \pm 0.0000218 \end{aligned}\right\} \qquad (4)$$

Applying these corrections, reducing from the median magnitude to maximum light, and adding the correction (3), the heliocentric light-elements that represent all the observations to date are:

$$\left.\begin{aligned} \text{Max.} = \text{J.D. } 2414856.451 + 0^{d}566831\,E \\ -0^{d}024 \sin (0^{\circ}0340\,E - 104^{\circ}5). \end{aligned}\right\} \qquad (5)$$

[1] The large probable error of the period is accounted for in Section V.

The representation by this formula is shown by the residuals in the last column of Table V and by the deviations from the sine curve in Fig. 2. They will be discussed further in Section V.

IV. CHANGES IN THE FORM OF THE LIGHT-CURVE

That a wide variation exists in the form of the light-curve at different maxima has been shown directly for a few cluster-type variables,[1] and the oscillations in the time of the median magnitude all but prove the same thing for others of this class.[2]

Fig. 3 contains diagrams of maxima of RR Lyrae which show that for it also the curve changes its shape very conspicuously and rapidly. The two upper curves are taken from Wendell's earliest observations. The maxima are only seven periods apart, but the interval of time when the star was brighter than $7^{M}.0$ is in one case $1^{h}55^{m}$, and in the other nearly 40 per cent less. The lower pair of curves in Fig. 3 are from the Lick Observatory measures by Kiess. The interval when the brightness exceeded $7^{M}.3$ is about twice as great at the first maximum as at the second, which follows by seven periods. Moreover, the brightness at maximum differs by $0^{M}.15$.[3] If the photographic observations by Martin chanced to refer to maxima of the "faint" kind, the apparent equality of photographic and visual ranges may have no real significance.[4]

It is obvious from the diagrams that the determination of the time of median magnitude involves directly the shape of the curve. It suggests that oscillations in that time indicate variations in shape rather than actual oscillations in the epoch of maximum light. Apparently the recovery from a delayed outburst is more rapid than from an early one; that is, the factors that cause the late rise to maximum light generally allow an early decline, and vice versa. These peaked and round-topped curves are shown more completely in the figures accompanying the earlier paper relative to XX Cygni.

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[1] XX Cygni, *loc. cit.;* SW Draconis and SU Draconis, *Astronomische Nachrichten*, 184, 241, 1910.

[2] ST Ophiuchi, SW Andromedae, RR Lyrae, *Mt. Wilson Contr.*, No. 92, p. 5; *Astrophysical Journal*, 40, 452, 1914.

[3] If we should attribute the difference in maximum magnitude to a variation in the comparison star, the curves would still be conspicuously dissimilar.

[4] Similarly, perhaps, we should not accept finally the hump on the photographic curve three hours after maximum, for there remains the strong possibility that some of the measures near that phase refer to narrow maxima and others to the wider variety.

V. OSCILLATIONS IN THE TIME OF THE RISE TO MAXIMUM LIGHT

From the least-squares solution for the light-elements the probable error of a unit-weight determination of the time of median magnitude is $\pm 0^{d}0167 = \pm 24$ minutes. For the average determination listed in Table V, therefore, the probable error is more than ten minutes and for none is it less than seven minutes. Such large uncertainties in the observations, however, are clearly impossible. In many of Wendell's measures of the rise to maximum light the error in determining the time of median magnitude cannot well exceed two minutes, and from an inspection of all the plotted curves on which Table V is based the probable error of unit weight would be estimated at less than ±8 minutes. The curves given in Fig. 4 illustrate this point fully. All the plotted measures are from Wendell except the last, which is taken from the writer's observations. The short heavy vertical line near each series indicates the time of the median magnitude for that maximum, computed from the adopted light-elements (5).

There can be no doubt that a real irregularity is present. An attempt was made to find a uniform period for the variation that would satisfy all the observations. This failed in part, perhaps because of insufficient data, but it seems that for the whole series the oscillation is roughly periodic with a varying amplitude. For the first 14 months of Wendell's measures, however, the variation is definitely periodic. Throughout that interval the following simple formula represents the residuals, $O-C_3$, with considerable accuracy:

$$(O-C_3)-0^{d}004-0^{d}013 \cos \frac{2\pi}{70}(E-45)=0. \qquad (6)$$

Table VI contains the data relative to this correction. The residuals from (6) are given in the fifth column, and the last column contains the difference between the weighted squared residuals before and after correcting. The existence of the periodic variation is completely established. The sum of the squares of the residuals is decreased from 0.007762 to 0.001034, which gives, in this interval, for the new probable error of an observation of weight unity ±9 minutes. The variation is shown diagrammatically in Fig. 5.

TABLE VI

THE APPLICATION OF FORMULA (6)

No.	Epoch	Weight	$O-C_3$	$O-C_4$	$(pvv)_3-(pvv)_4$
1	100	2	$+0^{d}.014$	$+0^{d}.007$	+0.000294
2	114	7	+ .012	− .005	+ .000833
3	121	6	+ .020	+ .005	+ .002250
4	135	2	− .002	− .003	− .000010
5	144	9	− .007	.000	+ .000441
6	160	3	− .003	+ .001	+ .000024
7	195	4	+ .012	.000	+ .000576
8	197	4	+ .007	− .003	+ .000160
9	218	9	− .009	.000	+ .000729
10	225	8	− .010	− .002	+ .000768
11	241	6	+ .002	− .006	− .000192
12	285	8	− .008	.000	+ .000512
13	518	2	+ .009	+ .004	+ .000130
14	555	4	+ .007	+ .006	+ .000052
15	562	2	− .002	+ .003	− .000010
16	578	3	− .006	.000	+ .000108
17	765	7	+0.005	+0.004	+0.000063

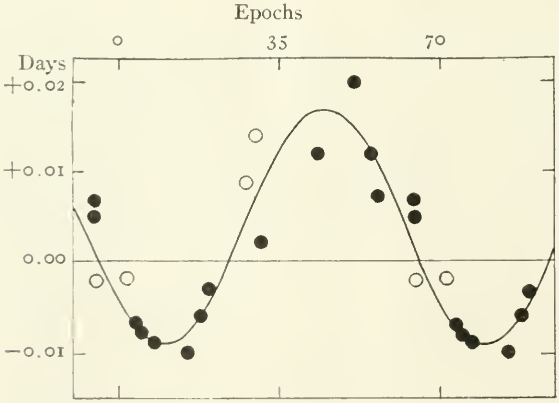

FIG. 5.—Short-period variation in time of median magnitude.

Eleven oscillations are involved; the period is 70 epochs or 40 days, and the amplitude is 37 minutes.

If we make the reasonable assumption that the errors of all the observations could be reduced proportionately as much as these earlier ones by analogous corrections for short-period irregularities in the median magnitude, then the probable errors from the least-squares solutions for T and P, given by (4), would be reduced from $\pm 0\overset{\mathrm{d}}{.}0011$ and $\pm 0\overset{\mathrm{d}}{.}000022$ to $\pm 0\overset{\mathrm{d}}{.}0004$ and $\pm 0\overset{\mathrm{d}}{.}000008$, respectively.

VI. SUMMARY

The present paper discusses in some detail the peculiarities of the light-variations of the Cepheid variable RR Lyrae, and considers the variations of its spectrum as observed at Mount Wilson. The star is the brightest of the sub-group with periods of a half-day. It is shown in Section II to be a variable in spectrum as well as in light and velocity. The computations of the third section demonstrate upon the basis of the work of nine observers throughout an interval of fifteen years that the mean period is affected by a slow variation which may complete its cycle in 16.5 years. The fourth section contains diagrams to show that the maxima of the light-curve are not constant in form, and in the fifth the closely analogous phenomenon of variations in the time of the rise to maximum light is proved to exist.

Mount Wilson Solar Observatory
December 1915

NOTES ON CERTAIN ULTRA-VIOLET SPECTRA

By F. A. SAUNDERS

Information about many spectra in the region near λ 2000 is scanty, or lacking altogether. Here both quartz and the air begin to absorb, and the reflection of speculum metal falls off, while the gelatine of ordinary photographic plates offers a further obstacle to successful investigation. It is the purpose of this paper to present a few observations in this part of the spectrum, which have been made at various times and with various experimental arrangements.

All the results here given were obtained by photographs taken on Schumann plates, using a one-meter concave grating, purchased through the generosity of the Committee of the Rumford Fund. This grating was mounted, either in air in an ordinary way, or inclosed in a brass tube a little more than a meter long, and 10 cm in diameter, which could be exhausted. In this latter arrangement, the slit and the plate were placed just outside the tube, at one end, the tube being sealed with suitable quartz plates.

In case the spark was used as a source of light, it was placed as close as possible to the slit. The light had then to travel a distance of about a millimeter through the air to the first quartz plate. This plate was 1.5 mm thick. The light then went down the one-meter tube, and on its return from the grating it passed through about 5 mm more of quartz, and a few tenths of a millimeter more of air, to the plate. The tube was kept at a cathode-ray vacuum, as shown by a small discharge tube, sealed in by a side connection, and was washed out repeatedly with hydrogen before exhausting, so that the residue of gas in it was transparent to the light to be photographed.

In case a vacuum arc was the source, a small and convenient form of water-cooled chamber with removable electrodes was very kindly loaned to me by Professor Paschen (this part of the work having been done in his laboratory at Tübingen). The positive electrode was itself separately water-cooled, and the negative was movable by means of a screw, for making contact. Aluminium was usually chosen as the material for the negative electrode, mainly because it furnishes good standard lines in this part of the spectrum; the positive (lower) terminal was tipped either with a rod of the metal to be studied, or by a cup-shaped piece in which (for low-melting metals, such as In, Tl, Cd, Pb, etc.) a drop of the molten metal could be placed. The exposure came to an end when all this metal had boiled away. A 60-volt direct-current arc was used; higher voltages made the arc wander away often from the tips of the terminals. An image of the arc was cast on the slit by a quartz lens located in a projecting side tube, which was long enough so that the entire course of the rays outside the grating tube was through gas at the same low pressure as that about the arc itself. The results here presented for Ca and Mg came in part from a more ordinary experimental arrangement, in which the grating and source were used in air. The vacuum-grating arrangement gave results to λ 1670, the sudden increase in absorption due to air at λ 1850 being quite inconspicuous, due to the shortness of the path in air.

The vacuum arc spectra of Mg, Ca, Zn, Cd, Al, In, Tl, Pb, Sn, and Cu were investigated. The plates were sometimes underexposed, as it was difficult to run the arc very long. Cu gave no new results. The "vacuum lamp"[1] of Mg and of Ca was also used, and gave beautifully sharp lines, though not many of them in this region. The spark spectra of Mg, Ca, Zn, Cd, Al, In, and Tl were also obtained. Certain common impurity lines showed themselves in some of these spectra, but are not here mentioned, lacking positive identification. In every case great care was taken to eliminate from the lists lines which could be identified as due to impurities. The wave-lengths chosen as standards were Paschen's measurements of the Zn lines near λ 2100; Runge's of the Al lines near λ 1850, and Wolff's[2] value for Al, λ 1670. In general there is a great difference between the arc in air and the vacuum arc, the latter giving a spectrum nearly the same as that from the spark. In Tl and in Ca, for instance, the ends of important series come in this region and are rather prominent in the spectrum of the arc in air, but usually absent from the other spectra altogether.

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[1] *Astrophysical Journal*, **40**, 377, 1914.

[2] Wolff, *Annalen der Physik*, **42**, 825, 1913.

The results for each element are presented in detail below; the wave-lengths are given in international units, reduced to vacuum. Through most of this part of the spectrum it has been assumed that the wave-lengths on the Rowland scale are too great by 0.06 units.

ALUMINIUM

The vacuum arc spectrum of Al proved to be practically identical with the spark spectrum as given by Lyman. The prominent single line at λ 1670 is perhaps a little stronger relative to the triplet near λ 1720 in the arc than in the spark. This is to be expected if these lines really belong to single-line and triplet series systems, as their appearance would indicate.

MAGNESIUM

In a recent (1913) Tübingen dissertation by E. Lorenser, there appears a single-line series of the "principal" type in the spectrum of Mg, with its first line at λ 2852, the flame line of this element. The second line of this series was not observed by Lorenser, but was calculated by him to be at λ 2026.46 I.A.vac. Some time earlier I had obtained a photograph of this line, which is here reproduced (Fig. 1*a*), showing its heavy reversal, the line thereby declaring itself as a line of the principal series. The arc in air between Mg rods was used as source. The same line shows also in Fig. 1*b*, Ca arc in air, the line not being reversed, since there was not much Mg in the source. Sharp photographs of this line were obtained with vacuum sources, and from these its wave-length has been determined to be λ 2026.48 I.A.vac., differing from Lorenser's calculated value by less than the probable error of measurement. It is worth noting that there is a Zn line whose wave-length is about 0.3 greater, but this is a "pair" line, strong in the spark spectrum and very weak in the arc in air. This is just opposite to the behavior of the Mg line, so that they almost never occur together. Eder and Valenta give a Cd line here also, but it seems likely that this was due to Zn.

Photographs with the vacuum arc, and with the Mg "vapor lamp" also, show the next line of this same Mg series, for which the value λ 1828.06 is obtained. This agrees with the value given by Lyman, who was the first to measure this line. The spark spectrum of Mg was photographed in this region, but showed no new lines.

CALCIUM

The measurements of the Ca spectrum in the region of λ 2000 given in an earlier paper[1] have since been improved by getting better photographs. One of these is of sufficient interest to be worth reproducing (Fig. 1*b*). It was taken from an arc between two pieces of metallic Ca, of which the positive one was on the point of melting in air. A short-focus concave grating and a Schumann plate were used. The reproduction, enlarged fourfold, shows the reversed lines of the single-line principal series (called SL1 before), heavily overexposed. The emitted light from these lines in the case of the last four fuses together into a continuous background, upon which the reversals stand out as absorption lines. The character of these lines in this photograph is strong evidence for classifying them together as a principal series. The first two lines of this

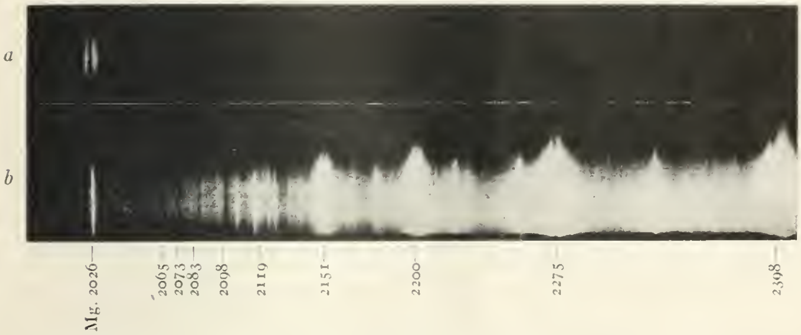

Fig. 1.—Region of λ 2000

a. Mg arc in air, λ 2026 only.

b. Ca arc in air, showing principal series of single lines.

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[1] *Astrophysical Journal*, 32, 153, 1910.

series are the flame line λ 4226 and a line at λ 2721.76 Rowland (air), which is peculiar in two ways. First, it was discovered by Rowland[1] but was overlooked by later observers, and has not yet found its way into the lists. Second, its intensity is abnormally low as a member of the series. Yet, it must be accepted as one, because its wave-length is precisely that predicted[2] by a simple shift from a line (λ 6718) of another series (SL2), in exactly the same way as are the other lines of this series. This abnormality of intensity is not absolutely unique, as something of the sort occurs in the spectrum of potassium; and it appears that in other sources, such as the "vapor lamp" of Ca, the abnormality is not so striking.

The list below gives the remeasured values of some of the series lines mentioned above, and also several new lines in this part of the spectrum which appear to belong to Ca, and to be members of new single-line series.

λ I.A. Vac.	Intensity	Source	Remarks
2428.77	1	Arc in air	New
2392.94	2	Arc in air	New
2330.02	2	Arc in air	New
2258.07	1	Arc in air	New
2222.59	1	Arc in air	New
2211.86	0	Vapor lamp	New ?Ca
2187.66	1	Arc in air	New
2179.44	1	Arc in air	New
2167.57	0	Arc in air	New ?Sr
2132.51	0	Arc in air	New
2098.14	1	Arc in air	Prin. series, remeasurement
2083.38	1	Arc in air	Prin. series, remeasurement
2073.26	0	Arc in air	Prin. series, remeasurement
2065.42	0	Arc in air	Prin. series, new
1840.26	6	Vapor lamp	Remeasurement
1838.13	5	Vapor lamp	Remeasurement

ZINC

The spectrum of Zn obtained from the vacuum arc included only the stronger lines shown in the spark spectrum, in the same relative intensities, possibly on account of underexposure. The spark spectrum was measured in part, with the following results:

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[1] *Astronomy and Astrophysics*, **12**, 321, 1893.

[2] *Physical Review*, **1**, 332, 1913.

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2253.55	1	1969.59	1
2139.27*	9	1964.76	1
2104.92	1	1919.04	2
2102.88	3	1839.19	1
2100.53*	9	1833.79	1
2097.44	1	1767.84	2
2087.66	3h	1757.25	1h
2079.57	1	1752.6	0
2064.93	7	1749.59	2
2062.57*	9	1745.73	0
2026.19*	10	1742.98	2
2012.55	1	1706.89†	0
1987.17	1	1689.04	0
1982.21	2	1673.09	1

* Paschen's value, used as standard.

† ?Zn.

The observations of Handke (see Kayser's *Spectroscopy*, Vol. VI) are the only ones extending below λ 1919. The list here given does not agree with his.

CADMIUM

The spectrum of Cd was obtained in the same way as that of Zn, and there was the same similarity between the vacuum arc spectrum and the spark spectrum. The table below of the spark spectrum is in part covered by the measurements of L. and E. Bloch[1] but there are many unexplained differences between the two lists.

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2265.61	9	1856.62	3
2240.53	1	1855.83	1
2195.30	9	1844.58	1
2145.05*	10	1793.06	1
2112.18	2	1789.04	0
2056.80	0	1772.89	2
2027.64	1	1768.54	0
2004.54	2	1747.77	2
1987.84	1	1721.76	0
1903.37	0	1707.04†	0
1874.06	6	....	....

* Used as standard: value of Kayser and Runge.

† ?Cd.

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[1] *Journal de Physique*, 4, 622, 1914.

INDIUM

The spark and vacuum arc spectra of indium were found to be as follows:

λ I. A. Vac.	Int. Arc	Int. Spark	λ I. A. Vac.	Int. Arc	Int. Spark
2351.45		2	1820.38		1
2321.83		1	1774.68		2
2300.81	5	7	1770.57	1	5
2154.81		1	1748.65	1	5
2079.28	4	7	1716.28		1
1977.44	2	5	1699.86		2
1966.69	2	6			

THALLIUM

The spark and vacuum arc spectra of thallium were found to be as follows:

λ I. A. Vac.	Int. Arc	Int. Spark	λ I. A. Vac.	Int. Arc	Int. Spark
2469.76		4	2207.58		1
2452.70		8	2139.98		2
2395.27		1	1908.68	10	5
2380.34	3	6	1892.72	1	3
2365.65*		2	1881.19		2
2298.95	4	9	1871.47		1
2243.66		1	1837.96		1
2238.59		1	1828.00		2
2210.46		1	1814.72	1	4

* Not Cd.

LEAD

The vacuum arc spectrum of lead is as follows:

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2402.62	6	2170.60	8 ✓
2400.17	1	2115.63	2
2394.52	7	2112.37	1
2389.34	1	2053.83	2
2332.97	2	2022.64	0
2254.68	0	1904.88	1
2247.53	7	1868.59	0
2238.03	3	1822.06	8
2204.18	10	1796.53	3 ✓
2190.08	1	1726.71	1
2176.23	0	1682.54	0 ✓

The relative intensities in the vacuum arc are not the same as those in the arc in air, but are approximately those of the spark spectrum.

Kayser and Runge found in the spectrum of Pb related groups of lines, one of which is reproduced from another by the addition of a constant frequency-difference. The new lines here given made it worth while to test the spectrum again for such groupings, and some new results were obtained. For instance, the lines λλ 5201, 5005, and 4340 form part of an additional group; as do also, λλ 2833 2170, 2053, 2022, 1904, and 1868. Apparently also λλ 4062 and 2613 belong to Kayser and Runge's first and third groups respectively; λλ 2823 and 2614 to their second and third, and λλ 7228, 4057 and 3639 to their first, second, and third. The careful measurements of Klein[1] were used as a basis for this part of the work.

TIN

Observations similar to those made on Pb were made also with Sn, with the following results (vacuum arc spectrum):

λ I. A. Vac.	Intensity	λ I. A. Vac.	Intensity
2355.57	5	2199.93	3
2335.53	2	2195.16*	1
2317.93	2	2152.08	4
2287.28	2	2149.43	1
2269.65	6	2114.74	2
2246.73	4	2092.14	1
2232.37	1	1811.29	3
2210.38	4		

* Not Cd.

In the same manner as with Pb, the groups of lines found by Kayser and Runge in the spectrum of Sn were examined, with similar results. To the first, second, and third groups of Kayser and Runge should be added λ 3662, λ 2790, and λ 2661 respectively. Part of a fourth group is formed by λ 5631, λ 4524, λ 3655, and λ 3141. Several other pairs of unclassified lines have the separation of the first and second groups, or of the second and third, but, as some of these coincidences may be accidental, it may not be worth while to give all the details.

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[1] Klein, Bonn Dissertation, 1913.

SUMMARY

The vacuum arc spectra of Mg, Ca, Zn, Cd, Al, In, Tl, Pb, and Sn have been examined from λ 2300 to λ 1670, as have also the spark spectra of Mg, Ca, Zn, Cd, Al, In, and Tl. The results herewith presented are in some cases of interest in their connection with series or other relationships among the lines.

Vassar College
January 1916

MINOR CONTRIBUTIONS AND NOTES

THE ALTITUDE OF AURORA BOREALIS SEEN FROM BOSSEKOP DURING THE SPRING OF 1913

In the November 1913 issue of this *Journal* (38, 311) I gave a short account of an expedition to Bossekop which I undertook in the spring of 1913 for the purpose of completing the results of my expedition to the same place in 1910.

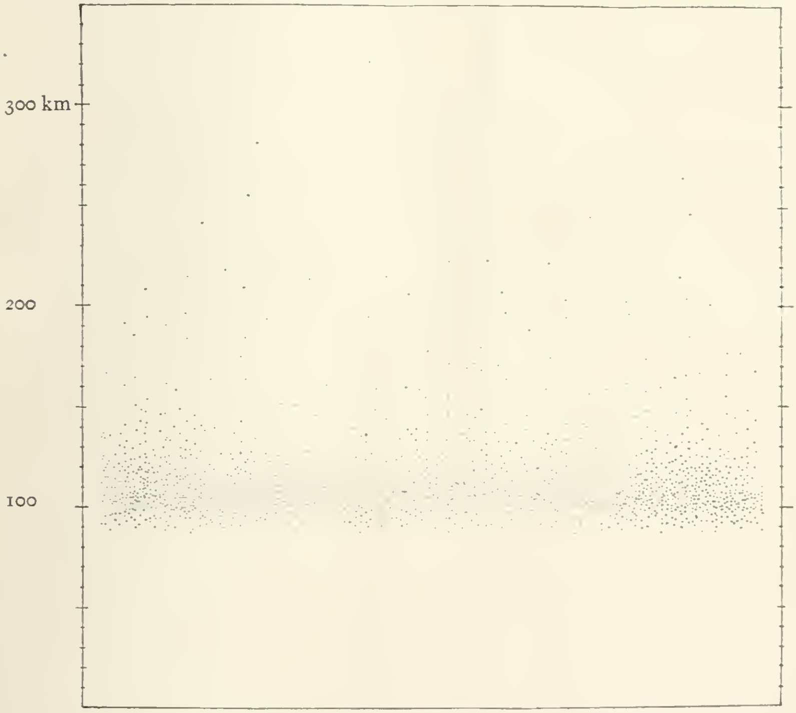

FIG. 1. Altitudes of selected points of Auroras

The aim of the expedition in 1913 was mainly to obtain a greater number of good photographs of the aurora for the determination of its height and situation in space. A very great number of simultaneous photographs were taken from the two stations Bossekop and Store Korsnes, which are connected by telephone and are situated 27.5 km apart. My assistant, the meteorologist Bernt Johannes Birkeland, made the observations at Store Korsnes and I those at Bossekop.

The material has now been worked up and about 2500 parallaxes of selected points of the auroras have been calculated for the determination of the situation and altitude of these points above the earth's surface. They are all located within a distance not exceeding 800 km from Bossekop and mainly in the directions west, north, and east.

On Fig. 1 are seen all the altitudes calculated, each altitude marked by a point. The number of points giving the same altitude are distributed as follows:

TABLE I

Altitude in Kilometers	Number of Points	Altitude in Kilometers	Number of Points	Altitude in Kilometers	Number of Points	Altitude in Kilometers	Number of Points
85......	0	109......	75	133......	13	157......	4
86......	1	110......	73	134......	16	158......	3
87......	2	111......	70	135......	12	159......	6
88......	8	112......	72	136......	8	160......	5
89......	11	113......	72	137......	16	161......	2
90......	21	114......	55	138......	10	162......	5
91......	25	115......	67	139......	17	163......	0
92......	28	116......	39	140......	6	164......	4
93......	30	117......	49	141......	8	165......	2
94......	34	118......	54	142......	9	166......	4
95......	36	119......	39	143......	7	167......	2
96......	41	120......	52	144......	3	168......	1
97......	43	121......	24	145......	4	169......	3
98......	48	122......	40	146......	9	170......	2
99......	66	123......	24	147......	9	171......	3
100......	70	124......	28	148......	8	172......	3
101......	84	125......	33	149......	7	173......	1
102......	79	126......	25	150......	0	174......	2
103......	91	127......	27	151......	2	175......	1
104......	72	128......	24	152......	5	176......	1
105......	101	129......	18	153......	3	177......	4
106......	99	130......	15	154......	2	178......	1
107......	95	131......	11	155......	4	179......	0
108......	85	132......	17	156......	4	180......	4

Over 180 km there are some altitudes which can be seen on the figure; they are derived particularly from auroral rays.

The numbers here given may perhaps be somewhat modified in details through renewed measuring and computation, but I think the average values are well founded.

We shall not here enter into a detailed discussion of the material, nor into a comparison between the observations and the theory, because these questions are taken up in a series of articles now in process of publication in *Terrestrial Magnetism and Atmospheric Electricity*.

CARL STÖRMER

UNIVERSITY, CHRISTIANIA
February 11, 1916

NOTE ON MR. SHAPLEY'S STUDIES OF THE PERIODS OF ECLIPSING VARIABLES

In the *Astrophysical Journal* for May, 1915 (**41**, 291), and in the *Publications of the Astronomical Society of the Pacific* for June, 1915 (**27**, 135), Mr. Harlow Shapley has made use of his catalogue of eclipsing binary stars in order to investigate whether the two types of periods I have suspected from an inspection of the records of spectroscopic binaries[1] are also indicated in the previous class of variables.

In my opinion it was not to be expected that the eclipsing binaries should give much evidence of the grouping in question. At the time when my article was written I had access to the periods of about 80 eclipsing binaries, the evidence from which (p. 12, *loc. cit.*) was thought by me to be in favor of grouping of the periods.

I had various reasons for being satisfied with that evidence, reasons sufficient to satisfy me also with the very vague signs of the grouping (i.e., the curves of skew-frequency) that have come out of Mr. Shapley's recent statistical studies.

Those reasons are based chiefly on the fact that the eclipsing variables are binaries presented for observation by a process of selection that is very much less favorable for the tracing of two groups of periods than is the case with the spectroscopic binaries. Such different modes of selection, within two statistical series that are to be compared, often cause trouble in such investigations, and the mind of the statistician must always be on the alert for the occurrence. In the present instance the mode of selection unduly favors the observation of short-period doubles in case of the eclipsing systems. The selection is produced in the following way: first, the photometric methods are better adapted for the detection and characterization of short-period variables than the spectroscopic methods; second, the probability that a system of double stars will be seen from the earth as an eclipsing system, very rapidly decreases with increasing distance of the components, hence also with increasing period. Also the fact that the systems of small mass are less liable to eclipse than systems of great mass will tend to connect the eclipsing character with shortness of period, for the period is inversely proportional to the square root of the mass.

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[1] S. D. Wicksell, "Contributions to the Statistics of Spectroscopic Binary Stars," *Meddelande från Lunds Observatorium*, No. 63, 1913.

Hence it is a priori to be expected that a catalogue of eclipsing binaries will contain mainly those of short periods, and that a long-period group after its eventual separation will be indicated only slightly or not at all. Thus the distribution of eclipsing variables will not justify any conclusions regarding the distribution of periods of double stars in general. The whole thing may plainly be illustrated by plotting the curves of *relative* frequency for the logarithm of the period of eclipsing and spectroscopic binaries. The much greater height and steepness (i.e., the smaller dispersion) of the eclipsing curve for short periods is a practical illustration of the statements above.

Finally I will again call attention to a property of the eclipsing binaries first detected by Shapley, which, if not directly indicating two groups of periods, or rather of orbits, may at least be well explained by assuming that they exist. I refer to the tendency of eclipsing variables to show two groups according to density. It is clear that if two groups of orbital dimensions exist, two groups of densities should also appear among eclipsing binaries, while the eclipsing character of a system of great orbital radius makes it probable that the volumes of the components are also large. Otherwise the components would eclipse only if the inclination were confined within a very narrow range, and the eclipses should be very short, in comparison with the period, and very hard to detect. If the volumes are great, the densities must on the whole be small, as the long-period stars are not generally of very great mass. In the article cited I have already pointed out that Shapley's group of low density emanates from the long-period systems, a fact that I deem very well worthy of notice, as it is in contradiction to the explanation given by Shapley himself for this phenomenon.

On the whole I must say that the results of the statistical studies of Shapley are of a character rather to strengthen the evidence in favor of the existence of two groups of periods among the close double stars. To avoid misinterpretation I must finally assert that processes of selection similar to those here described for the eclipsing binaries are of course also at work on the spectroscopic binaries; but the significant difference lies in the fact that the force of the selection is so much stronger in the former category. This fact is clearly illustrated by the following argument: suppose that by spectroscopic and photometric methods we had discovered and determined the period of all double stars in the heavens down to the seventh magnitude and all shorter than a certain period. Then, taking out all systems that are eclipsing, the above-mentioned second mode of selection should still offset their periods, i.e., eclipses will be relatively less frequent among systems with large orbits (periods) than among systems with small orbits. Thus going back to the present state of the catalogues it is seen that there is always a preponderance in the selection in favor of short periods in case of the eclipsing variables.

SVEN D. WICKSELL

UNIVERSITY OBSERVATORY
LUND, SWEDEN
January 2, 1916

ADDENDA AND ERRATA FOR VAN MAANEN'S LIST OF STARS WITH PROPER MOTIONS EXCEEDING 0″.50 ANNUALLY[1]

The following numbers of the list in *Mt. Wilson Contr.*, No. 96,[2] (several of which were marked doubtful) should be excluded: Nos. 6, 7, 63, 69, 130, 167, 196, 226, 267,[3] 330, 331, 364, 384, 389, 390, 441, 442, 480, 514, and 526.

Further, in the case of

No. 25 read $\phi =$ 66°.6
“ 62 “ “ = 110.4
“ 159 “ “ = 206.3
“ 162 “ “ = 209.3
“ 166 “ “ = 179.0
“ 187 delete Sp. Bin.
“ 247 read $\phi =$ 291°.6

No. 269 read $\phi =$ 268°.0
“ 325 add β G.C. 7166
“ 365 read $\phi =$ 304°.7
“ 440 “ “ = 11.7
“ 499 “ “ = 61.9
“ 508 add β G.C. 12274

Attention may be drawn to Porter's note in *Astronomical Journal*, **20**, 46, 1915, which gives several stars whose proper motions, as indicated by later observations, are a little below the limit of half a second.

Besides the additions to the original list which appear in the accompanying table, 12 stars with proper motions in excess of 0″.50 have just been given by Wolf in *Astronomische Nachrichten*, **201**, 345, 1915.

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[1] This is appended to Mt. Wilson Contr., No. 111.

[2] *Astrophysical Journal*, **41**, 187, 1915.

[3] *Astronomische Nachrichten*, **201**, 271, 1915.

Stars to Be Added to the List in *Mt. Wilson Contr.* No. 96; This Journal, 41, 187, 1915

Name	B.D.	Mag.	α 1900	δ 1900	μ	φ	Authority
Comp. to Lal. 248	....	10.7*	0^{h}12^{m}43^s	+43°28′	2.″83	82.°9	O. Struve
Anonymous	....	13 *	2 8 14	+15 31	1.1–	90.0	Wolf
A. Oe. 2526	+64°312	7.5*	2 9 12	+64 30	0.50	228.5	Po.
Comp. to Pi 2^{h}123	....	11.5†	2 30 46	+ 6 24	2.63	48.9	Van Maanen
A. Oe. 3303	+51°658	9.1	2 50 18	+52 5	0.50	107.9	Po.
A. Oe. 4961	+52°857	8.7	4 29 47	+52 42	0.53	149.7	Po.
B.D.+17°1320	+17°1320	9.5	6 31 26	+17 38	0.88	293.–	Wolf
B.D.+33°1505	+33°1505	9.3	7 12 58	+33 2	0.57	127.–	Hertzsprung
B.D.+33°1694	+33°1694	9.2	8 18 54	+32 58	0.69	173.–	Hertzsprung
Schroeter 899	+67°552	9.3	8 27 23	+67 38	1.09	270.0	Schroeter
B.D.+77°361	+77°361	9.2	9 6 24	+77 40	1.02	268.0	Jones
Anonymous	....	9.5*	10 56 56	+ 1 19	0.7–	38.–	Wolf
Anonymous	....	13 *	11 2 9	+ 2 9	0.67	146.–	Wolf
Anonymous	....	10.5*	11 4 56	– 2 14	0.49	166.–	Wolf
Anonymous	....	12.5*	11 10 58	+ 8 34	0.81	261.5	Wolf
Anonymous	....	10 *	11 18 39	+ 9 7	1.05	278.7	Wolf
Comp. to Lal. 27744	....	7.5	15 8 50	– 0 58	1.38	247.4	β G.C. 7166
Comp. to W.B. 23^{h}175	....	8.4	23 11 54	–14 22	1.31	202.9	β G.C. 12274

† Visual magnitude. The photographic magnitude is 13.2. These values were kindly determined by Mr. Shapley.

Adriaan Van Maanen

Mount Wilson Solar Observatory

Reviews

Die Kultur der Gegenwart, Dritter Teil, Dritte Abteilung: Physik. Edited by E. Warburg. Leipzig and Berlin: B. G. Teubner; 1915. 4to. Pp. 762. M. 22; bound, M. 24.

This volume contains thirty-six articles on the principal phases of modern physics, as follows: 1, "Die Mechanik im Rahmen der allgemeinen Physik," by E. Wiechert. 2, "Historische Entwicklung und kulturelle Beziehungen," by Felix Auerbach. 3, "Thermometrie," by E. Warburg. 4, "Kalorimetrie," by L. Holborn. 5, "Entwicklung der Thermodynamik," by F. Henning. 6, "Mechanische und thermische Eigenschaften der Materie in den drei Aggregatzuständen," by L. Holborn. 7, "Umwandlungspunkte, Erscheinungen bei koexistierenden Phasen," by L. Holborn. 8, "Wärmeleitung," by W. Jaeger. 9, "Wärmestrahlung," by Heinrich Rubens. 10, "Theorie der Wärmestrahlung," by W. Wien. 11, "Experimentelle Atomistik," by Ernst Dorn. 12, "Theoretische Atomistik," by A Einstein. 13, "Entwicklung der Elektrizitätslehre bis zum Siege der Faradayschen Anschauungen," by Franz Richarz. 14, "Die Entdeckungen von Maxwell und Hertz," by Ernst Lecher. 15, "Die Maxwellsche Theorie und die Elektronentheorie," by H. A. Lorentz. 16, "Ältere und neuere Theorien des Magnetismus," by R. Gans. 17, "Die Energie degradierenden Vorgänge im elektromagnetischen Feld," by E. Gumlich. 18, "Die drahtlose Telegraphie," by Ferdinand Braun. 19, "Schwingungen gekoppelter Systeme," by Max Wien. 20, "Das elektrische Leitungsvermögen," by H. Starke. 21, "Die Kathodenstrahlen," by W. Kaufmann. 22, "Die positiven Strahlen," by E. Gehrcke, and O. Reichenheim. 23, "Die Röntgenstrahlen," by W. Kaufmann. 24, "Entdeckungsgeschichte und Grundtatsachen der Radioaktivität," by J. Elster and H. Geitel. 25, "Radioaktive Strahlungen und Umwandlungen," by Stefan Meyer and Egon von Schweidler. 26, "Entwicklung der Wellenlehre des Lichtes," by Otto Wiener. 27, "Neuere Fortschritte der geometrischen Optik," by Otto Lummer. 28, "Spektralanalyse," by F. Exner. 29, "Struktur der Spektrallinien," by E. Gehrcke. 30, "Magnetooptik," P. Zeeman. 31, "Verhältnis der Präzisionsmessungen zu den allgemeinen Zielen der Physik," by E. Warburg. 32, "Die Erhaltung der Energie und die Vermehrung der

Entropie," by F. Hasenöhrl. 33, "Das Prinzip der kleinsten Wirkung," by Max Planck. 34, "Die Relativitätstheorie," by Albert Einstein. 35, "Phänomenologische und atomistische Betrachtungsweise," by W. Voigt. 36, "Verhältnis der Theorien zueinander," by M. Planck.

Each article is a very brief but thoroughly correct and modern statement of the subject which it discusses. The book is intended primarily for the general academic public. The physicist who expects to find here a great store of modern data in any particular field will be disappointed, as the articles are necessarily very brief and are written in popular style, as free as possible from technicalities and practically without equations. They may be read with profit, however, by physicists who desire a brief, scholarly summary. The strongest appeal of the book should be to workers in allied sciences—chemistry, astronomy, technology, biology, etc.—who have been unable to keep up with the rapid development of physics in recent years.

H. G. G.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language may be accepted for publication, but unless a wish to the contrary is expressed by the author, they usually will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts typewritten, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

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ON AN APPARATUS AND METHOD FOR THERMO-ELECTRIC MEASUREMENTS IN PHOTOGRAPHIC PHOTOMETRY. I

By HARLAN TRUE STETSON

The desirability of purely physical methods for the determination of the magnitudes of stars has long been felt among astronomical workers in the field of photometry. Although the introduction of polarizing and wedge photometers has greatly increased the precision of visual work, systematic errors of considerable proportions may nevertheless escape detection where in the end the eye is the sole interpreter of stellar brightness. Aside from the unavoidable differences in spectral sensitiveness of individual eyes, the errors introduced in consequence of the well-known "Purkinje effect" involve considerable uncertainty in the photometric observations over wide ranges of magnitudes.

From the very introduction of the dry plate into astronomy it was seen that the size of the stellar image upon the plate might be taken as an index of the star's magnitude. As early as 1857 Bond[1] made use of the parabolic formula

$$Pt+Q=y^2$$

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[1] *Astronomische Nachrichten*, **49**, 81, 1857.

to demonstrate the relation between exposure time t and the diameter y of the photographic image, P and Q being constants of the plate used. Later investigations of Charlier[1] showed close agreement between magnitudes and measured diameters when the relation was logarithmically expressed:

$$m=a-b\log D.$$

Investigations at Greenwich and elsewhere have seemed to indicate that a square-root relation was applicable to a wider range of conditions as regards plates and instruments than the logarithmic expression would satisfy, and accordingly the well-known form has found wide acceptance:

$$m=a-b\sqrt{D}.$$

Given the relation above, the precision of results depends upon the accuracy with which D may be measured. Since the images at best show no well-defined periphery, the principal source of error in measuring is the uncertainty of locating the extremities of the diameter to be measured. The difficulty is augmented if the images are elongated or poorly defined. Again, the same eye may pass different judgments upon large and small images in the same field. The amount of agreement obtained by different observers using the same method is commendable and indicates the degree of reliability of the results.

Other methods for the reduction of magnitudes include the scale method of estimation employed extensively at the Harvard College Observatory,[2] and the method of extra-focal images.[3] In the latter case, the plates being taken at a considerable distance from the focus, the star-disks are appreciably all of the same size and the difference of magnitude is determined by comparison of the opacity of the images with a calibrated photographic wedge, by means of a Hartmann micro-photometer or other similar device.

The encouraging results obtained in the use of the selenium cell for the direct measurements at the telescope as developed by

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[1] *Publikationen der Astronomischen Gesellschaft*, No. 19.

[2] *Harvard Annals*, **18**, 120; **71**, 4.

[3] *Astrophysical Journal*, **26**, 244, 1907.

Minchin,[1] and by Stebbins,[2] and the still more recent investigations with the photo-electric cell, particularly those of Elster and Geitel,[3] Guthnick,[4] and others, indicate a far-reaching advance in the sensitiveness of photometric methods. At present, however, such methods are confined chiefly to stars of the first few magnitudes and lose advantages otherwise gained by photography.

The possibility of the application of some physical method for the measurements of photographic magnitudes from the rapidly accumulating collection of astronomical negatives has often been suggested, and emphasized particularly on various occasions by E. C. Pickering[5] in connection with the photometric researches of the Harvard Observatory. Accordingly, a series of experiments was begun by the writer in 1911 at the Wilder Laboratory of Dartmouth College, in search of suitable apparatus for the problem in hand. It is with the results obtained from this series of experiments and their application to the determination of stellar magnitudes that the present paper has to deal.

DESCRIPTION OF APPARATUS

The general principle involved is to measure the energy absorbed from a beam of light by the silver grains in the stellar image on a photographic plate, and to interpret such absorption in terms of stellar magnitude. The thermopile, bolometer, radio-micrometer, and radiometer as "detectors" were respectively considered. After some preliminary experiments and deliberation the thermopile was selected for the work as the instrument best combining simplicity and sensitiveness with convenience of manipulation.

It was seen from the outset that the apparatus in question would be more serviceable if adapted to the measurement of original negatives, rather than made dependent upon positive copies. Accordingly the scheme adopted was to restrict the region of the negative in the immediate vicinity of the star to be measured by a very small circular diaphragm, allowing the unobstructed light-beam to register its energy in a corresponding manner. When a successful combination of light-source, diaphragm, projecting lenses, and thermopile had been obtained, the whole apparatus was assembled, and placed in a convenient portable form. The diagram (Fig. 1) will serve to make clear the arrangement, and is reproduced as a suitable design for a more permanent and elegant form of the apparatus herein described.

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[1] *Proc. Roy. Soc.*, **58**, 142, 1895.

[2] *Astrophysical Journal*, **32**, 185, 1910.

[3] *Physikalische Zeitschrift*, **12**, 609, 1911; **14**, 1287, 1913.

[4] *Astronomische Nachrichten*, **196**, 357, 1913.

[5] *Harvard Annals*, **71**, 5.

Light from a source *L* passes through the condenser *C* and is brought to a focus on the surface of a metallic plate *S*, forming the stage of the instrument, upon which the photographic plate is laid. In the center of the stage is placed a small pinhole-diaphragm *d*, which serves the purpose of isolating a small portion of the plate in the immediate neighborhood of the stellar image to be measured. A projection lens *P* of short focal length is then used to project an image of *d*, and the stellar image with which it is in contact, upon the surface of a thermopile *T* connected to a galvanometer. Stellar images of different sizes corresponding to known magnitudes are placed in turn upon the diaphragm and the corresponding change in the galvanometer deflection is noted. This affords means for the calibration of the instrument. A detailed description of the method of reduction will be presented shortly. As considerable difficulty was encountered before the instrument could be relied upon for consistent readings with the degree of accuracy sought, the essential features in the design of the apparatus in its final form will be mentioned.

In regard to the thermo-element, it was seen at once that a surface-element would be necessary, of small heat-capacity, hence short period, and of the maximum sensitiveness. After futile efforts to obtain a suitable element in the market, the requirements were met by a thermopile specially designed and constructed for the purpose by W. W. Coblentz, of the Bureau of Standards, Washington. The element is a bismuth-silver combination of symmetrical design, having a circular receiving surface 5 mm in diameter, and a resistance of 2.3 ohms.[1] Tests reveal a sensitiveness

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[1] For a detailed description of the construction of bismuth-silver thermo-couples, see "Instruments and Methods Used in Radiometry," by W. W. Coblentz, *Bulletin of the Bureau of Standards*, **9**, 15, 1912.

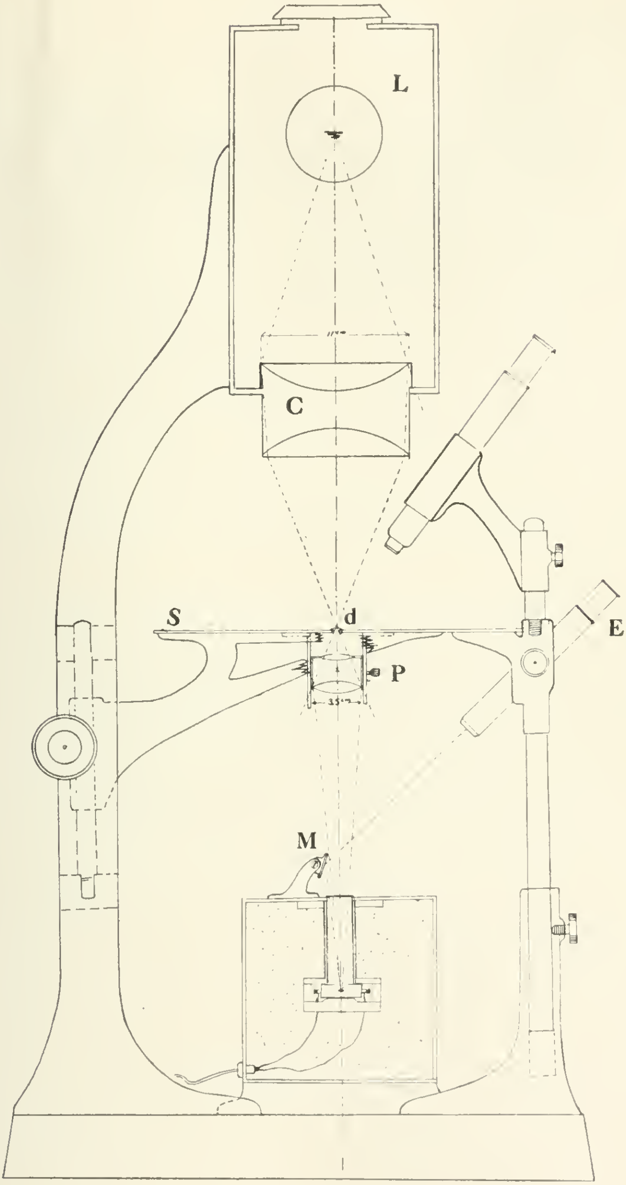

Fig. 1

of one microwatt per 0.18 microvolt, and a period of but a few seconds' duration. The element was incased in a brass receptacle and the whole mounted in the interior of a box of nonconducting material packed with cotton wool, the opening to the well through which the radiation passes being closed with a thin glass plate. Such insulation has proved wholly satisfactory under all ordinary conditions of temperature. With the seclusion of the thermopile, it became necessary to introduce a small mirror at *M* to reflect light from the element into a telescope of short focus at *E*, which serves to locate the stellar image upon the receiver.

Any error due to imperfect centering of the stellar image upon the thermopile is avoided by using the finding eyepiece and making several settings for each star. The agreement is generally better among the image-readings than among the plate-readings, as the latter are distributed more or less uniformly about the image in question, and represent the transparency for slightly different parts of the film.

As at best but a small part of the light from the source is transmitted through the restricting diaphragm *d* (Fig. 1), it is seen that the light-source used must combine intensity with smallness of dimension. The arc, though complying beautifully with these requirements, becomes entirely unsuited for the purpose, on account of its marked unsteadiness. Since it was hoped for the sake of compactness and convenience to use a direct-deflection method, an incandescent lamp operated from a storage-battery of 50 volts was sought. The most convenient luminant tested thus far has been found to be a 50 c.-p. stereopticon-bulb with a carbon filament spirally wound. Such a source when projected may be adjusted to give a satisfactory illumination over the diaphragm, uniformity of light being obtained by so orienting the bulb that projected images of the neighboring coils of the filament overlap slightly. The Nernst lamp at first appeared most promising, but in the final arrangement was abandoned in favor of the type described. Interviews with lamp-makers in consideration of filaments of special design have given little promise of improvement. The ideal source would be an incandescent lamp with a disk or spherical source some two or three millimeters in diameter, of about the intrinsic brightness of the carbon lamp. The 50-volt stereopticon-bulb has the advantages of a filament of large radiating surface, and ease of duplication, and when operated from storage-batteries gives very satisfactory results.

In general any change in voltage is kept under control by connecting a precision voltmeter across the terminals of the lamp. An adjustable rheostat is provided for regulating the voltage when necessary. With storage-batteries of sufficient capacity, little error is introduced from this source, and any progressive or gradual fluctuations in voltage are duly corrected for in the method of reduction, as the plate-readings take into account both variation in the transparency of the film and any gradual change in the intensity of the source. As the plate-readings for each star are necessary in any instance, and form a convenient check on the readings, further precautions beyond the employment of storage-batteries and a sensitive voltmeter seem unnecessary as regards the source.

The essential feature of the diaphragm at *d* (Fig. 1) is the restriction of the cone of light to the approximate dimensions of the stellar image under examination. By the use of two such diaphragms used interchangeably, a wide range of magnitudes is covered for any given plate. In practice the diameters of the apertures most generally used were Nos. 60 and 75 on the standard drill gauge, corresponding to about 1.0 mm and 0.5 mm, respectively.

As originally designed, the diaphragms employed were drilled in a sliding strip of brass flush with the stage of the instrument. Since the ordinary commercial plates are slightly concave, it was found that a large and troublesome source of error was being introduced, the effect of the concavity providing a variable distance between the stellar images and the diaphragm. This varied seriously the dimensions of the cone of light transmitted to the thermopile for stars of equal magnitude when located on different parts of the concave surface. After several experiments, a conical design for the diaphragm was adopted, projecting slightly above the level of the stage of the instrument and thus insuring an invariable contact with the film under all cricumstances (Fig. 2). Any error due to the variation of the slight tilt of the plate thus made necessary was carefully looked for but found negligible.

In the design of the projecting lens two factors have been borne in mind: first, the requisite focal length to produce proper magnifying power within the limits of adjustment; and, secondly, utilization of all the principal rays of the cone emanating from *d*. Change of diaphragm in general will be accompanied by change in the projecting lens in order that the projecting image may cover but not overspread the receiving surface of the thermopile. Two projecting lenses are thus provided corresponding to the two diaphragms most generally employed.

The complete instrument and auxiliary apparatus consisting of galvanometer, resistance box, volt- and amperemeters are shown as in use in Fig. 3.

Adjustments are provided for the light-source in three coordinates. The stage and projecting lens are also adjustable for centering and focusing. A series of diaphragms are provided and may be placed interchangeably upon the stage.

Fig. 2

In the selection of a galvanometer, several factors enter. Theoretically the thermopile is working most efficiently when the resistance of the galvanometer equals its own. For this reason as well as for its high sensitivity, the Thompson type is that usually employed in thermo-electric measurements. On the other hand, convenience of manipulation and freedom from outside disturbances favor the D'Arsonval or moving-coil type whenever such can be adapted to the end sought. Most satisfactory results have been obtained in the present investigation with a Leeds and Northrup galvanometer, Type H, having a resistance of 50 ohms and a sensitivity of 0.75 microvolt. The critical damping resistance of such an instrument is 120 ohms, and is obtained by using external resistance in series with galvanometer and thermopile. With diaphragm No. 60 and a photographic plate of normal density, these conditions afford deflections conveniently large and within the limits of the scale. To obtain corresponding deflections with smaller diaphragms the external resistance is necessarily reduced, increasing the sensitivity of the galvanometer and lengthening its period.

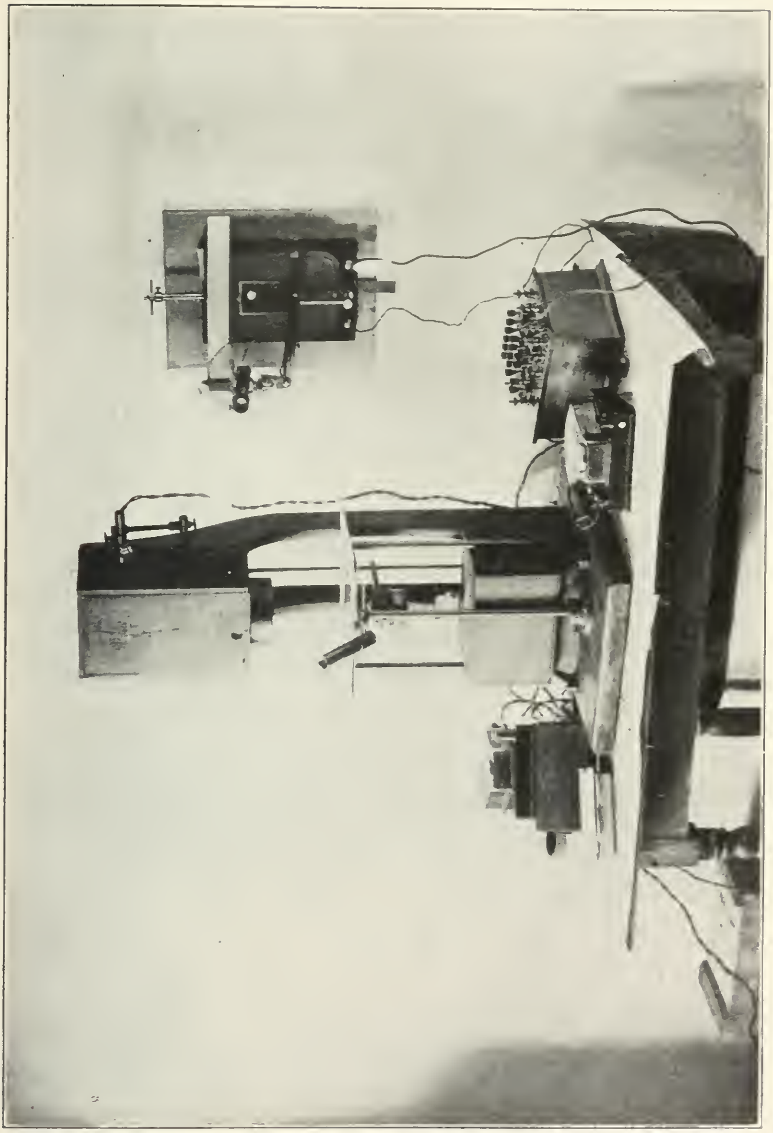

Fig. 3.—The thermo-electric photometer and auxiliary apparatus

As few connections as possible should be made between the galvanometer and thermopile, care being taken to avoid unlike metals, to use ordinary means for maintaining uniform temperature and freedom from draughts, and to protect all binding-posts and connectors with cotton wool.

METHODS OF MEASUREMENT AND REDUCTION

In measuring a plate for reduction, a diaphragm somewhat larger than the stellar images to be measured is selected and put in place upon the stage of the instrument. Since the construction of the diaphragm is such that it projects slightly above the level of the stage-platform to insure contact with the film, the plate is always placed film side down with care. The stellar image in question is placed over the diaphragm, and brought to the center of the latter by viewing its image upon the thermopile by means of the finding eyepiece. As soon as the galvanometer has come to a steady deflection the reading is noted. The stellar image is then moved just off the diaphragm and the reading of the galvanometer again noted as an index of the amount of energy transmitted through the unobstructed portion of the plate and film.

Since the deflections of the galvanometer are taken proportional to the difference of potential impressed, and the difference of potential maintained by the thermopile is directly proportional to the heating effect, then the amount of energy absorbed by the stellar image may be expressed as

$$q=K(D-D'),$$

where D and D' represent the two readings of the galvanometer and K is a constant of proportionality.

Again, since the size and density of the stellar image upon the plate increase with the brightness and hence with the magnitude of the star, which will be denoted by m, for any given exposure we may write:

$$m=f(D-D'),$$

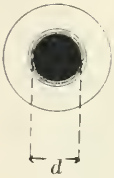

FIG.

and the problem of reduction is the problem of ascribing the form of the function existing for given conditions. Let A be the area of the free aperture of the diaphragm (Fig. 4). Consider the stellar image which, because of its size and blackness, absorbs energy, as having an effective diameter d equal to that of an opaque disk absorbing the same amount of energy. If ρ = the energy transmitted per unit-area, then we have the relation

$$\frac{\rho A}{\rho\left(A-\frac{\pi}{4}d^2\right)}=\frac{D}{D'} \qquad (1)$$

where D and D' represent the galvanometer-deflections corresponding to the plate-readings taken with free aperture, and with a stellar image interposed, respectively. The above may be simplified and written

$$\frac{A}{\frac{\pi}{4}d^2}=\frac{D}{D-D'} \qquad (2)$$

whence

$$d^2=\frac{4A}{\pi}\left(\frac{D-D'}{D}\right)=c\delta \qquad (3)$$

where δ represents the difference in deflection between the free and image-readings for a given star per centimeter of deflection for free aperture, and c is the constant of proportionality.

If we assume the well-known relation between magnitude and measured diameters to hold in the present instance, we have

$$m=a-b\sqrt{d}\,,$$

or

$$d=\left(\frac{m-a}{b}\right)^2.$$

Hence, substituting in (3), we obtain

$$\left(\frac{m-a}{b}\right)^4=c\delta$$

$$m-a=bc^{\frac{1}{4}}\delta^{\frac{1}{4}}\,;$$

or, writing new constants,

$$m = a - \beta\delta^{\frac{1}{4}} . \tag{4}$$

This formula, though indirectly derived from the square-root formula, stands merely as an empirical expression in an attempt at the solution of $m = f(\delta)$. It will be observed that in defining δ as the ratio $\frac{D-D'}{D}$ no assumption as to the constancy of D is involved. The value of D will depend upon the transparency of the plate-film, thickness of the glass, voltage of the source, resistance in the galvanometer-circuit, etc.

Since the reduction involves only the ratio $\frac{D-D'}{D}$, it will be noted that a change in D due to a change in intensity will not affect the results, provided conditions remain constant throughout the measurement of D and D' for a given star. In like manner, variation in the transparency of glass and film over different parts of the plate will be largely if not wholly eliminated, since an increase in absorption though reducing the deflections does not alter this ratio. This was verified experimentally by taking readings on stellar images on a given plate in the usual manner, then remeasuring the same stars with absorbing media interposed in the path of light to the thermopile. Although D was reduced about 50 per cent, the ratio $\frac{D-D'}{D}$ remained the same for each star within the limits of accidental errors. Similar results were obtained from measurements of the same stellar images for different values of voltage.

The first plates reduced were those taken at the Dearborn Observatory, with the 18-inch refractor, a color-filter, and Cramer Isochromatic plates. It was known from careful measurements that the square-root formula gave good results for measured diameters on these plates.

The close approach to a straight-line relation between magnitudes of the Pleiades and the fourth root of the galvanometer-deflections obtained by measuring images on the thermopile is shown graphically in Fig. 5, and justifies the reduction-formula experimentally. Müller and Kempf's[1] values for the magnitudes are laid off as abscissae, and the fourth root of the corresponding galvanometer-deflections as ordinates. The constants α and β of (4) may be taken at once from the graph, and used for the determination of any unknown magnitude on the plate.

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[1] *Astronomische Nachrichten*, 150, 203, 1899.

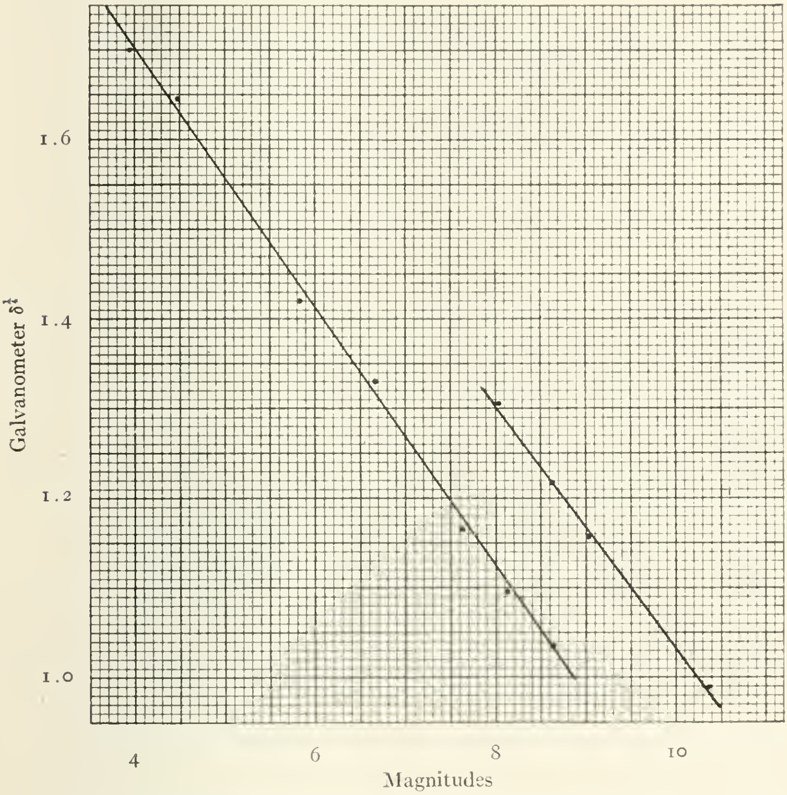

FIG. 5.—From Plate A 140 taken with 18-inch refractor and color-filter at Dearborn Observatory.

As the stellar image becomes smaller relative to any particular diaphragm, the proportional amount of energy absorbed becomes less and the consequent change in the galvanometer-deflection decreases, as is evident from the algebraic expression $m=\alpha-\beta\ \delta^{\frac{1}{4}}$. For small values of δ the change in $\delta^{\frac{1}{4}}$ becomes less appreciable and therefore this increase of error is compensated for by substituting a smaller diaphragm and proceeding as before.

If Δm represents a known difference in magnitude between two stars, and $\Delta\delta^{\frac{1}{4}}$ the corresponding difference in $\delta^{\frac{1}{4}}$ for the two readings, then, for any given diaphragm, the slope of the straight-line relation becomes

$$\frac{\Delta m}{\Delta\delta^{\frac{1}{4}}}=\beta .$$

For the same Δm measured with a second diaphragm we have similarly

$$\frac{\Delta m}{\Delta\delta'^{\frac{1}{4}}}=\beta' ,$$

whence the relation between the slopes of the two plots becomes

$$\frac{\beta}{\beta'}=\frac{\Delta\delta^{\frac{1}{4}}}{\Delta\delta'^{\frac{1}{4}}} , \tag{5}$$

and readings taken with any diaphragm may be reduced to the corresponding scale of any other when two stars are measured in common with the two diaphragms.

In Fig. 5 the graphs are drawn separately for each diaphragm on the same scale and indicate clearly the use of the smaller aperture for the fainter magnitudes. It should be added that with the substitution of the smaller diaphragm, the sensitivity of the galvanometer was increased by lowering the external resistance from 120 to 40 ohms.

In practice it is found that a single diaphragm may be relied upon for a range of three or four magnitudes in measurement, and that the use of two different diaphragms is usually sufficient to cover the entire range of measurable magnitudes on a single plate.

To understand more fully the rôle of the diaphragm and conditions of sensitivity in measurement, let R be the radius of any given aperture used for the diaphragm, and r the effective radius of the stellar image. From Fig. 4 and equations (1) and (2) we may write

$$\frac{\pi R^2}{\pi r^2}=\frac{D}{D-D'} ,$$

or, writing δ for $\frac{D-D'}{D}$ as before, we have

$$\delta=\frac{r^2}{R^2} ; \tag{6}$$

whence by differentiation

$$\frac{d\delta}{dr}=\frac{2r}{R^2}. \tag{7}$$

The sensitivity of the arrangement represented by the derivative $\frac{d\delta}{dr}$ is therefore directly proportional to r, since R is constant for any given diaphragm.

A varying degree of sensitivity dependent upon the size of the image might seem at first a serious and troublesome disadvantage. As will be shown, however, the difficulty is more apparent than real, and may be controlled within limits usually exceeded by accidental errors.

As the sensitiveness of the instrument falls as a result of small values of r relative to R, a new diaphragm of smaller dimensions is substituted. Reference to equation (7) will show that such a change causes an increase in the sensitivity, inasmuch as $\frac{d\delta}{dr}$ varies inversely with R^2, although the free deflection D will be less than before in consequence of the corresponding change in the amount of energy admitted to the thermopile. This change in D, however, may be conveniently compensated for by decreasing the resistance in the galvanometer-circuit until D regains its former value. Although theoretically $\frac{d\delta}{dr}$ depends only on r and R and is quite independent of the magnitude of D, nevertheless in practice it will be seen that the accuracy with which δ may be observed is greater for large values of D. For this reason the resistance in the galvanometer-circuit is always adjusted to maintain D conveniently large for a given set of readings.

Let us suppose for the sake of comparison that we were to measure transparent stellar images upon an opaque ground, such as would be the case in measuring a glass positive instead of the original negative. In this case the galvanometer-deflection D, due to the energy transmitted by the stellar image, would be directly proportional to the area of the image or to r^2, in the form

$$D=Kr^2,$$

the derivative of which would indicate the sensitiveness of the arrangement

$$\frac{dD}{dr}=2Kr\,.$$

Comparing this expression with equation (7) above, we see that as before the sensitiveness varies directly with the radius of the image without the advantage of the controlling factor R^2. Experimental tests with both positives and negatives revealed at once the increased sensitiveness to be obtained with the use of the interchangeable diaphragms on negative plates.

Aside from the advantage of using the original plate, moreover, the ease of manipulation in locating and setting upon the several stellar images is much facilitated by using the plates with dark images against a clear background, as is the case in the negative form. Again, since readings taken from positive plates would be subject to uncertain corrections for opacity of the background and for varying degrees of transparency of glass and film, the advantage lies in the clear glass background, where the condition of the film is constantly under observation and the galvanometer-deflections are large.

Since the sensitiveness of the thermo-electric photometer, if we may so term the apparatus, has been shown to be a function of r for any particular diaphragm, it becomes important to investigate the changing effect of error in δ upon the resulting magnitude.

Making use of the equation of reduction (4),

$$m=\alpha-\beta\delta^{\frac{1}{4}}\,,$$

and differentiating, we obtain

$$\frac{dm}{d\delta}=-\frac{\beta}{4\delta^{\frac{3}{4}}}\,.$$

If the galvanometer-deflections are recorded in centimeters, the value of β for the Dearborn plates is very nearly 6.5. The same is true for Cramer plates and color-filter used with the 6-inch Zeiss doublet of the Yerkes Observatory. If we assume for the error of the mean of several settings on a star an accidental error not greater than $\Delta\delta = 0.01$ cm, a very plausible assumption, we may express the resulting error in stellar magnitudes as

$$\Delta m = \frac{6.5 \times 0.01}{4\delta^{\frac{3}{4}}} = 0.0162\ \delta^{-\frac{3}{4}}.$$

A curve was therefore to be platted for $\beta = 6.5$ using values of δ as abscissae and the corresponding computed value of Δm as ordinates (Fig. 6). Similarly, a family of such curves could be constructed with the parameter β. Such a curve serves at once to indicate the degree of precision attainable with given conditions of plate, galvanometer, and diaphragm. As will be seen, for example, when δ, the fall in galvanometer-deflection due to the interposition of the stellar image, is equal to 1 cm, the error in magnitude resulting from a reading-error of 0.01 cm, will be $0^{\mathrm{M}}016$ and will remain less than $0^{\mathrm{M}}01$ if δ does not fall under 1.6 cm. Unfortunately, systematic

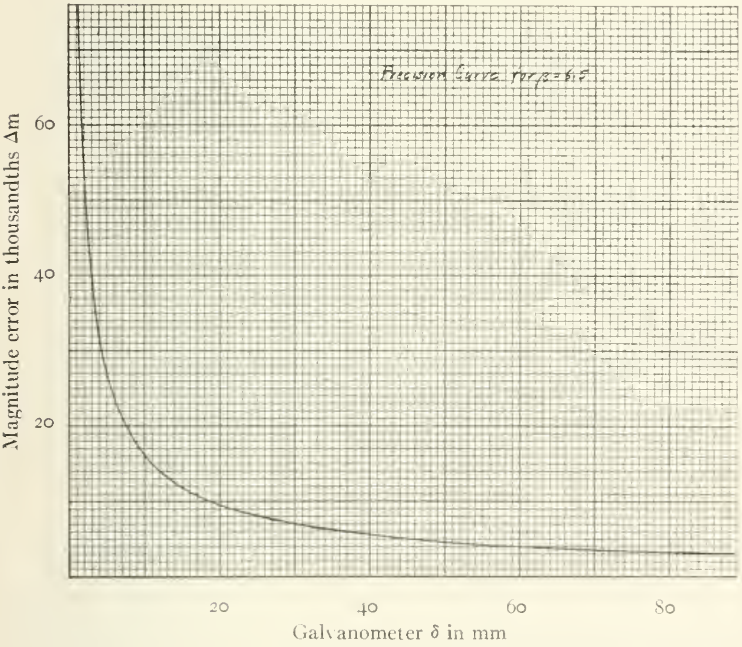

FIG. 6.—Precision curve for $\beta = 6.5$

errors due to conditions of atmospheric transparency or irregularities in "seeing" and plate-sensitiveness seldom warrant higher precision in measurement, though such should be easily obtainable if the stellar images are not too minute to afford larger values of δ. Except in the case of the faintest images, readings are seldom made when $\delta < 1$ cm. When, therefore, in a series of measurements δ reaches the limiting value, a smaller diaphragm is substituted, and the sensitiveness of the galvanometer increased by lowering the variable resistance, with the resulting increase in δ for the same star.

In the actual reduction of plate-magnitudes and the platting against a known or assumed scale of magnitudes as is shown for the Dearborn plate in Fig. 5, care must be taken to differentiate between errors of measurement and systematic errors often neglected, such as correction to the center of the plate, color-curve of the lens, and the relation of color-equation to plate and filter used with a given outfit. As many of these quantities are at best uncertain, it is seen that we could hardly expect a given plate to show residuals not larger than those attributable to errors of measurement. In the interpretation of the curve in Fig. 5 it is to be noted that no correction to the center was attempted, and the approximation to the straight line indicates the degree of agreement between the color-filter and the Cramer plate combination, and the Müller and Kempf "eye."

MEASURES OF THE MAGNITUDES OF STARS OF THE PLEIADES

In investigations with the thermo-electric photometer at the Yerkes Observatory, plates have been measured, taken with each of the several instruments regularly in use, the 6-inch Zeiss camera, the 2-foot reflector, and the 40-inch refractor. Fig. 7 shows the results of platting measurements of the Pleiades in precisely the same manner as was done in Fig. 5. Inasmuch as the field of the Zeiss doublet shows no appreciable correction for distances less than 0.05 from the center,[1] this somewhat uncertain quantity may be considered as inappreciable in the case of the Pleiades plates where the group is centrally located.

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[1] *Astrophysical Journal*, 36, 179, 1912.

In this comparison the agreement with Müller and Kempf's values is at once apparent. The value of the constant β of the reduction-formula as determined by the graph is 6.49 against 6.50 for the Dearborn plates.

To indicate clearly the method of measurement and reduction as well as the amount of agreement among individual settings, a specimen reduction-sheet is shown for Plate UV 1261 (Table I).

The first column gives the Bessel designation of the star. The second column gives the galvanometer-readings in centimeters, when the stellar image is located centrally on the diaphragm, and the third column shows the readings corresponding to the free film of the plate in the immediate neighborhood of the image. Three settings are usually taken as sufficient for each star, and the mean is used. The plate-readings serve in each instance to check any progressive errors in the transparency of the film or glass as well as the effect of possible change in the voltage of the light-source. Although a slight fogging of the background seems to have little effect upon the size or blackness of the stellar image, it enters of course with full value upon the transparency of the background with which each image is compared, and so needs to be eliminated. To

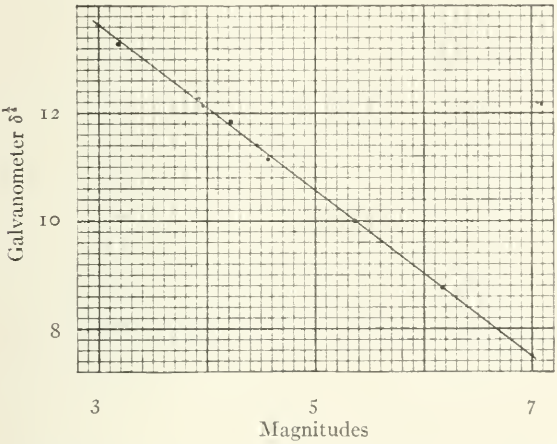

FIG. 7.—From Plate UV 1261, taken with Zeiss doublet and color-filter at Yerkes Observatory.

TABLE I

PLATE* UV 1261, PLEIADES. MEASURED DECEMBER 17, 1914

Volts, 51.5. Amperes, 3.8. Galvanometer Resistance, 40. Diaphragm No. 75

STAR	GALVANOMETER DEFLECTION			δ	$\delta^{\frac{1}{4}}$	MAGNITUDE		
	D'	D	$D-D'$			Müller and Kempf	Computed	Residual
η..........	9.83 cm 9.80 9.75	14.20 cm 14.21 14.16						
	9.79	14.19	4.40 cm	3.10	1.326	3.19	3.23	+.04
f..........	10.88 10.94 10.90	14.10 14.08 14.08						
	10.91	14.09	3.18	2.26	1.225	3.92	3.90	−.02
b..........	10.83 10.83 10.88	13.80 13.88 13.81						
	10.85	13.83	2.98	2.16	1.212	3.96	3.98	+.02
c..........	11.40 11.38	14.18 14.19						
	11.39	14.18	2.79	1.97	1.185	4.21	4.16	−.05
d..........	11.56 11.58 11.69	13.89 13.99 14.04						
	11.61	13.97	2.36	1.69	1.139	4.48	4.46	−.02
e..........	11.02 11.00 11.00	13.10 12.95 12.91						
	11.00	12.98	1.98	1.53	1.114	4.57	4.62	+.05
h..........	12.69 12.70	14.12 14.10						
	12.70	14.11	1.41	1.00	1.000	5.38	5.36	−.02
k..........	12.78 12.79	13.53 13.59						
	12.78	13.56	0.78	0.58	0.874	6.17	6.18	+.01

$\beta=-6.50$ $\alpha=11.86$ p.e. $=\pm 0.022$

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* Cramer Instantaneous Isochromatic with visual luminosity filter β 10.

avoid decimals it is found convenient to take for δ ten times the ratio $\frac{D-D'}{D}$. The quantity δ may now be defined as the difference in galvanometer-deflections between the stellar image and its background for a standard plate-reading of 10 cm. Column five therefore gives

$$\delta = \frac{D-D'}{D} \times 10\,.$$

The fourth root of δ is entered for each star in column six and is easily obtained by use of a table of squares and square roots of numbers. If the number whose fourth root is desired is located in the column marked "Square," its fourth root may at once be taken from the column marked "Square Root"—such tables for numbers from 1 to 1600 are usually found in the numerous engineers' handbooks and require little interpolation.

The last three columns give the magnitudes found by Müller and Kempf, the magnitudes computed from the constants $\alpha = 11.86$, $\beta = -6.50$, obtained from the least-square solution, and the resulting residuals, indicating close agreement between the magnitudes photo-visually determined with the Zeiss camera and Müller and Kempf's scale, the probable error for a single star being $0^{\mathrm{M}}.022$.

Ordinary plates taken without light-filter with the 2-foot reflector of the Yerkes Observatory were reduced in a similar manner and justified again the use of the fourth-root law. The results of reducing Pleiades-magnitudes from Plate R 531 is shown graphically in Fig. 8. The photographic magnitudes platted as abscissae are those of Münch determined at Potsdam. The solid dots indicate the reduced magnitudes after correction to the center of the plate. Although the aperture of the mirror was reduced to 12 inches for the particular exposure in question, the field is still subject to correction for distances greater than $\rho = 4'$ and amounts to $0^{\mathrm{M}}.2$ or more at a distance of $\rho = 20'$. The method of calibration of this field correction was similar to that previously described in this *Journal*[1] and needs no detailed description here. In the present instance Plate R 2594 was selected, which contained four sets of images taken across the optical axis. The absorption of these images was measured with the thermopile in the usual manner and the $\delta^{\frac{1}{4}}$ corresponding to each image was platted against the distance of the images from the center of the plate, expressed in minutes of arc. Fig. 9 shows the resulting graph. The departures from the mean curve may be attributed to changes in "seeing," atmospheric transparency, and small errors in the equality of the times of exposure. These curves are all similar and express the variation of $\delta^{\frac{1}{4}}$ with distance from the optical axis for four different-sized images corresponding to four different values of $\delta^{\frac{1}{4}}$ for any given distance from the axis. Within the range of images measured, no particular

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[1] *Astrophysical Journal*, 38, 224, 1913.

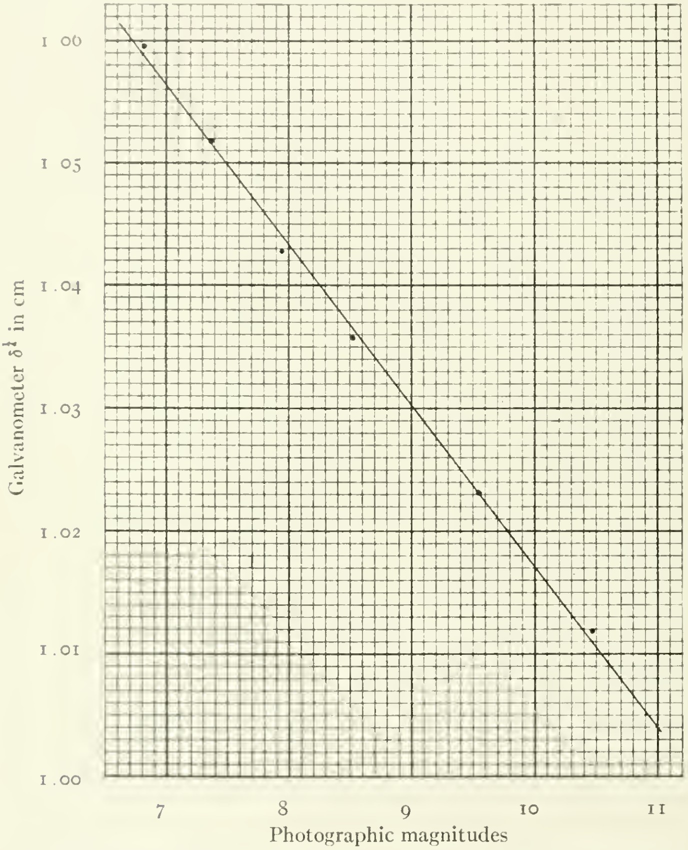

Fig. 8.—From Plate R 531, taken with 2-foot reflector at Yerkes Observatory

dependence of the form or size of correction upon the size of the image seems apparent. It must be admitted that corrections made in this manner seem at best artificial, and at present it seems exceedingly doubtful if corrections taken from certain plates used experimentally can be applied generally to any other plate without large assumptions as to the order of accuracy maintained. In applying corrections from the curves of Fig. 9 to the magnitudes of R 531, Fig. 8, the constant β of equation (4) was so chosen as to reduce to a minimum the residuals of the corrected magnitudes of R 531.

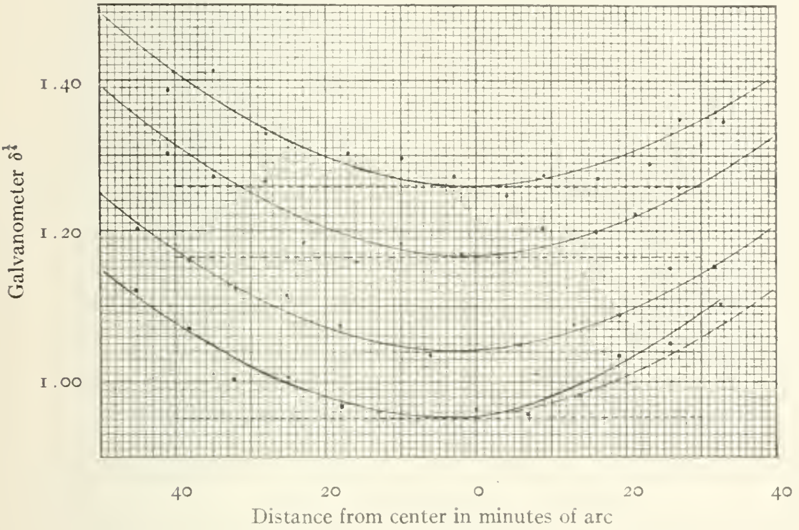

FIG. 9.—Calibration curves for 2-foot reflector from Plate R 2594

This gives a value of $0\overset{\mathrm{M}}{.}026$ for a change in $\delta^{\frac{1}{4}}$ of 0.01 unit. Stars 24 and 27 (Bessel) receive the largest corrections, being 17′ and 26′ from the axis, respectively.

The whole matter of finding suitable methods of correction to the center of the plate is one of the most obstinate difficulties in the way of exact photometry on an absolute scale. A high degree of precision in the measurement of photographic images will therefore best be rewarded in the investigation of relative magnitude-changes or in the photometry of small areas where the field under investigation is restricted to the vicinity of the optical axis.

Other difficulties in the way of more accordant results from various observers is the lack of uniformity of equipment and spectral sensitiveness of the plates used. "Photographic magnitudes," as at present published arè understood to be the results of measurement of the stellar images on "ordinary" (blue-sensitive) dry plates exposed at the focus of a reflector or a photographic refractor (corrected to blue rays). Unfortunately the difficulties of non-uniformity in color-sensitiveness of the various dry plates on the market is greatly accentuated by the wide divergence in the optical equipment employed. Unless a color-filter is used, the uncorrected, and out-of-focus rays will materially affect the size and blackness of the stellar image on the plate. Moreover, the effect will be quite different for differing spectral types, the difficulty being exaggerated for focal settings on either side of the normal position. Again, a photographic objective of special Jena glass will undoubtedly behave somewhat differently from a photographic lens of the usual flint and crown combination. In the case of the reflector, where radiations of all wave-lengths are combined, the effect must invariably differ from the case of any refracting telescope. These divergences, small or large, will become more apparent with greater refinement in the measurement of the photographic images and indicate further fields of investigation in stellar photometry.

It would seem that, for agreement on a precise scale for magnitudes, the reflector would be the ideal instrument for photography, as the question of a color-curve would thus be entirely eliminated so far as the instrument is concerned. The chief difficulty remaining would then be the adoption of a standard plate of given spectral sensitiveness, or. in want of such plates, emplcying a prescribed filter of known absorption which would produce as far as possible a certain effect for a given brand of plates.

For purposes of completeness in the application of the fourth-root law for the reduction of thermopile-measures, tests were made upon plates taken with the 40-inch refractor through color-filter and with isochromatic plates. Fig. 10 shows the magnitudes of the Pleiades on Müller and Kempf's scale platted against the corresponding $\delta^{\frac{1}{4}}$ of the galvanometer-readings. From other series of measurements covering a variety of times of exposure, the slope of the line appears quite independent of the time of exposure, at least within the range of exposures used.

Although considerable unpublished material exists relative to the magnitude-corrections to the center of the plate for the 40-inch refractor, and further investigations were made using the thermopile method for plate measurement, consistent results have not been obtained which may be trusted within the degree of precision desired. Owing to rapid and uncertain changes in seeing and the great focal length, it has been found practically impossible to use the simple method employed for the calibration of the reflector plates. Inasmuch as the stars of the Pleiades platted above cover a considerable area on the plate, the residuals indicated are doubtless attributable to this troublesome and uncertain correction to center which has yet to be applied. A still further difficulty in the way of applying this correction is the uncertainty of locating with exactness the position of this so-called "center." Although a rough

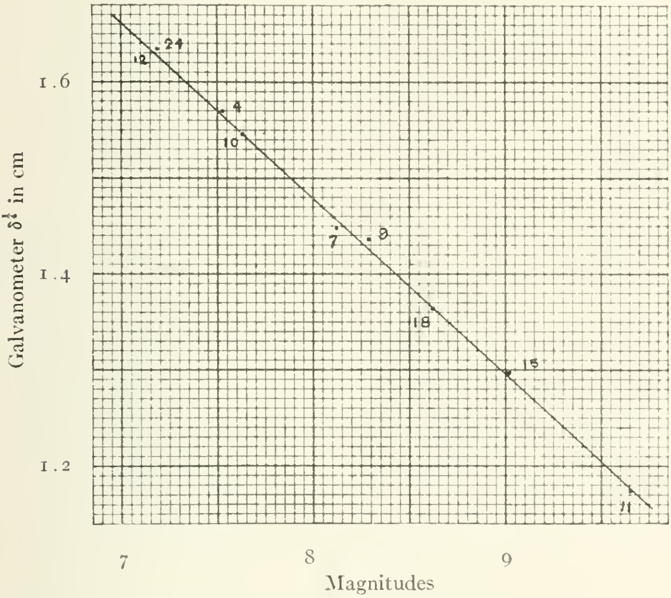

FIG. 10.—From Plate 015, taken with 40-inch refractor and color-filter at Yerkes Observatory.

approximation for the position of the optical axis with a given plate may be obtained from the intersection of the prolonged major axis of the elongated stellar images near the edge of the plate, the errors introduced by the uncertainty in the location will easily amount to several hundredths of a magnitude at large distances from the axis. Further, the stability of the position of this "center" from plate to plate should not be assumed without careful investigation.

The investigations of Schlesinger[1] in locating the optical axis in the case of the 40-inch refractor, and the subsequent precautions taken in obtaining plates for stellar parallax, may not be without application to work of precision in the line of photographic photometry.

SOURCES OF ERROR IN THE PLATE

Enough has already been said in regard to sources of error for which the telescope is responsible. To a large degree these errors are susceptible of evaluation and correction, if not of elimination. There remains, however, one serious source of error in all photographic photometry which has its effect in purely relative measurement as well as in determinations on a standard scale. Such is the plate itself. Residual errors due to a lack of uniformity in the sensitiveness and thickness of the emulsion on the plates must exist, and it remains to eliminate as far as possible dangers from this source. Fortunately the magnitude of such errors has not been large enough to interfere seriously with the tenth's place on the magnitude-scale, and for this reason such errors have in general aroused little attention. If, however, we hope to push forward another decimal place in astronomical photometry, here lies one of the fields for further investigation and research.

Neglecting the region of the plate within 1 cm from the edge, Parkhurst[2] found from several hundred measures on Seed and Cramer plates at the Yerkes Observatory that the effect of local variations in sensitiveness of film would in general be less than $0\overset{\mathrm{M}}{.}04$, and under most unfavorable circumstances might rise to $0\overset{\mathrm{M}}{.}1$.

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[1] *Astrophysical Journal*, 32, 376–378, 1910.

[2] *Astrophysical Journal*, 31, 20, 1910.

This error could doubtless be much reduced by employing plates especially coated on plate glass. The concavity of ordinary commercial plates introduces another source of error, and has been investigated by Parkhurst, who estimates the probable effect for focal measures to be of the order of $0\overset{\mathrm{M}}{.}01$ or $0\overset{\mathrm{M}}{.}02$.

Investigations by Hartmann[1] reveal somewhat larger sources of error than those found by Parkhurst.

In a further study it is purposed by means of the thermopile to investigate extensively a large number of plates of both domestic and foreign manufacture on both commercial and plate glass to determine, if possible, the maximum degree of precision which may be relied upon in the use of the plates now upon the market. When one considers the extreme sensitiveness of the photographic plate as a detector of radiation, and the possibility of obtaining measurable images of faint stars hopelessly beyond the range of any other known "radiometer," the importance of perfecting such a medium for work of precision cannot be overemphasized. If the co-operation of observatories and other scientific institutions using large numbers of plates could be secured, it would doubtless be possible through special manufacture to secure plates much superior to any of the commercial brands now upon the market.

METHOD OF EXTRA-FOCAL MEASUREMENT

The adaptation of the thermo-electric photometer to the direct measurement of plate-opacities is at once apparent. For such work, since the area of the region to be measured is presumably more than sufficient to cover the diaphragm, a large aperture may be used and the galvanometer-resistance reduced for greater speed of manipulation. For such work galvanometer δ's are taken as directly proportional to plate-opacities. If extra-focal stellar images are being measured, correction for a lack of uniformity in the absorption of the glass can be made by taking readings on the free glass in close proximity to the image, as is done in the measurement of focal plates, already described. Here, as in the other case, precision of results rests, first, on the assumption of uniform sensitiveness of the photographic film, and secondly, on the assumption that the absorption of the glass for the region immediately surrounding the stellar image is the same for the region beneath the image under measurement.

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[1] "Ueber die Konstanz der Empfindlichkeit innerhalb einer photographischen Platte," *Eders Jahrbuch für Photographie*, 1906.

The advantage which the thermopile has over the "microphotometer" for extra-focal work is the elimination of the personal equation by the shifting of responsibility for correct judgment of opacities from the eye to a purely mechanical device. The order of accuracy of setting is about the same for the two methods, though, of course, the results of the Hartmann method are influenced by the skill of the observer.

To illustrate the degree of blackening due to various exposures, a plate containing a series of "standard squares," produced by simultaneous exposures to light-sources of known ratio of intensity, was measured with the thermo-electric photometer. The graph in Fig. 11 shows the relative blackening as measured by the galvanometer-deflections platted against the relative intensities expressed in magnitude-differences. Since the exposures were all made simultaneously, the validity of the reciprocity law $E=It^p$ does not enter, the values of E being to each other as the intensities of the source. It should be stated that the plates used for this test were selected from among those used by Parkhurst for another purpose and described by him in an earlier publication.[1]

Readings were taken upon the clear background of the plate adjacent to each image as usual, and subtracted from the galvanometer-readings for the image alone. Assuming that the galvanometer-deflections so corrected will be directly proportional to the transparency of the image, we should expect to find a nearly linear relation between the deflections so recorded and the logarithm of the "exposure" E, or a direct proportion between the quantity E as expressed in magnitudes. Data concerning the several apertures admitting the light to the several images have already been published (*loc. cit.*), together with the relative magnitudes. It will be observed that an approximate relation holds over a range of about one magnitude only.

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[1] *Astrophysical Journal*, 26, 244, 1907.

An extra-focal plate (blue-sensitive) of the Pleiades taken with the Zeiss camera was then measured with the thermo-electric photometer. The graph obtained from platting magnitudes against the logarithm of the galvanometer-deflections is shown in Fig. 12. The bending of the curve for the brighter stars due to overexposure indicates the comparatively narrow limits within which the law of blackening may be applied. In extra-focal work several exposures properly timed will ordinarily be needed to cover the desired range for a given field. The same limitation applies of course equally well to measurements of extra-focal images by any other method.

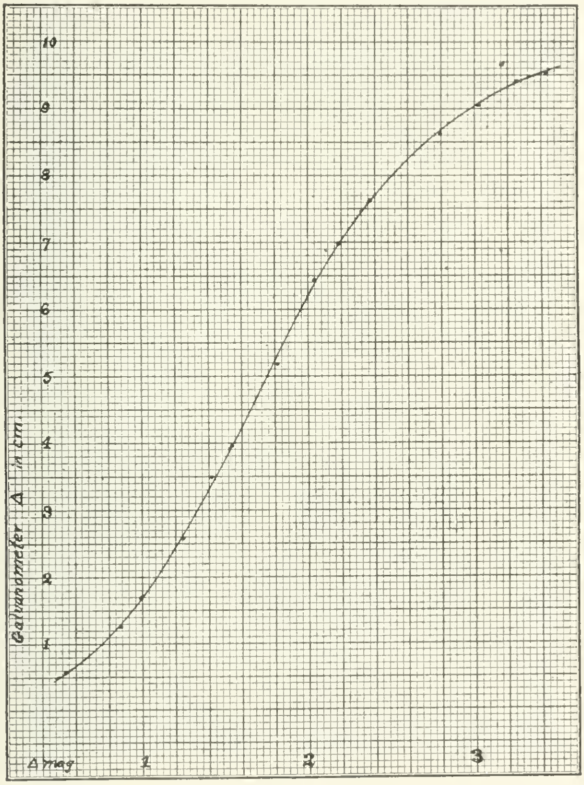

Fig. 11.—Curve showing plate opacities in terms of galvanometer deflections

SPECTRO-PHOTOMETRY

The idea of measuring relative intensities in spectral plates by means similar to the foregoing is not new. The extended investigations of Koch[1] resulting in the so-called "self-registering" microphotometer have proved not only the feasibility of the method but the valuable results which may be obtained by such methods of measurement, especially in the study of laboratory spectra.[2] The apparatus under present consideration, when provided with a

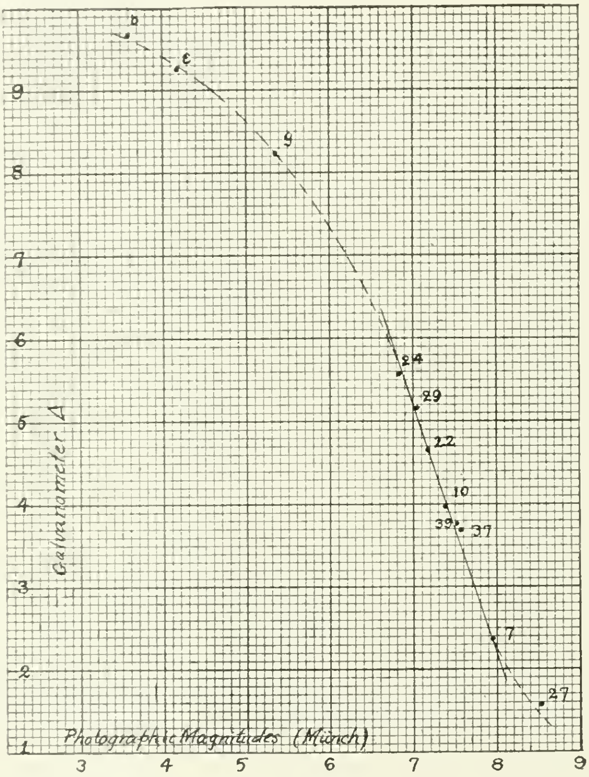

FIG. 12.—Extra-focal measures of magnitudes of stars of the Pleiades (the designation is that of Bessel).

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[1] *Annalen der Physik*, 39, 705, 1912.

[2] *Astrophysical Journal*, 39, 213, 1914.

micrometer-screw stage, becomes at once adapted to such measurements. When so used, the small circular diaphragm is replaced by a small adjustable slit, which all but touches the under and film-side of the spectral plate to be measured. Arrangement is then made whereby a sliding platform moved by an accurately cut micrometer-screw may be attached to the main stage of the instrument. The spectral plate is held to the latter by clips in the usual manner, with the film down and in close contact with the slit-diaphragm. The spectrum is then exposed by setting the micrometer-screw for a given reading and noting the galvanometer-deflection corresponding to the opacity of the negative, then advancing the plate a known fraction of a millimeter, and recording the corresponding galvanometer-reading against the screw-setting, continuing the process till the region in question has been covered. This arrangement, though lacking the elegance of the "registering micro-photometer," has the advantage of simplicity, the thermopile replacing as detector the somewhat more troublesome photo-electric cell as used in the more elaborate apparatus of Koch.

In the case of objective-prism photographs, when the linear dimensions of the stellar spectra are not great, the task of mapping the intensity-curve is not difficult and may be accomplished in a comparatively short time. After the spectrogram has been properly lined up on the stage no further attention is necessary other than noting the reading of the galvanometer after each turn of the screw. Results are shown for measurements of an objective-prism plate of the Pleiades taken with the Zeiss ultra-violet doublet previously mentioned. The graphs in Fig. 13 show the intensity-curves for the photographic spectra of the three stars Alcyone, Taygeta, and Merope. Screw-readings are platted as abscissae against the direct readings of the galvanometer as ordinates, the plate being advanced 0.01 mm for each setting. The plate for which the curves are drawn is an enlarged positive, the hydrogen lines appearing dark and accordingly represented in the graph by depressions corresponding to the lower values of the galvanometer-reading. The slit is purposely adjusted somewhat wider than the absorption lines in order to minimize their effect and give a somewhat smoother form for the entire curve of distribution of energy. No shift in the point of maximum radiation is apparent, but the center of gravity of the area formed by the curve and its horizontal base-line is appreciably nearer the violet end for Alcyone than is the case for the other two stars. This would seem to indicate that the bright star is relatively somewhat "bluer" than the other two in question. Careful investigations of spectral types in this manner may afford a more refined method for establishing color-indices and make possible important quantitative measurements in spectro-photometry.

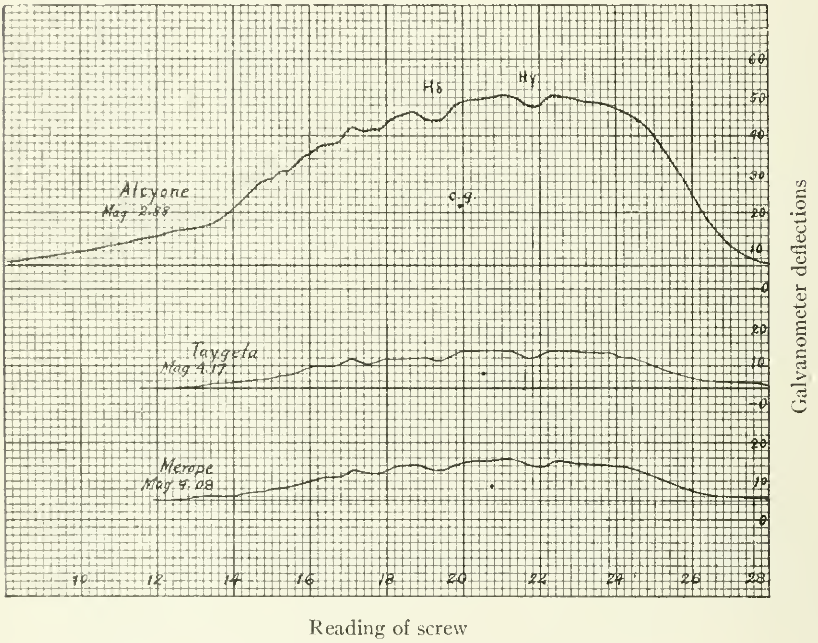

FIG. 13.—Spectral intensities of three stars of the Pleiades

By narrowing the slit in the case of the arrangement described above, it becomes possible to determine with considerable degree of accuracy the mean position of broad and diffuse lines, which are so difficult of measurement by the usual methods. Further, by a special modification of the design of the thermopile it should become possible to measure with considerable rapidity the location of a given line, if its spectral intensity is not a question of importance. Such a modification would form an important accessory to the instrument and would prove of special value in the determination of the radial velocities of stars of early type.

A continuation of this subject of thermo-electric methods of measurements, with an application to variable stars, will appear in a subsequent number of this *Journal*.

YERKES OBSERVATORY
March 1916

THE NATURE OF THE CONSTANT-ERROR TERM FOUND IN THE DETERMINATION OF THE SOLAR MOTION FROM RADIAL VELOCITIES

By C. D. PERRINE

INTRODUCTION

I have stated the suspicion that the term K found in the determination of the solar motion from radial velocities was in reality not a constant term but that it was due to unrepresented motion. That suspicion resulted from the finding of peculiarities in the residual velocities in the various spectral classes in different regions of the sky. It is highly important to know whether this term is in reality some form of constant error (using this expression in a broad sense) or whether it is due to motion.

The term K is in effect the *average algebraic residual after taking out the solar motion.* If there is an excess of positive or negative velocities after eliminating the solar motion—whether this excess is due to a few large or many small velocities or whether to a few or many isolated regions of the sky—this excess divided among the stars used will give a constant term. The consistency with which such a constant residual is shown in different parts of the sky should be a good test of the reality of the phenomena as a constant error or physical effect of some kind.

ELIMINATION OF THE SOLAR MOTION

It seems desirable to consider for a moment the reality of the so-called solar motion, elementary though it be. If we consider the evidence afforded by the proper motions, as well as the generally good agreement of the values of the solar motion derived from such heterogeneous data as the widely different average velocities of the different spectral classes and in turn the consistency of the velocities within these spectral classes after the elimination of the solar motion, there seems no reason to doubt that the so-called solar motion is in reality an actual relative translation of the sun among the naked-eye stars. As to what the real motion may be of this mass of stars with respect to others or to the spiral nebulae, planetary nebulae, globular clusters, or other super-systems, we are not concerned for the present.

The question of what value of the solar motion is to be used for any special investigation must depend upon the nature of that particular investigation, whether the point of reference is the center of gravity of a certain class or group of stars, or of all the stars. In the present and similar general investigations it appears preferable to use a common motion rather than the values found for each class separately.

There is some uncertainty in regard to the position of the apex of the solar motion. It is, however, very unlikely that the uncertainty is great enough to affect the general conclusions contained in this paper. The velocity seems even less doubtful. My own investigations convince me of the general correctness of Campbell's conclusion that the velocity of the sun is close to 19.5 km per second in a direction not very far from $18^h, +30°$, referred to the stars which we are discussing.

The observed velocities of the stars have been freed from the solar motion on this hypothesis. Owing to the undoubted connection of some of the principal systematic motions of the stars with the Milky Way, the first investigation related to it. The comparison of galactic and non-galactic regions also seems an efficient test, as these regions appear to be but little affected by uncertainties in the amount and direction of the solar motion.

The residuals were first grouped for the galactic and non-galactic regions separately and the results are given in Table I.

TABLE I

CLASS	GALACTIC			NON-GALACTIC			R_1-R
	R	Mean V	No. of Stars	R_1	Mean V	No. of Stars	
	km	km		km	km		km
B–B5.....	+5.2	8.3	163	+4.3	5.4	17	−0.9
B8–B9....	+1.1	6.0	31	+1.2	6.5	14	+0.1
B all.....	+4.5	8.0	194	+2.9	5.9	31	−1.6
A........	−1.4	12.2	113	+0.7	9.6	99	+2.1
F........	−2.1	16.0	115	+2.1	13.2	85	+4.2
G........	−0.6	25.5	89	+0.9	15.6	64	+1.5
K........	+2.7	17.9	283	+4.9	16.9	160	+2.2
M........	+3.0	17.0	42	+7.5	17.6	38	+4.5

The galactic limits were arbitrarily assumed at ±40°, the limits used in earlier investigations. These are somewhat outside the real limits of the Milky Way and were used purposely in order to show as clearly as possible any differences between the two regions. This practice was considered preferable to the adoption of narrower limits on account of the relatively small number of stars. Such limits should at least not exaggerate any real differences. *R* is the mean algebraic residual for the group, and mean *V*, the average velocity, the signs being ignored.

The first thing to attract attention in these results is the algebraically smaller values of *R* for the galactic than for the non-galactic regions in all except the stars of class B, which exhibit a contrary condition.[1] This table also shows the markedly larger average velocities which exist in the galactic stars of the earlier classes, to which attention has been called by others. Our present interest in this table lies chiefly in the sizes of the discordances which are shown in the residual velocities of galactic as compared with non-galactic stars. If the term *K* is in reality a constant error it must from the nature of the case be independent of the region of the sky. The facts as derived from Table I seem to be that these residuals are different for the galactic and non-galactic regions, not only in the three classes which show large values for the *K* term, but also for the other three classes as well, that these differences are of considerable size, and that they appear to be systematic.

It may be noted that the algebraic mean of the galactic and non-galactic values of *R* are essentially the same as the *K* term for the three classes which have exhibited the latter. For the other three classes which showed no appreciable *K* term the means of the values of *R* are essentially zero.

In order to test whether there is any considerable constancy in smaller regions, the sky was arbitrarily divided into 72 regions, each 2^h in right ascension by 30° in declination, and the B stars of $3^M_.0$ and fainter in each region were combined into one mean. The angular distance from the assumed solar apex was determined, using the mean α and δ of the group. Although not strictly rigorous, this practice is amply accurate for such an investigation as this. Some of the regions were combined where there were a very few stars and they fell within a square 30° on the side or but little larger. A few single stars were omitted because of their isolation.

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[1] The same phenomena appear in the residuals of the B stars which were derived by eliminating the term *K* as well as the solar motion, (V_1).

In order to eliminate to a large extent the effect of the larger velocities near the vertices of the ellipsoid, all stars within 40° of those vertices were omitted. There resulted 129 stars in 20 groups. Solutions were made for the solar velocity, preferential motion toward 7^h, $+64°$, and the K term. The results, together with two other solutions to be explained later, are given in Table II. The value of K derived from all of the 20 regions is in good agreement with that found from all of the B stars.

TABLE II

No. of Equations	No. of Stars	$V_\odot$	V_S	K
		km	km	km
20	129	-20.5	$+0.8$	$+4.5$
13	89	-19.8	$+0.4$	$+0.8$
7	40	-20.1	$+0.1$	$+11.3$

An examination of the residuals of the different groups, however, showed a marked tendency to divide themselves into two classes, those with positive signs and numerically averaging about as large as the K value and those of about the same size but with negative signs. This feature was so pronounced that separate solutions were made for the two groups, with results as shown in the second and third lines of Table II.

One-third of the regions (and less than one-third of the stars) have produced the positive residuals which give rise to the constant-error term, two-thirds of the regions showing only a very small excess of positive residuals.

As the value of the solar velocity is in good agreement for all of the solutions, we are forced to the conclusion that the differences are practically all in the excesses which give rise to the K term. An examination of the residuals from the third solution indicates that *three* different values of K (instead of two) would represent these

20 regions quite closely. The grouping of the stars used in this investigation by areas of considerable extent frees the foregoing values of K from criticism on the ground that they are simply accidental values. Individual stars would be open to such criticism, but it can scarcely hold for groups even as small as these when they represent areas of any considerable extent. It seems evident that there is a lack of anything like the uniformity which we should expect from a strictly constant error. The peculiar tendency to fall into two or three fairly well-defined values of K in these 20 regions is perhaps significant.

An examination of the numbers of the stars with large inherent velocities of approach and recession indicates a preponderance of positive velocities in classes B, K, and M and the contrary for classes A, F, and G. All of the stars of classes B, K, and M show a similar excess of positive velocities, equality for classes A and G, and slightly the contrary for class F. All of the 1300 stars show a decided excess of positive velocities. If the K term is in reality some form of constant error, its application tends to obliterate these peculiarities, so that the foregoing evidence is not wholly independent.

Table III contains the results of an examination of the residual velocities (R) and mean inherent radial velocities (V) of the stars within 40° of the apex and antapex of the sun's way after elimination of the solar motion, using the value of 19.5 km toward 18^h, $+30°$ and omitting K.

TABLE III

	Apex			Antapex		
	R	Mean V	No. Stars	R	Mean V	No. Stars
		km		km	km	
B	$+2.2$	8.1	21	$+5.7$	7.2	51
A	$+2.1$	9.1	28	-1.7	12.2	22
F	-2.2	15.7	27	-1.4	14.9	27
G	-6.4	16.9	24	-5.0	24.6	14
K	-2.3	18.0	53	$+8.3$	16.9	66
M	-12.3	18.1	7	$+6.2$	20.4	12

Table IV contains the results of an examination of the *numbers* of stars (and their average inherent velocities), *moving toward* the solar apex and antapex, i.e., positive velocities in the antapex and negative in the apex region are considered as moving *toward* the antapex and *vice versa* for the apex region.

TABLE IV

	Stars Moving Toward			
	Apex		Antapex	
	No. Stars	Mean V	No. Stars	Mean V
		km		km
B	22	6.7	50	7.8
A	23	13.5	27	7.9
F	26	15.5	27	15.7
G	22	15.1	16	26.0
K	47	14.8	72	20.4
M	4	26.4	15	17.7

The peculiar and possibly significant thing about these results is that in the three classes yielding large K terms (B, K, and M) we not only find an actual excess of stars in the antapex regions of all three classes but roughly the same number of stars *moving toward the antapex* as the number in that region. Further than this, we find that the ratios between the numbers of stars *moving toward the antapex* and in the opposite direction in these three classes are roughly proportional to the values of K obtained and more closely by the expression

$$K=1+\frac{\text{No. stars moving toward antapex}}{\text{No. stars moving toward apex}}\ .$$

It is further to be observed that in the regions discussed the stars of classes B, K, and M (those with large K terms) show generally large excesses of positive residuals toward the antapex region. Class A shows a considerable excess in the opposite sense and classes F and G very nearly balance. It seems premature to attempt a discussion of these points until the fundamental one of the nature of the K term is better understood. It may be pointed out, however, that such peculiarities as the above should be studied in connection with other evidence bearing on the nature of this term.

There appears to be no reason a priori to consider the residuals which we have found as being constant error any more than motion of some kind. While the physical explanation of pressure which has been offered has a firm basis, it remains to be shown that there is any such effect in the stars with large K terms, and, if so, any connection with the observed residuals. Neither do I see any reason to assume that there is a K term *superposed* upon peculiar or systematic motions. It is certainly most desirable to ascertain whether there are effects of pressure in any class of stars, but up to the present time I do not think that we are justified in assuming that the radial velocities in general are affected in such a way.

So far as the peculiar residuals which have been found in different portions of the sky are concerned, there appears no reason why they may not be some form of motion. We have no reason to assume that the motions of considerable groups of stars in opposite portions of the sky are the same. In fact, the accepted ideas of star-streaming, increase of velocity with increase of supposed age of the stars, and the magnitude-velocity equation would lead us to expect some such general differences of motion in widely separated portions of the sky.

Whether the differences found are large enough to disprove *constancy* sufficiently to require the abandonment of the term K as constant may be open to question. In my opinion the lack of uniformity in the residuals derived from galactic and non-galactic regions and from areas of considerable extent, taken in connection with other eccentricities of residuals which have been observed, is sufficient to show the necessity of proving a physical effect or a strictly constant error in the stars of classes B, K, and M before ascribing the residuals observed to other than motion.

Should it be confirmed that the peculiarities observed are in reality due to motion and not to a constant error, the results given in the tables are extremely suggestive. These results indicate:

1. That the non-galactic stars of all six classes are moving outward as a whole with respect to the instantaneous position of the sun, the velocities of the B, K, and M stars being greatest.

2. That the *galactic* stars of classes A, F, and G as a whole are approaching the observer with small velocities, while the galactic stars of classes B, K, and M are moving away.

3. That the outward movement of the galactic stars is in excess for the B stars, but that in all the other classes the greater outward movement is in the non-galactic stars.

4. That the peculiar preference of the stars of classes B, K, and M for the general region of the antapex of solar motion, taken in connection with the proper motions of these classes, and peculiarities in the residual velocities in the regions of the apex and antapex, lead to the belief that they may be indicative of some far-reaching structural principle in the arrangement of these classes of stars.

What effect uncertainties of the solar motion either in amount or direction may have upon the above indications is not known. The interpretation of the phenomena with which we are dealing is a matter requiring great care and a due reserve is felt with respect to any explanations suggested.

CONCLUSIONS

1. After eliminating the solar motion from the stars of each spectral class the residuals are different for the galactic and non-galactic regions in all of them.

2. The algebraic mean of these residuals for galactic and non-galactic regions agrees reasonably well with the constant-error term (K) found for the same spectral classes in the solutions for the solar motion.

3. The differences between galactic and non-galactic residuals are of considerable size and appear to be systematic.

4. Other large differences of an apparently systematic nature have been found in large areas of the sky, which add to the suspicion of irregularity in the tendency of motions in different regions.

5. The results of this investigation indicate a lack of the uniformity in the residuals giving rise to the term K which we should expect from a strictly constant error or physical effect, and until such effects are proved to be present in the observed velocities, the doubt exists that the peculiarities observed are other than motions.

Observatorio Nacional Argentino, Córdoba
October 4, 1915

NOTE ADDED BY AUTHOR, UNDER DATE DECEMBER 6, 1915

Subsequent investigations have disclosed large and apparently systematic differences between the values of the *K* term as derived from the northern and southern stars separately. The values obtained and the corresponding values of $V_\odot$ are given below:

	$V_\odot$		K	
	North	South	North	South
	km	km	km	km
All spectral classes 2.9 and brighter	−10.8	−21.4	−5.7	+3.4
B 3.0 and fainter.	−24.8	−20.6	+9.6	+5.6
A 3.0 and fainter.	−23.0	−19.7	+3.8	+2.9
A 3.0 and fainter, omitting ellipsoid vertices.	−16.6	−22.2	0.0	+8.3
A 3.0 and fainter, omitting ellipsoid vertices and 2 large values.	−17.2	−16.0	−0.9	+5.4
F and G 3.0 and fainter, galactic.	−22.9	−14.5	+1.1	−4.3

A MORSE OPTICAL PYROMETER ADAPTED TO A WIDE RANGE OF LABORATORY USES

By W. E. FORSYTHE

The following is a description of an optical pyrometer of the Morse type (generally known as the Holborn-Kurlbaum pyrometer) that has been constructed for use in Nela Research Laboratory. It is thought to have some advantages over existing forms. A photograph of the complete instrument is shown in Fig. 1. In Fig. 2 is shown diagrammatically the arrangement of the different parts.

The objective lens *A* is an achromatic lens 3.5 cm in diameter. If the pyrometer were to be used only with a particular so-called monochromatic filter (e.g., red glass) before the eyepiece, there would be no need of having anything but simple lenses. However, it is quite often desirable to use different filters before the eyepiece, in which case quite a change in focus would be required if only simple lenses were used.

As may be seen from the diagram, the tube holding the objective lens is not a part of the tube to which the rack and pinion are attached. The reason for making it a separate tube was to increase the range of magnification. For the same reason this tube is made in two parts, there being a joint at the point marked *B*. Thus with only a comparatively short rack for focusing, adjustments from a magnification of little more than two times down to a focus for parallel light can be obtained. This is accomplished by moving the tube holding the objective lens backward or forward in the holder *C*. The tube can also be taken apart at the point *B* and only the short piece containing the objective lens used. The lens can also be reversed in this short tube and the tube reversed in the holder *C*. Thus the lens can be placed very near the diaphragm *D*. At this position the pyrometer is focused for parallel light. This feature has been found quite desirable. The pyrometer may be calibrated by comparison with a black body where large magnification cannot be used conveniently, and the pyrometer may also be used to measure the brightness of either distant objects or, by a slight change, to measure the brightness of a lamp filament which is only about 0.1 mm in diameter.

The pyrometer lamp *K* has a tungsten filament somewhat of the shape shown in Fig. 3. The pointer *P* and the small bend in the filament indicate the point of the filament under observation. The filaments used are of three sizes (0.1 mm, 0.061 mm, and 0.037 mm in diameter), but for the most part the second size is used, as with this the disappearance is satisfactory. The length of these filaments is approximately shown in Fig. 3. The (0.061 mm) filament requires about 0.46 amperes at about 3.3 volts to apparently match in brightness the black body at the temperature of melting palladium. The pyrometer lamp bulb is about 5 cm in diameter. These bulbs were selected with very great care in order to obtain bulbs as free from defects as possible.

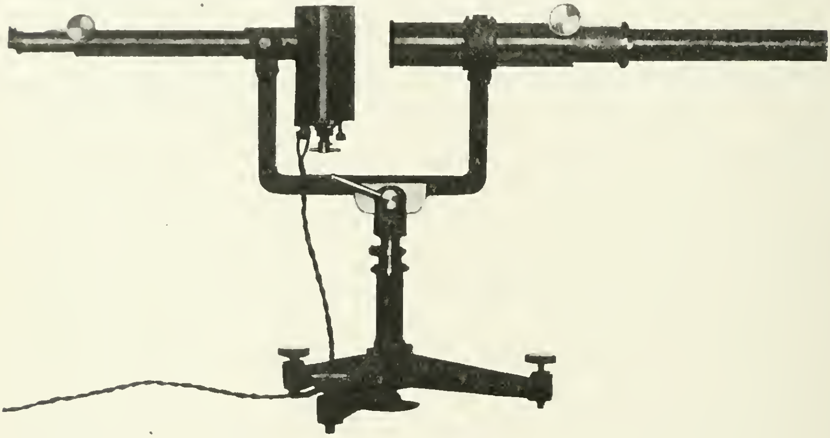

FIG. 1.—Optical Pyrometer

The pyrometer lamp is so mounted that it may be moved across the field of view by a screw motion. Thus the particular part of the filament that it is desired to use may be easily brought into the center of the field of view. In addition to this the mounting is so arranged that all other possible adjustments, such as raising and lowering, turning and tipping the bulb, can be easily obtained.

As it is sometimes very desirable to have the pyrometer filament horizontal and at other times vertical, the pyrometer lamp-holder *F* and the eyepiece telescope were made so as to rotate in the collar *G*. By this means the pyrometer filament can be set at any angle. This has been found very convenient when measuring the temperature of a lamp filament. Owing to the variation from the cosine law of emission,[1] it is desirable to have the pyrometer filament parallel to the background filament. In this case it is the brightness of the central part of the filament that is measured rather than the mean brightness across the filament. This average brightness has been shown to be about 3 per cent greater than the normal brightness.

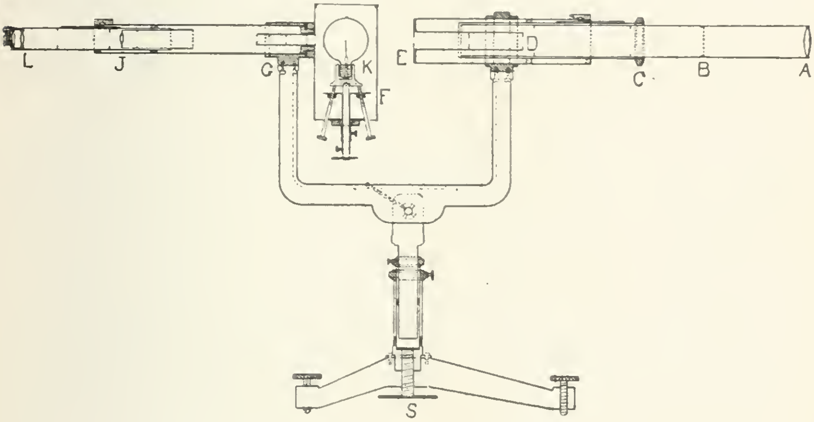

Fig. 2.—Optical Pyrometer

It is sometimes desirable to use two different pyrometer lamps in the same optical pyrometer. This is necessary when two different observers have to use the same pyrometer but desire to use different pyrometer lamps. It is also desirable to have a second pyrometer lamp to use occasionally to check the calibration of the pyrometer lamp that is generally used. To be sure, one pyrometer lamp can be removed and the other put in its place, but a much more accurate procedure is to have the holder large enough so that both lamps can be kept in the holder at all times. The holder for the pyrometer lamps can be brought exactly into place. This can be accomplished by turning the holder or by sliding it along guides. If the lamps are kept in the holder, they will be more likely to be used in the exact position in which they were calibrated, which is very important for accurate work.

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[1] *Astrophysical Journal*, 36, 345, 1912.

The pyrometer lamps with tungsten filaments have a very long life as they are used in this kind of work. They are seldom operated at a brightness above that necessary to match that of a black body at the temperature of melting palladium (1822° K). A particular lamp that has been in use for about two years, and has during the greater part of that time been used almost every day, shows an exceptionally good performance. When it was first calibrated two years ago it required 0.4573 ampere to apparently match the black body at the temperature of melting palladium. A recent calibration showed that it now required 0.4578 ampere to apparently match the black body at the same temperature. As this lamp has been operated for about 300 hours in all, it is seen to be very satisfactory. This pyrometer lamp has a 0.062 mm filament.

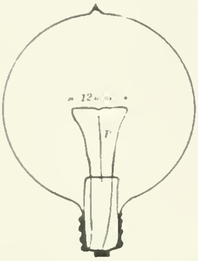

FIG. 3

As a ready means of checking the calibration of the pyrometer lamp a large filament (0.25 mm) tungsten lamp, standardized by comparison with the black body by the substitution method, is used. By using this lamp as a secondary standard the necessity of setting up and calibrating a standard black body can often be avoided. The calibration of the pyrometer lamp can be checked much more readily by means of this secondary standard than with the black body.

The opening at E was made for two purposes: (1) to allow the lamp-holder and eyepiece telescope to revolve, as was pointed out above; and (2) to permit the insertion of a rotating sector, since it has been shown[1] that if a rotating sector disk is used to reduce the intensity of the light incident upon the pyrometer lamp it should be placed as near the pyrometer lamp as possible. Though for the most part it is much better to use rotating sectors to reduce the intensity of the incident radiation, owing to the difficulty of securing satisfactory filters of neutral tint, it is sometimes necessary to use such filters. This is especially true when using light of the shorter wave-lengths, as the ratio of change of brightness for a given change in temperature is much greater for shorter wave-lengths. A holder for the absorbing glasses is located at *E*.

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[1] *Astrophysical Journal*, 42, 303, 1915.

The telescope, consisting of the objective lens *J* and the eyepiece *L*, is used to observe the pyrometer filament and the projected background. This telescope is so constructed that the magnification can be altered. Thus it is possible to use the greatest magnification allowable and still have a good disappearance of the pyrometer filament against the background. One objection that has been raised against tungsten pyrometer filaments is that they disappear against the background too easily. This is due for the most part to the optical system used to observe the pyrometer filament. With an optical system of good resolving power and large magnification it is often impossible to get an apparent disappearance of the pyrometer filament against the background. It has been pointed out[1] that in order to obtain a disappearance it is sometimes necessary to use an eyepiece with less resolving power than would otherwise be convenient. It is not to be understood that the trouble with disappearance is limited to tungsten pyrometer filaments. With carbon pyrometer filaments it is often difficult to obtain a disappearance. Tungsten and carbon pyrometer filaments of the same size behave differently, owing to their difference in deviation from Lambert's cosine law.[2]

At *D* is located the diaphragm that has been shown[3] to be necessary in order to avoid an error due to diffraction at the pyrometer filament. This error is caused by changing the solid angle of the radiation incident upon the pyrometer filament, due to changing the position of the objective lens. As the solid angle is changed, the diffraction around the filament will be changed. If, for example, this solid angle is increased, more light will be diffracted around the filament. The filament will thus require less current to apparently match in brightness a particular background. The size of this diaphragm is so chosen that it is completely filled by the cone of light from the objective lens even when the lens is moved to the farthest distance possible from the pyrometer filament.

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[1] *Physical Review* (N.S.), 4, 163, 1914.

[2] *Astrophysical Journal*, 36, 345, 1912.

[3] *Physical Review* (N.S.), 4, 163, 1914.

The real limiting diaphragm is located at *H*. The upper limit to the size of this diaphragm must be so chosen that it is smaller than the cross-section of the cone of rays limited by the diaphragm *D*. A limiting diaphragm is necessary, as the apparent brightness of any radiating body depends upon the solid angle through which the radiation passes before it enters the eye. Thus, with a limiting diaphragm, the brightness of any radiating solid, as observed with a telescope, is independent of its distance so long as the limiting diaphragm is filled. This same argument applies to the pyrometer filament, and the two should always have the same limiting cone. What really determines the size of this diaphragm is the resolving power of the eyepiece telescope. The size of this diaphragm must be so chosen that with the magnification necessary to use, disappearance of the pyrometer filament can be obtained. The foregoing diaphragms *D* and *H* were respectively 1 cm and 2½ mm. The other diaphragms, shown in the eyepiece telescope and the objective telescope, are for the purpose of lessening the stray light. The support for the pyrometer, as shown in Fig. 2, is so constructed that the instrument can be rotated, raised or lowered, or set at any angle with the horizontal. The pyrometer can also be raised a small amount by means of the screw *S*.

This pyrometer has been used in connection with the less portable forms of the Holborn-Kurlbaum optical pyrometer previously described.[1] The accuracy obtained with this pyrometer is about the same as with the larger forms.

It is interesting to note that the principle of this pyrometer (generally referred to as the Holborn-Kurlbaum pyrometer), patented by Morse in this country about 1900, was discovered and used back in 1888 in one of the lamp factories. At that time a certain furnace was operated at such a temperature that it matched the standard lamp at about 80 volts.

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[1] *Physical Review* (N.S.), 4, 163, 1914.

The author does not desire to claim originality for the above-described instrument. The pyrometer as constructed is the outgrowth of several years of use of such instruments both in this laboratory and in the University of Wisconsin. Different members of this laboratory as well as others elsewhere have given many valuable suggestions.

Nela Research Laboratory
National Lamp Works of General Electric Company
Nela Park, Cleveland, Ohio
April 1, 1916

A NOTE ON SPECTROPHOTOGRAPHY

By M. LUCKIESH

Two ideal photographic emulsions that would meet with high favor in spectrophotography are, respectively, one that is equally sensitive to radiant energy of all wave-lengths, and another that is sensitive to radiant energy of various wave-lengths in quite the same relative proportions as the average eye. It is well known that no such ideal emulsions are available. In fact, no emulsions are available which even approximately satisfy the foregoing conditions excepting for relatively short spectral ranges. It is a common procedure to resort to colored filters for the purpose of obtaining the same final results as would be obtained by the ideal emulsions. Obviously the functions of the colored filters is to suppress, in proper proportions, energy of various wave-lengths to which the emulsion is more sensitive, thus "evening up" the photographic action in the emulsion throughout the desired spectral range, or accomplishing other desired results in the same manner. It is not difficult to make colored filters which roughly compensate for the lack of uniform spectral sensibility of the emulsion, but it is usually a long and tedious process to make a filter for a given photographic emulsion which accurately accomplishes the same final result as either of the ideal emulsions mentioned above. Further complications sometimes arise in spectrophotography owing to the lack of uniform spectral distribution of energy in the illuminant used in obtaining spectrograms of transmitted and reflected light, and also owing to the effect of prismatic dispersion in spectrographs in which prisms are employed. Templates may be used instead of colored filters, and for accurate work are perhaps as easily made as colored filters.

The method described here has been employed by the writer on various occasions during the past few years with considerable success and satisfaction. The procedure is so simple that it is probable that it has been used by others, but, inasmuch as the writer is not aware that this is a fact, it appears worthy of a brief description. The method consists of making a spectrophotographic filter through which spectra are photographed. The method will be discussed first for a grating spectrograph. A photographic plate which is to be employed for spectrophotography is placed in its ordinary position in the spectrograph and the spectrum of a light-source yielding a continuous spectrum, such as the tungsten lamp, is photographed. Several spectrograms obtained by different exposures are photographed upon the plate. The latter is developed, fixed, dried, and replaced in its original position in the plate-holder. Another plate of the same kind is placed on top of the first plate and spectra are now photographed upon the second plate through the negatives on the first plate. The photographs should be made on the first plate with the sensitive side reversed from its normal position. This makes it possible to place the sensitive side of the second plate in contact with that of the first plate, thus avoiding the errors that might arise in refocusing the instrument, as would be necessary if the sensitive films in both cases were not in the same plane. This method compensates fairly well for the non-uniformity in the spectral sensibility of the emulsion

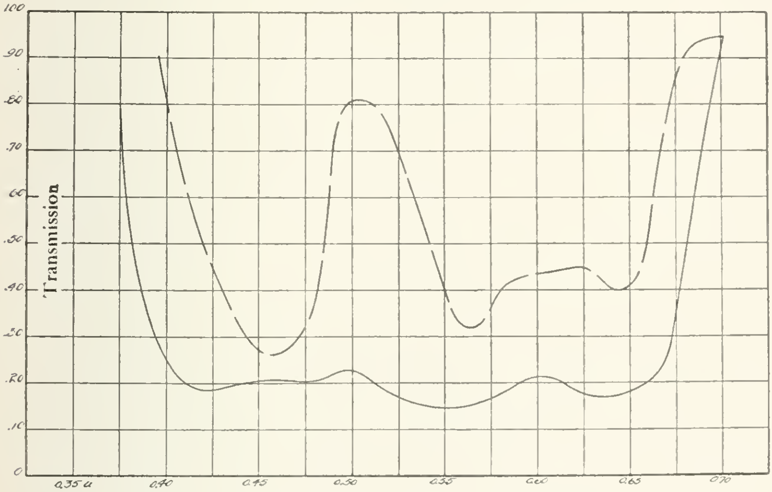

Fig. 1.—Broken line: Transmission of spectrographic filter. Full line: Transmission of a continuous spectrum through the filter, with Cramer Spectrum plate and grating spectrograph.

and for the non-uniform spectral distribution of energy in the illuminant. There is a best relation between the exposures on the first plate and the uniformity of the photographic effect on the second plate throughout the range of wave-lengths to which the plate is sensitive. After ascertaining the best density for the "spectrophotographic filter," the writer uniformly exposes the first plate completely across its width by moving the plate-holder at a uniform and proper speed across the opening. After developing, fixing, and drying, the wave-length scale is ruled upon the "filter," providing that it has not been photographed during the original exposure. The result obtained by this simple procedure closely approximates that which would be obtained by means of an ideal emulsion of uniform spectral sensibility and an illuminant of uniform spectral energy-distribution.

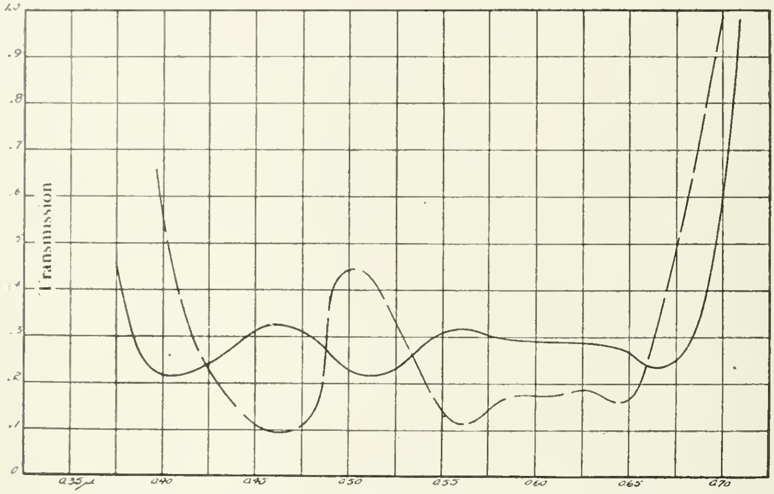

FIG. 2.—Same as Fig. 1, except that longer exposures were given. "Filter" was more dense.

To illustrate the results, actual spectrograms might be used, but a much better idea of the compensatory effects of the spectrophotographic filters is obtained by means of photometric data obtained by measuring the transmission of the "filters" and of the final spectrograms at different wave-lengths. The data might ordinarily be presented in terms of density or opacity, the former being the logarithm of the opacity and the latter the reciprocal of the transmission coefficient, but, inasmuch as the eye usually estimates transmission coefficients in judging photographic transparencies, the spectral transmission curves of the "filters" and the final spectrograms are presented.

In Fig. 1 the broken line represents the spectral transmission curve of the spectrogram of incandescent tungsten light (the lamp operating at about 20 lumens per watt) on a Cramer Spectrum

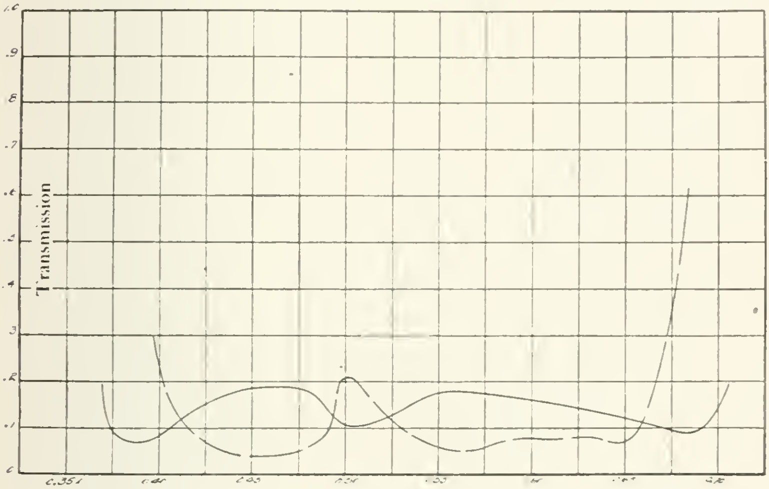

FIG. 3.—Same as Fig. 2, except that the filter was still more dense

plate. The negative was of rather light density. The full line represents the spectral transmission-curve of the spectrogram of the same illuminant taken through the first negative with the latter in contact with the second plate and in its proper position according to the wave-length scale. It is seen that throughout most of the region of sensitivity for ordinary exposures, from 0.4 μ to 0.67 μ, the photographic effect is nearly constant and therefore this represents a satisfactory correcting screen. The absorption of the glass, emulsion, and any slight fog has been eliminated in the measurement.

In Fig. 2 the broken line represents the spectral transmission curve of a spectrogram of the illuminant on a Cramer Spectrum plate, the negative in this case being more dense than in the previous case. The full line shows the effect on the spectrogram of the same illuminant taken through the first negative and indicates an overcorrection; that is, the maxima and minima of the full-line curve correspond respectively to the minima and maxima of the broken-line curve.

In Fig. 3 the two curves represent data corresponding to those shown in the previous cases, but the exposures were greater. The overcorrection is again evident.

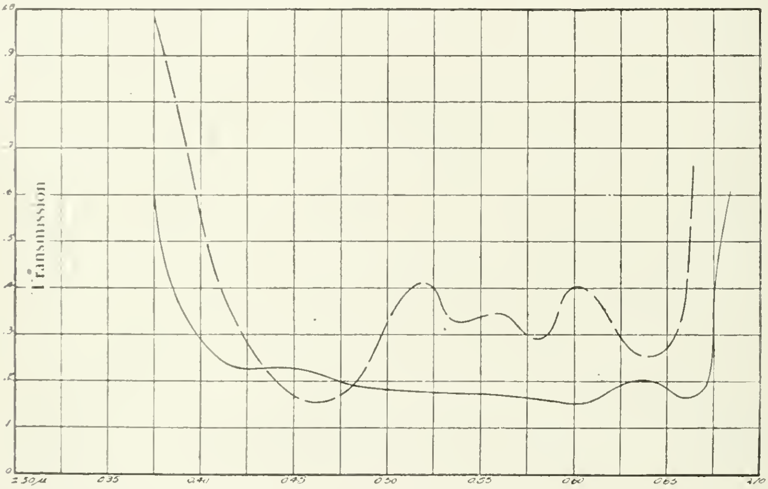

Fig. 4.—Similar results with a Wratten & Wainwright Panchromatic plate

In Fig. 4 are shown similar results with a Wratten & Wainwright Panchromatic plate. The spectrophotographic correcting-filter in this case is quite as satisfactory as that shown in Fig. 1.

It is thus seen that with a grating spectrograph the compensation for the non-uniform spectral energy-distribution in the illuminant and for the non-uniform spectral sensibility of the photographic emulsion is satisfactory. For qualitative studies this method has proved highly satisfactory by practically eliminating any misinterpretation of the spectrograms which often arises owing to the insensitive spectral regions encountered with photographic plates. For quantitative measurements the method is quite helpful because, with a proper spectrophotographic filter, all densities throughout a large spectral range fall on the straight portion of the intensity-density curves. In much work the slopes of the intensity-density curves for energy of different wave-lengths may be assumed equal.

The results obtained with a prism spectrograph are shown in the remaining illustrations. In this case the variable dispersion of the prism is often annoying, owing to the extreme diminution of energy-density in the short-wave region of the spectrum. In Fig. 5 the broken line shows the spectral transmission curve of a spectrogram of the same illuminant on a Seed 26 plate. The resulting spectrogram through this spectrophotographic filter is shown by the full line. This plate is not very sensitive to energy of longer wave-lengths than 0.50 μ, and the glass prism begins to absorb energy quite appreciably at 0.35 μ, becoming practically opaque at 0.30 μ. This is an ordinary plate and therefore its spectral range is considerably shorter than the panchromatic plates. However, it is seen that even in a prism spectrograph the compensating method which has been described is quite effective

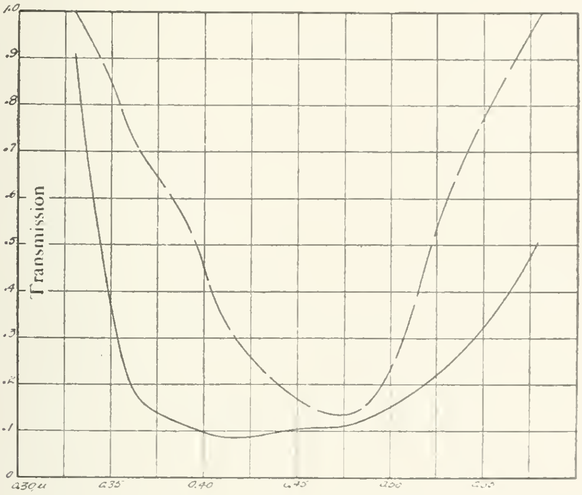

FIG. 5.—Similar results with a glass prism spectrograph and Seed 26 plates

throughout the region of chief sensibility of the photographic emulsion.

In Fig. 6 similar results are shown for greater exposures. The correction is perhaps slightly better than in the preceding case.

The method has been used successfully with various kinds of photographic plates and with prism and grating spectrographs. Promising results have been obtained with a quartz spectrograph as far as 0.30 μ, using a source giving a continuous spectrum. In a great deal of spectrophotographic work, such as the examination of glasses for protecting the eyes, the region of most importance is the near ultra-violet, 0.29 μ to 0.40 μ, which is transmitted by glass. Here a method which compensates as does the foregoing is quite desirable.

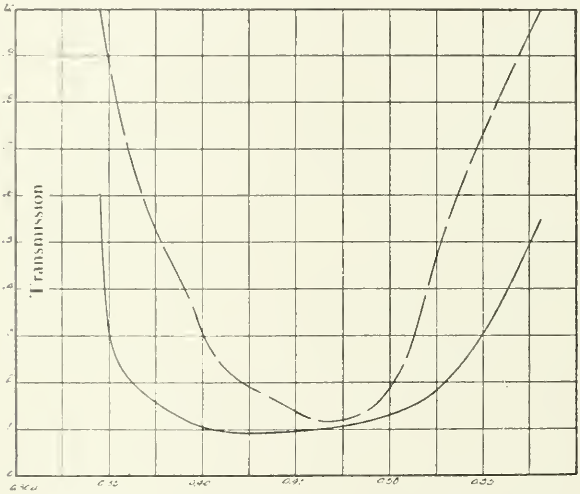

Fig. 6.—Similar to Fig. 5, with a more dense "filter"

SUMMARY

A simple method has been described for making a spectrophotographic filter to be used with a grating instrument for obtaining practically the same results as would be obtained in spectrum photography with an ideal photographic emulsion of uniform sensibility throughout its chief range of sensitiveness and with an illuminant emitting equal amounts of energy of all wave-lengths to which the emulsion is chiefly sensitive. The method is equally applicable to prism instruments, in which case the effect of prismatic dispersion is largely compensated, as well as the non-uniform sensibility of the emulsion and the non-uniform spectral distribution of energy in the illuminant.

The method has been used with both prism and grating spectrographs and the details and some results are presented.

Nela Research Laboratory
National Lamp Works of General Electric Company
Nela Park, Cleveland, Ohio
March 24, 1916

MONOCHROMATIC PHOTOGRAPHY OF JUPITER AND SATURN[1]

By R. W. WOOD

In the *Astrophysical Journal*, **36**, 75, 1912 I gave an account of the results obtained by photographing the moon through ray-filters transmitting limited regions of the spectrum. Marked differences in the distribution of light and shade were observed in the different pictures, the most conspicuous being an extensive dark deposit around the crater Aristarchus, which appeared in the picture made by means of ultra-violet light but was quite absent in the one made by the yellow rays. Laboratory experiments on the appearance of various rocks and minerals, when photographed through these same ray-filters, made it appear probable that the dark spot was due to sulphur or some sulphur-bearing rock.

It seemed quite likely that results of even greater interest would be obtained if the method of monochromatic photography were applied in the case of planets, but instrumental facilities were not at my disposal at the time.

These earlier experiments were made with a 16-inch nickeled glass mirror of 26 feet focus, mounted on the equatorial of the Princeton Observatory, which unfortunately was not adapted to the very accurate guiding necessary in work of this nature. As it appeared probable that increased definition and magnification would bring out regions of smaller area showing selective reflection, and as there was every reason to suppose that the planets would show peculiarities worthy of study, I determined to repeat and improve upon the earlier experiments. In this work I have been aided by a grant of $200 from the Gould Fund of the National Academy. The preliminary experiments were made at East Hampton, during the past summer.

The Naval Observatory placed at my disposal a very good Gaertner coelostat, which was used in conjunction with a horizontal reflecting telescope of 56 feet focal length and 16 inches aperture, figured by Mellish and nickeled by the electrolysis of a dilute solution of nickel fluoborate as described in the *Astrophysical Journal* for November 1915 (**42**, 365). The nickel deposit was necessary, since silver reflects only about 4 per cent of the light in the ultra-violet region of the spectrum utilized in the work.

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[1] *Contributions from the Mount Wilson Solar Observatory.* No. 113.

A new mirror cell was made for the coelostat, large enough to carry a 16-inch flat, which was also nickeled, and the instrument mounted on a very solid pier of brick and cement, sheltered by a small house with a hinged roof. A double-slide carrier for the plate-holder and ray-filters, with two slow motions and two eyepieces for following, was made of sheet brass, and mounted on a heavy pedestal of iron.

The photographs made with this apparatus, while distinctly better than those made with the 26-foot mirror, failed to show the very fine details which were desired. This was due partly to bad seeing and partly to the difficulty of following a rather rapid periodic oscillation or drift of the image originating in some small error or errors in the gear wheels which could not be located or remedied.

On this account it appeared best to study the behavior of the various ray-filters, and the spectrum range of the light after reflection from two or more surfaces of silver or nickel and transmission through the filters, rather than to waste time in trying to secure a more accurate drive of the coelostat. These experiments showed that it would be feasible to complete the work with a large reflecting telescope with silvered mirrors, using as an ultra-violet screen a cell filled with dense bromine vapor. Such a cell transmits the entire ultra-violet region below λ 3500, while silver reflects the region λ 3300–3500 sufficiently well to make photography possible in this region with silver-on-glass mirrors.

Photographs were made of the moon and Jupiter with infra-red, yellow, violet, and ultra-violet light, and the times of exposure, with the screen, were determined for both silvered and nickeled mirrors. As a result of this preliminary work, I was able to get satisfactory results at once with the 60-inch reflector of the Mount Wilson Observatory, which was placed at my disposal for four nights in the latter part of October.

With this instrument a very complete set of photographs was made of the moon, Jupiter, and Saturn, by light of the four regions of the spectrum referred to above. Results of very great interest were obtained in the case of the two planets, for dark belts were shown by violet and ultra-violet light of which no trace appeared visually or on the plates made by yellow light, while the infra-red pictures showed scarcely any trace of the belts ordinarily seen. By applying the methods of three-color printing to these plates we can produce a colored positive, in which the belts of selectively absorbing gases appear strongly colored; in other words, we produce an effect somewhat similar to what we should see if our eyes were sensitive to a wider spectral range.

In the earlier experiments, made several years ago, I used for the ultra-violet ray-filter a rather thick deposit of metallic silver on a thin plate of quartz or uviol glass. Silver films are, however, extremely opaque to the selectively transmitted region, which lies between λ 3000 and λ 3200, so much so that an exposure of two or three minutes is required for the full moon. With bromine vapor I found, however, that a fully exposed plate could be obtained in eight or ten seconds, and that if we use in addition a thin cell filled with a very dilute solution of potassium chromate, we obtain a transmitted region practically the same as with a silver film, but of about tenfold intensity. The silver film was accordingly abandoned in the present work and the bromine cell used exclusively in its place.

The cell was made by cutting a section from a square glass bottle and grinding the edges flat. A small hole was then drilled through the wall for the introduction of the bromine. The uviol plates (1 mm in thickness) were cemented with beeswax, which was found by Ribaud, in the course of his investigations on the absorption spectrum of bromine, to withstand the action of the vapor fairly well. A piece of asbestos paper, impregnated with a saturated solution of calcium chloride and strongly heated to drive off all of the water, was placed in the cell before the plates were cemented in position. This was necessary owing to the dampness at East Hampton, for, even when the bromine was thoroughly dried, moisture deposited on the inner surface of the uviol plates when the cell cooled off at night. After several hours there appeared also a deposit which resembled a light dew but which did not dry off when the plates were removed. It appeared to be of a greasy nature, and I attributed it to some volatile compound formed by the action of the bromine on the wax. I subsequently found it on clean glass plates which had been standing on my desk for several days, and was quite at a loss for an explanation until Dr. Mees, of the Eastman research laboratory, called my attention to the fact that beeswax sublimes quite rapidly, a circumstance which I had not suspected, as it is one of the constituents of a supposedly reliable vacuum cement. The trouble was minimized by applying the wax to the outside of the cell only, after the glass plates were placed against the ends. The wax was rolled into thin cylinders between the fingers, and melted against the seam with a hot glass rod 2 mm in diameter. Cells prepared in this way showed no trace of the deposit for fifteen or twenty hours, and, as they were cleaned every evening and freshly filled, no further trouble was experienced from this source. Four or five drops of liquid bromine were introduced with a capillary pipette, and after vaporization was complete and most of the air expelled, which required a minute or two, the hole was closed with a plug of wax, touched with a hot glass rod.

The cell was 5 cm in thickness and was clamped in position in front of the plate-holder by means of two brass set-screws. Exposure with the quartz spectrograph showed that such a cell was transparent from λ 3500 to the end of the solar spectrum (λ = 2900), while it cut off completely all of the less refrangible rays capable of affecting an ordinary photographic plate.

If a more restricted region of the spectrum is required we may add a thin cell, say a millimeter or two thick, containing a very dilute solution of chromate of potash; the yellow color should be barely perceptible by daylight. The correct dilution can be determined by trial only, the quickest way being to use sunlight filtered through the bromine cell, a quartz spectrograph, and a plate of uranium glass. If the solution is not extremely dilute, it removes all of the ultra-violet. If just right, it removes the region λ 3250–3500, the combination transmitting the same region as a thick silver film, but with a much greater intensity (about tenfold). If the chromate-of-potash cell is used, it is necessary to give three or four times the exposure required with the bromine cell alone.

The infra-red screen transmitted the region above λ 7000, the yellow screen everything above λ 5000, and the violet the region between λ 4000 and λ 4500. These screens were prepared from stained films of gelatine made by the Eastman company, and mounted in balsam between thin sheets of plate glass. They were of such a size that they could be substituted for the thick bromine cell, and were held in place by the same set-screws.

The work at Mount Wilson was commenced on October 22. The double-slide carrier with its attachments for the bromine cell was adapted to the 60-inch telescope, so that the work could be taken up without the construction of any new apparatus. The early hours of the evening were spent in making plates of the moon, which was full at the time, through the four ray-filters, while the rest of the night was given over to the planets.

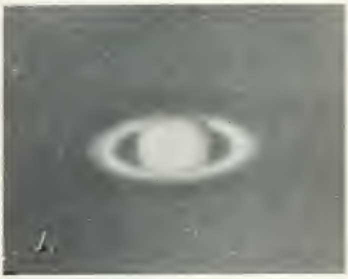
The telescope was used at the 80-foot focus, and, as the mirrors were silvered, the region of the spectrum utilized for the ultra-violet photography was slightly less refrangible than in the earlier work with the silver film. Preliminary experiments had been made at East Hampton with a quartz spectrograph, on the spectrum of sunlight after three reflections from silver and transmission through the bromine cell (the conditions which obtained with the 60-inch telescope).

The most interesting results were obtained in the case of Saturn. The picture taken through the infra-red screen showed the ball of the planet practically devoid of surface markings, there being only the merest trace of the belts ordinarily seen. Through the yellow screen the planet presented its usual visual appearance, the narrow belts showing distinctly. On the plates made with the violet ray-filter (transmission λ 4000–λ 4500) a very broad dark belt surrounded the planet's equator, occupying the region of the planet which was brightest in yellow light. In addition to this dark equatorial belt, a dark polar cap of considerable size appeared in the pictures. So different were the two pictures that, were it not for the ring, it would have been difficult to believe that they represented the same object. In ultra-violet light the appearance was much the same, but the dark belt was not quite as wide, the bright region between the polar cap and the belt being distinctly broader. Photographs made with the four monochromatic filters are reproduced in Plate IV.

Two hypotheses suggest themselves in explanation of the dark belt. We may be dealing with a fine mist or dust which forms an

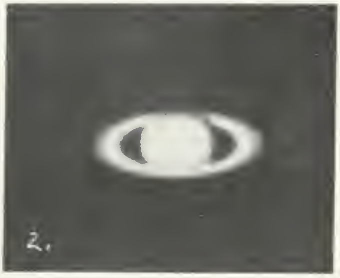
PLATE IV

1



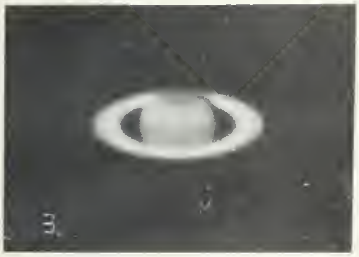
Infra-red

2



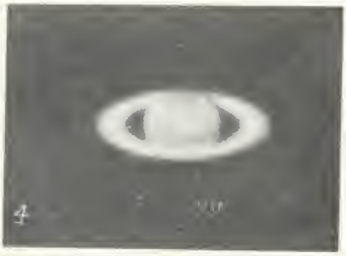
Yellow

3



Violet

4



Ultra-violet

Monochromatic Photographs of Saturn

extension of the crape ring down to the ball of the planet. This hypothesis appears to be favored by the circumstance that, on the negatives made by the violet and ultra-violet light, the sky between the ball of the planet and the ring is distinctly denser than the region just outside. This would indicate that the region inside of the ring is filled with some material which reflects the short wave-lengths to a slight degree. No trace of this darkening appears on any of the plates made with the yellow screen, even on one that was many times overexposed, which appears to show that the phenomenon is real. This peculiarity was, however, not detected until the work was finished, and I should prefer to verify it or have it verified before recording it as an established fact. The luminosity is much too feeble to show in the prints.

Measurements of the density of the plates between the ring and the ball of the planet and the sky just outside of the ring have been made for me by Mr. Harold D. Babcock, of the Mount Wilson staff, with the Hartmann photometer. His results showed greater density between the ring and ball on both plates (yellow and ultra-violet, but the density was greater in the latter). His values for the densities are as follows:

Yellow Negative		Ultra-Violet Negative	
Sky outside of ring	1.1	Sky outside of ring	1.05
Sky between ring and ball	1.17	Sky between ring and ball	1.27

One's first thought is that slight darkening of the region between the ring and ball may be the result of halation. It appears to me, however, that if this were the case we should expect the effect to be more marked with yellow light, which penetrates the film and the glass plate and is reflected back, while the ultra-violet light does not even get through the photographic film, as is shown by the circumstance that the image does not appear on the reverse side of the film with prolonged development, as is the case with ordinary light.

I do not, however, believe that the dark belt is in reality due to the absorption of a dust ring, for measurements made by Mr. Ellerman showed that the belt extended higher up on the ball of the planet than the line of intersection of the plane of the rings. Moreover, it seems highly probable that the belt and the dark polar cap are to be explained in the same way.

The second hypothesis assumes the existence in the planet's atmosphere of some substance capable of absorbing violet and ultra-violet light. This material might be a fine mist or dust, or some gas capable of absorbing the more refrangible part of the spectrum. Such a gas would be of a pale yellow color, and sulphur vapor and chlorine naturally occur to us. I have, however, examined the absorption spectrum of both of these gases in quartz bulbs, with the result that the absorption appears to be much stronger in the ultra-violet than in the violet, which is in disagreement with the circumstance that the band appears wider in violet than in ultra-violet light.

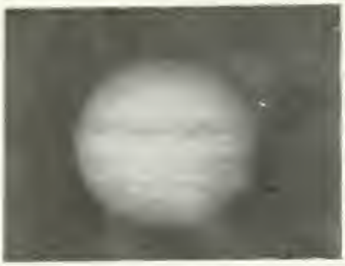
Two spectrograms of the planet have been made by Adams with the 60-inch reflector. In one, the slit of the instrument was parallel to the major axis of the ring and cut through the dark belt shown on the violet and ultra-violet photographs. In the other, the slit cut across the bright region above the belt. The spectra embraced the region between λ 5000 and λ 4000, and we should expect that between λ 4000 and λ 4500 the spectrum of the area occupied by the dark belt would be much darker (lighter in the negative) than the other. No difference could be detected, however, between the two negatives, except at the very end (λ 4000), where there was a very slight trace of the expected effect.

Of course it would be preferable to place the slit parallel to the short axis of the ellipse, and show the spectra of the two regions juxtaposed on the same plate, but the spectroscope available at the time did not permit of rotation. A quartz spectrograph is being planned for a more complete study of the question. The spectra were made in February 1916, and it is of course possible, though hardly probable, that the ultra-violet belt had in the meantime disappeared.

Another point of interest is the decrease in contrast between the inner and outer ring as the wave-length is decreased. This suggests that the outer ring contains so much finely divided matter that it shines in part by diffusion. On the infra-red photograph the ball of the planet is much brighter in comparison to the brightest part of the ring than on the violet and ultra-violet pictures. This again suggests a mist or dust in the planet's atmosphere which scatters the shorter wave-lengths. My infra-red photographs of landscapes

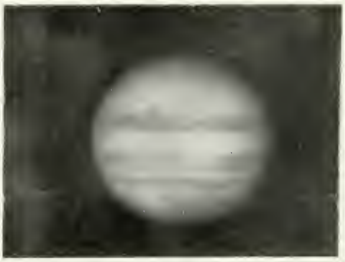
PLATE V

5



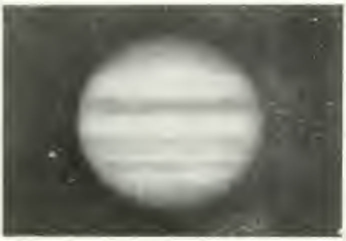
Infra-red

6



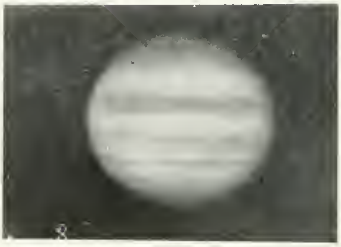
Yellow

7



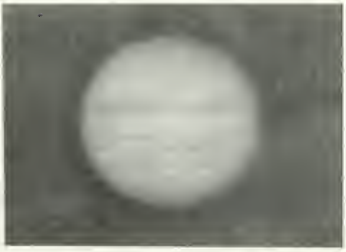
Violet

8



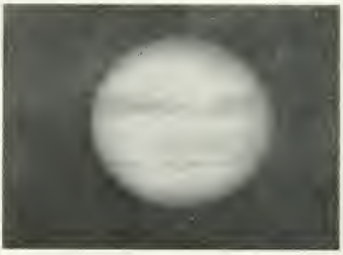
Ultra-violet

9



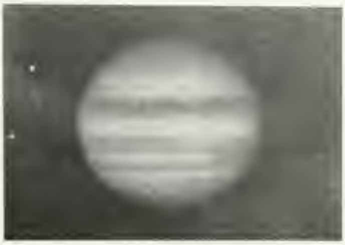
Infra-red

10



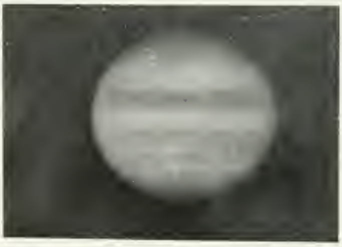
Yellow

11



Violet

12



Ultra-violet

Monochromatic Photographs of Jupiter

have clearly demonstrated that we can obtain clear photographs through a blue haze by means of the spectrum region above λ 7200. In these photographs it will be remembered that the blue sky comes out black and the grass and foliage show white. (See *Physical Optics*, 2d ed., p. 626.)

Photographs of Jupiter made with the four ray-filters showed differences almost as marked as those found in the case of Saturn, though not so striking by casual inspection. The rotation of the planet is so rapid that it was found necessary to make the series of pictures as quickly as possible, if comparable results were to be obtained. Following was accomplished by bringing one of the outer satellites on the cross-hair of the eyepiece in the movable plate-holder. The exposure times were two seconds for the violet and yellow negatives, seven seconds for the ultra-violet, and ninety seconds for the infra-red. The dark belts are scarcely visible on the infra-red plates, while the pictures made with violet light show them in the greatest contrast.

This circumstance made it appear probable that very marked color contrasts would appear if the planet were viewed through a screen which transmitted deep-red and violet light. The violet filter used in the work has this property, and through it the cloud belts appeared deep red on a violet background. Unfortunately the vivid color contrast obtained in this way is not of much help in the visual study of the planet, as the eye does not focus well for these two regions of the spectrum.

The appearance of the disk in the light of the four spectral regions specified is shown on Plate V. They were all made on the evening of October 24, 1915, between 7:30 and 9:30 Pacific time. The details are given in Table I.

The direction of rotation of the planet is from right to left. On three of the plates slight blemishes have been noticed which have nothing to do with the surface markings: a light spot on the broad dark belt near the left hand edge, Fig. 8; a small dark spot near the lower edge, Fig. 11; and a small dark spot on the broad dark belt above the bright equatorial belt.

The chief differences between the views of the planet made by light of different wave-lengths may be summarized as follows: The infra-red pictures show less darkening of the disk as we approach the limb, which is what would be expected on the theory that the darkening results from atmospheric absorption or scattering. The cloud belts moreover are very inconspicuous in these pictures. The two dark belts above the bright equatorial belt have a number of still darker spots on them which are most distinct in the yellow picture, less so in the violet, and almost invisible in the ultra-violet (Figs. 10, 11, 12). Above this dark belt, a double bright belt appears in the violet and ultra-violet pictures, which cannot be seen in the infra-red and yellow pictures, or, to speak more accurately, the dark belt between them is visible only in the violet and ultra-violet. The upper dark polar cap shades off gradually in the yellow and infra-red views, but is sharply terminated in the violet and ultra-violet, its lower edge marking the upper boundary of the double bright belt. These are the chief peculiarities. Observers who have made a long study of the surface markings of Jupiter will doubtless find other points of interest, and I shall be glad to lend the original plates to anyone who wishes to study them in greater detail.

TABLE I

Fig.	Ray-Filter	Time	Exposure	Plate
5	Infra-red	7:30	90 sec.	Spect. Panchrom.
6	Yellow	7:33	2	Cramer Iso. Inst.
7	Violet	8:15	2	Process
8	Ultra-violet	7:50	7	Process
9	Infra-red	9:20	90	Spect. Panchrom.
10	Yellow	9:23	2	Cramer Iso.
11	Violet	9:25	2	Process
12	Ultra-violet	9:30	7	Process

TRICHROMATIC PHOTOGRAPHS OF THE PLANETS

By applying the methods of trichromatic color-photography to the negatives made through the ray-filters, very beautiful and striking transparencies have been made, which represent, in strong color contrasts, the differences which can be seen by comparison of the monochromatic photographs. As is well known, if three negatives are made of an object through red, green, and blue ray-filters, and the positives made from them on bichromatized gelatine are stained with blue, magenta, and yellow dyes, these positives when superposed in register yield a fairly faithful reproduction of the object in its natural colors.

If we apply this same method to pictures made by yellow, violet, and ultra-violet light, we shall obtain a colored picture of the object as it would appear if the region of the eye's sensitiveness were shifted slightly toward the ultra-violet, or, to put the matter in astrophysical language, as the object would appear if it were receding from us with a velocity sufficient to cause ultra-violet to appear violet! Color-photographs of Saturn made in this way show the ball of the planet yellowish in color with a broad orange-red band (the violet and ultra-violet dark belt) surrounding the equator and a dark-red polar cap. The narrow belts, which can be seen visually and which appear on the negative made with yellow light, come out blue in the picture, the lower one being in coincidence with the upper edge of the dark belt. Owing to the fact that the dark belt is wider on the violet negative than on the ultra-violet one, the upper edge lacks yellow in the finished picture, and the superposed narrow blue belt appears purple in consequence (magenta+blue).

Color-contrasts also appear in the ring, as a result of the circumstance that the outer or fainter ring is relatively more luminous in ultra-violet light. The inner ring appears white, and the outer bluish green. Similar colored belts are shown in the pictures made of Jupiter. The photographs as a whole give us a vivid impression that we are really dealing with great belts of selectively absorbing, and hence colored, vapors or gases. They show us moreover that the absence of strong color-contrasts in the surface markings of the planets results merely from the limited range of the spectrum to which the eye is sensitive.

In view of the interesting results obtained with these two planets, it is my hope that similar observations will be made on Mars, on the occasion of its next near approach to the earth. I made one photograph of the planet with ultra-violet light in October, with the 60-inch reflector, but the disk was too small to show much of interest. Mr. Hoge was of very great assistance in helping me with the manipulation of the telescope.

JOHNS HOPKINS UNIVERSITY
February 1916

ORBIT OF THE SPECTROSCOPIC BINARY B.D. 78°412

By OLIVER J. LEE

The variable radial velocity of this star was announced in the *Astronomische Nachrichten*, **194**, 415, 1913. Its position is $\alpha = 12^h8^m$; $\delta = 78^\circ 10'$. In the *Harvard Revision* its visual magnitude is given as 5.12 and its type as A 5. The spectrum contains numerous good lines, and on fully exposed plates these give reliable velocities. Several causes account for the fact that most of the plates in this series are slightly or appreciably underexposed. The photographic faintness of the star, 5.3 magnitude, the rapid changes in radial velocity and the desire to record these changes in as short times of exposure as possible, the northerly position of the star, keeping it always well above the horizon, with the accompanying tendency to follow the star into lower altitudes than would usually be the case—all of these contribute to the frequent large plate residuals shown in the last column of the summary of observations (Table II). The 60 plates discussed below were all taken with the one-prism arrangement of the Bruce spectrograph. The lines given in Table I were used to obtain the velocities.

TABLE I

Element	Wave-Length	Element	Wave-Length	Element	Wave-Length
Fe.........	4045.975	Ti.........	4395.201	Fe........	4515.508
Fe.........	4063.759	Fe........	4404.927	Ti+.......	4549.767
Fe.........	4071.908	Fe........	4415.293	Ti.........	4572.156
Sr.........	4077.885	Mg........	4481.400	Fe........	4584.018
H	4101.900	Ti.........	4501.445	H.........	4861.527
H	4340.634	Fe........	4508.455		

The hydrogen lines were weighted considerably lower than the others, for which the weights will average nearly equal. In the column headed "Observer," B = Barrett; F = Frost; L = Lee; M = S. A. Mitchell; My = C. A. Maney. Mr. F. R. Sullivan assisted as usual in securing the plates.

TABLE II

Summary of Observations

Plate No.	Observer	G.M.T.	Phase	No. of Lines	Wt.	Velocity	Residual O.−C.
		1913				Km	Km
IB 3252........	L	Jan. 21.837	0.632	12	16	−55.6	+ 8.4
3270........	L, M	Feb. 3.802	0.887	11	18	−13.9	+ 6.0
3279........	B, L	5.800	0.343	12	14	+ 1.0	+ 8.4
3318........	L	Mar. 17.667	0.809	6	7	−31.3	+ 9.6
3329........	L	31.605	0.766	10	14	−55.2	− 5.7
3337........	L	April 7.672	0.207	12	15	+39.1	+ 5.6
3341........	L	15.638	0.547	11	13	−72.6	−15.6
3350........	L	16.890	0.528	11	15	−61.0	− 6.5
3352........	L	17.826	0.193	9	14	+46.4	+ 9.3
3356........	B	18.854	1.221	17	21	+69.9	+ 7.9
3358........	L	21.576	0.130	7	6	+39.0	−11.7
3363........	B	25.710	0.451	10	14	−42.4	− 3.8
3366........	L	28.577	0.776	11	10	−61.8	−13.4
3370........	L, B	28.813	1.012	13	16	+13.7	− 4.8
		1914					
3644........	L	Feb. 4.693	0.730	14	20	−64.3	− 7.6
3646........	L, F	4.815	0.852	9	15	−27.7	+ 2.0
3649........	F	4.994	1.031	13	21	+23.0	− 0.9
3652........	L	13.522	0.662	12	21	−64.7	− 1.8
3654........	L	13.650	0.790	12	17	−64.8	−19.4
3656........	L	13.775	0.915	13	12	− 9.9	+ 2.1
3659........	B	18.808	0.864	8	9	−17.3	+ 9.1
3660........	B	18.984	1.040	15	20	+26.2	− 0.2
3664........	L	20.660	0.174	12	18	+47.1	+ 5.6
3665........	L	20.696	0.210	14	17	+23.7	− 9.0
3666........	L	20.731	0.245	13	16	+21.7	− 1.3
3667........	L	20.768	0.282	14	18	+ 6.8	− 3.8
3670........	F	25.590	0.020	17	23	+70.0	+ 6.8
3671........	F	25.635	0.065	14	17	+56.9	− 3.3
3672........	F	25.681	0.111	12	16	+45.7	− 8.5
3674........	B	25.831	0.261	13	18	+18.2	+ 0.4
3675........	B	25.888	0.318	13	21	+ 4.3	+ 3.6
3677........	L, F	27.771	0.930	11	14	− 9.4	− 1.8
3709........	L	Mar. 20.917	0.469	5	5	−41.8	+ 0.7
3793........	F	Aug. 3.841	0.396	4	5	− 7.3	+15.7
3794........	F	3.889	0.444	3	4	−16.1	−20.6
3807........	F	14.603	0.990	6	6	+ 9.6	− 2.2
3808........	F	14.634	1.021	8	6	+16.5	− 4.9
3809........	F	14.672	1.059	7	5	+25.3	− 6.4
3810........	F	14.719	1.106	7	7	+43.1	− 0.5
		1915					
4116........	My	April 16.776	0.860	7	10	−27.7	0.0
4117........	B	16.820	0.904	9	12	− 9.9	+ 5.0
4118........	B	16.868	0.952	8	13	+ 1.5	+ 2.2
4119........	B	16.919	1.003	8	7	+ 5.0	−10.3
4128........	My	27.580	0.225	8	14	+31.1	+ 2.4
4129........	My	27.650	0.295	6	7	+ 0.7	− 6.0
4130........	My	27.711	0.356	9	12	−16.2	− 4.8
4131........	My	27.767	0.412	7	9	−27.0	+ 0.4
4132........	B	27.822	0.467	8	10	−38.2	+ 3.8
4159........	B	31.605	1.204	7	10	+68.4	+ 7.4

TABLE II—*Continued*

Plate No.	Observer	G.M.T.	Phase	No. of Lines	Wt.	Velocity	Residual O.−C.
		1913				Km	Km
4100	B	31.654	1.253	8	12	+65.0	+ 1.6
4101	B	31.707	0.035	10	16	+62.0	+ 0.8
4102	B	31.760	0.088	7	9	+54.3	− 3.4
4105	My	June 7.750	0.723	5	7	−41.6	+15.8
4106	My	7.803	0.776	6	10	−36.8	+11.6
4107	My	7.857	0.830	7	9	−32.2	+ 3.4
4185	My	July 5.621	0.632	9	11	−67.9	− 4.0
4186	My	5.677	0.688	7	8	−50.4	+10.6
4187	My	5.731	0.742	9	11	−57.6	− 2.9
4188	My	5.787	0.798	7	12	−39.0	+ 4.6
4189	My	5.842	0.853	7	9	−25.1	+ 4.4

The observations cover a time of 895 days corresponding to 705 periods. The period of 1.27100 days is considered definitive, and its error can hardly exceed ±2 units in the last place. With this period the observations were plotted and grouped into normal points with regard to phase, after the times of observation had been corrected for the light-equation. The smooth curve drawn to represent these points promised an appreciable eccentricity, and elliptical elements were derived by use of the graphical method of Lehmann Filhés. It was seen that a sine curve suited the observa-

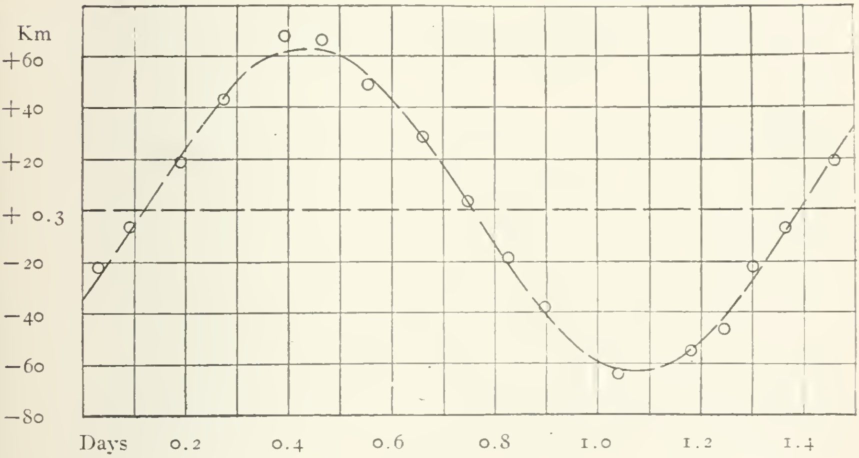
TABLE III

NORMAL PLACES

No.	Phase	Limits of Phase	Velocity	Residual O.−C.	Wt.
	Days		Km	Km	
1	0.027	$1^{d}25$–$0^{d}06$	+66.8	+3.8	0.4
2	0.114	0.06–0.19	+49.6	−4.1	.7
3	0.223	0.19–0.27	+29.3	+0.5	.9
4	0.309	0.27–0.35	+ 3.8	+0.7	.6
5	0.388	0.35–0.43	−18.2	+3.0	.3
6	0.458	0.43–0.52	−37.8	+2.3	.3
7	0.600	0.52–0.69	−63.9	−2.0	.8
8	0.743	0.69–0.78	−54.7	−0.5	.8
9	0.807	0.78–0.83	−46.2	−4.7	.4
10	0.863	0.83–0.89	−21.7	+5.3	.6
11	0.925	0.89–0.96	− 6.8	+1.8	.5
12	1.022	0.96–1.07	+19.1	−2.1	.8
13	1.106	1.10	+43.1	−0.5	.1
14	1.226	1.18–1.25	+68.2	+6.3	0.4

tions even better than the curve from these elements. An ephemeris for the normal points was computed from the following elements of a circular orbit and the differences from the observed values have been adjusted by a least-squares solution:

$$\begin{aligned} P &= 1.27100 \text{ days} \\ K &= 65.5 \text{ km} \\ \gamma &= +1.5 \text{ km} \\ T &= \text{J.D. } 2420685.266 \end{aligned}$$



Velocity-Curve of B.D. 78°412

The normal equations

$$\begin{array}{rrrl} 7.6\,x & -0.225\,y & -0.173\,z & = -9.02 \\ & 3.439 & +0.838 & = -8.01 \\ & & 4.174 & = -3.05 \end{array}$$

in which $x = \delta\gamma$, $y = \delta K$, $z = K\mu\delta T$ gives the following values of the corrections:

$$\begin{aligned} \delta\gamma &= -1.2 \text{ km} \\ \delta K &= -2.3 \text{ km} \\ \delta T &= -0.001 \text{ day} \end{aligned}$$

The final elements are:

$$P = 1.27100 \text{ days} \pm 0.00002 \text{ day}$$
$$\mu = 283^\circ.24$$
$$e = 0.00$$
$$K = 63.2 \text{ km} \pm 1.44 \text{ km}$$
$$\gamma = +0.3 \text{ km} \pm 0.94 \text{ km}$$
$$T = \text{J.D. } 2420685.265 \pm 0.0005$$
$$a \sin i = 1{,}104{,}000 \text{ km}$$

T is taken as a time of maximum velocity of recession.

The phases given in column 4 of Table II are referred to T as defined above. The quantities O. − C. in the last column of the same table are scaled off with the velocity-curve from the final elements. From them the probable error of a plate is ±5.2 km. Although this quantity is large and the probable error of a normal point is ±1.9 km, the orbit may be considered definitive.

Mr. Julius Lemkowitz, computer at this observatory, has kindly checked the least-squares solution.

Yerkes Observatory
May 4, 1916

THE
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AND ASTRONOMICAL PHYSICS

VOLUME XLIII JUNE 1916 NUMBER 5

ON AN APPARATUS AND METHOD FOR THERMO-ELECTRIC MEASUREMENTS IN PHOTOGRAPHIC PHOTOMETRY. II

APPLICATION TO VARIABLE STARS

By HARLAN TRUE STETSON

In the application of the thermo-electric method of measurement to the study of the light-curves of variable stars we are concerned chiefly with relative measurements, and difficulties incident to the determination of the "absolute scale" are of less importance in this field. Since in a given series of measurements the comparison stars and variable will be compared image for image, the uncertain correction to plate center will be largely eliminated. Again, we should expect the larger errors due to progressive change in atmospheric transparency to be minimized, since presumably variable and comparison stars would be similarly affected image for image.

The eclipsing variable U Cephei was selected for measurement by the new method for several reasons: the general character of the light-curve is well known, elements have been determined, and the period is short enough to enable a large portion of the curve to be obtained from a single plate.

Variable-star observers will recall that the light-curve of U Cephei has been determined by numerous investigators with somewhat conflicting results, indicating the possible existence of peculiarities yet to be determined. As early as 1884 observations by Wilsing[1] and also by Knott[2] indicated that the minima observed in the spring of the year appeared lower than those deduced from autumnal observations by two or three tenths of a magnitude. The period of the star being nearly $2\frac{1}{2}$ days (more exactly $2^{d}.493$), it is obvious that in lower latitudes two successive minima may never be seen from the same station. A given observer must content himself with observing *alternate* minima. Should we regard the period as double the foregoing value, the autumnal minima would then occupy parts of the light-curve opposite in phase to those corresponding to the minima observable in the spring of the year. In 1889 Chandler[3] published a light-curve showing the same peculiarity, indicating also a more rapid rate of increase than of decrease, corroborating the results of earlier observations by Schmidt. In Chandler's curve the lowest point is reached first, and a short inflection follows near the median of the curve. The reality of the seasonal variation, as well as that of the minor fluctuations, is regarded by Chandler as open to question. Bohlin,[4] working at Upsala from observations made in 1896, secured no foundation for any seasonal discrepancies, but obtained a light-curve with a slight elevation of about one-tenth of a magnitude at mid-minimum.

From an extended study of a large number of observations, Yendell[5] in 1903 came to the conclusion that all such irregularities were illusory and attributable to physiological causes, the seasonal variation depending upon the orientation of the observing field. He accordingly believed the light-curve to be symmetrical, with a constant minimum of two hours' duration due to the occurrence of an annular eclipse. That Yendell's statement, however, may not be regarded as final is evidenced by the fact that Bemporad,[6] of the Osservatorio di Capodimonte, in 1914 published light-curves for both the June and December minima indicating a difference of

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[1] *Astronomische Nachrichten*, **109**, 48, 1884. [2] *Ibid.*, p. 59.

[3] *Astronomical Journal*, **9**, 49, 1889.

[4] *Astronomische Nachrichten*, **157**, 293, 1902.

[5] *Astronomical Journal*, **23**, 213, 1903.

[6] *Astronomische Nachrichten*, **199**, 217, 1914.

$0^{\mathrm{M}}.25$ in the depth of the curve. The June curve indicates also a secondary fluctuation during the minimum phase, and the December curve suggests some similar peculiarity.

Still more recently Shapley,[1] in the discussion of a light-curve of U Cephei based upon Wendell's extensive observations, recognizes a non-symmetrical tendency of the primary minimum and attributes the downward slant of the bottom of the light-curve to the irregular luminosity of the dark companion. He adds also that the close proximity of the components suggests a prolateness which might well show its effect upon the light-curve, although Wendell's observations at maximum gave no definite indications of such a feature.

With such discussions in mind, any new method of approach to the problem becomes of interest, especially if it affords opportunity for measurements free from subjective errors. If any secondary fluctuation or other departures from the main curve do exist, it would seem that these could best be detected by a careful study of a single series of observations, since the very attempt to combine observations of different epochs and reducing to a mean curve would largely if not wholly mask any disturbance not commensurable with the principal period of the star. The advantage of photography in such a study is at once apparent, since it not only eliminates physiological difficulties, but makes possible a much larger number of independent observations during a given minimum than would be obtainable otherwise. With the thermoelectric photometer as a purely physical means for the measurement of the star-images, it would seem that any contributions through this means would be of special interest as a source of additional information of the behavior of the light-curve of this star.

Plates taken for measurement by the method of diameters were found in the collection of the Yerkes Observatory, and a light-curve was published by Parkhurst in 1906.[2] Plate R 71, taken on June 25, 1905, with the 2-foot reflector, was used as the basis of the results then published. The same plate was the one first selected for measurement by the present method. Comparison stars *f* (B.D. 81°30) and *g* (B.D. 81°27) were used. The exposures were made at approximately ten-minute intervals, the plate-carrier being moved by two revolutions of the screw after each exposure. Owing to the close proximity of the two series of images on the original plate, a second and somewhat enlarged negative was made from the original and used with the thermopile.

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[1] *Contributions from the Princeton University Observatory*, No. 3, p. 35, 1915.

[2] *Astrophysical Journal*, **23**, 79, 1906.

The use of enlarged second negatives in this connection has the advantage not only of separating images in close proximity, but of increasing the contrast between star and plate ground, which may be reduced to clear glass if desired. This in itself greatly increases the precision with which thermopile readings may be made, since it affords much larger values of the galvanometer deflections and, with a fine-grained film, makes possible a more uniform background. On the other hand, it is doubtful if the enlarging process does not amplify errors in the original plate as well as introduce new ones in each reproduction.

TABLE II

PLATE R 71. JUNE 25, 1905. U CEPHEI

Comparison Star B.D. 81°30, Magnitude 8.04
Comparison Star B.D. 81°27, Magnitude 8.73 $\beta = 2.80$

Image	Mag.	Time	Image	Mag.	Time
		G.M.T.			G.M.T.
1..........	7.43	$15^{\mathrm{h}}12^{\mathrm{m}}.0\pm$	11..........	9.15	$18^{\mathrm{h}}50^{\mathrm{m}}.0\pm$
2..........	7.88	43.4	12..........	9.12	19 19.6
3..........	7.98	16 00.9	13..........	8.90	45.6
4..........	8.07	20.5	14..........	8.55	20 04.4
5..........	8.27	37.6	15..........	8.53	16.3
6..........	8.72	59.8	16..........	8.23	28.2
7..........	9.14	17 20.8	17..........	8.33	36.0
8..........	9.30	39.7	18..........	8.21	44.0
9..........	9.20	18 02.7	19..........	8.07	50.0
10..........	9.07	19.2			

However, it is interesting to note that the resulting light-curve (Fig. 14) bears a striking similarity to that of Chandler to which reference has already been made. The times and magnitudes are given in Table II. The magnitudes of the comparison stars were taken as previously published, all points on the variable curve being referred to B.D. 81°30, of adopted magnitude 8.04.

Measurements were made on a number of other plates taken under widely differing circumstances, and indicated large residuals from the theoretical curve at minimum which could hardly be accounted for on grounds of errors in measurement. Repeated measuring gave the same results for a given plate with a surprising degree of accuracy.

Through the courtesy of Professor E. C. Pickering of the Harvard College Observatory a number of plates, taken with the "variable-star apparatus" attached to the 11-inch Draper telescope, were secured. By means of this attachment the variable was automatically photographed with a minute's exposure at short intervals, the plate being displaced mechanically a fraction of a millimeter after each exposure. In this manner a long series of images may be impressed upon a 5×8-inch ($12\frac{1}{2}\times 20$ cm) plate, covering a considerable portion of the minimum phase of the eclipse. The short exposure gave very small gray images of the variable when at minimum ($9^{\mathrm{M}}2$) which were difficult to measure. The photometer, however, was modified to meet the requirements, a

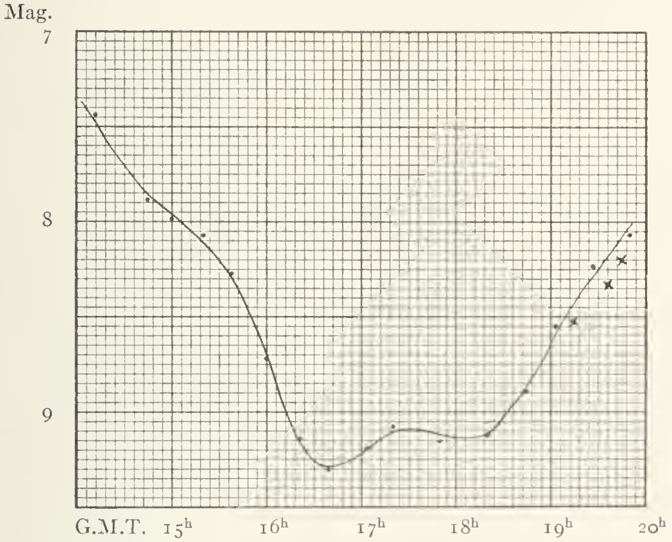

FIG. 14.—U Cephei, minimum of June 25, 1905. Plate R 71
x = points of half weight based on one comparison star

more efficient projection lens was substituted, a finding microscope was added above the stage, the galvanometer was removed to 2 m from the scale and read by a telescope magnifying 20 diameters. These improvements resulted in increasing the sensitiveness of the whole apparatus about five times and gave very good and consistent results.

Plates C 11686 and C 11687 were taken during the minimum of December 29, 1898. From among the images available, those best suited for measurement were selected. Care was taken to compare the measurement of each star-image with the film background in its own immediate vicinity, this precaution being observed to eliminate as far as possible any errors that might otherwise be introduced because of a changing transparency in the somewhat sky-fogged film. For the measurement of a single image not less than six settings were involved, the order of procedure being to set first on the image and then on the plate, alternating till the set of three pairs was secured. If the residuals appeared large for certain images, additional readings were sometimes taken in the order noted. The plate readings served also as a check on the voltage at the lamp terminals, and at the same time, when taken as indicated, eliminated any error introduced by the gradual decline, with consumption of current, of the potential of the storage battery.

As space would not permit the publication of the individual measures, a summary of the resulting data is given in Table III. The first column gives the number of the image on the plate, the second and third columns give the resulting $\Delta^{\frac{1}{4}}$ for the variable and comparison star, respectively, and the fifth column contains the differences of the second and third columns multiplied by the plate constant β, and therefore represents the differences of magnitude between the variable and comparison star. The value of β was determined from measures of stars B.D. 81°30 and B.D. 81°27, as was the case in the reduction of R 71. Columns 5 and 6 give the co-ordinates for plotting the light-curve, assuming the magnitude of B.D. 81°30 as 8.04 as before.

The graph resulting from these measures is shown in Fig. 15, and indicates a secondary fluctuation of about one-tenth of a magnitude, with an interval of 40 minutes between these apparent undulations. It is hard to account for this appearance on the grounds of systematic errors in measurement. The variable and comparison star were measured independently, and on different occasions, and it appears hardly likely that this curve is the delineation of accidental errors. Such a result might seem attributable to peculiarities in the plate or to atmospheric disturbances. Unfortunately, the overlapping of several rows of images on the plates restricted the use of suitable images other than those of comparison star B.D. 81°30.

TABLE III

Plates C 11686 and C 11687. Date of Plates, December 29, 1898

Comparison Star B.D. 81°30

Plate	Image	$\Delta^{\frac{1}{4}}$		Diff.	Δ Mag.	Mag.	G.M.T.
		U	*f*				
C 11686	1	0.750	1.054	0.304	1.220	9.260	$10^h59^m23^s.9$
	11	0.790	1.144	.354	1.436	9.476	11 9 23.9
	21	0.830	1.191	.361	1.465	9.505	19 23.9
	31	0.790	1.122	.332	1.345	9.385	29 23.9
	41	0.781	1.005	.324	1.315	9.355	39 23.9
	52	0.790	1.136	.346	1.404	9.444	50 23.9
	62	0.801	1.162	.361	1.465	9.505	12 0 23.9
	72	0.845	1.174	.329	1.338	9.378	10 23.9
	83	0.837	1.163	.326	1.322	9.362	21 23.9
	92	0.815	1.150	.335	1.360	9.400	30 23.9
	102	0.740	1.122	.382	1.551	9.591	40 23.9
	112	0.781	1.140	.359	1.457	9.497	50 23.9
	122	0.760	1.118	.358	1.453	9.493	13 00 23.9
C 11687	2	0.833	1.116	.283	1.145	9.188	9 23.8
	12	0.919	1.166	.247	1.003	9.043	19 23.8
	22	0.946	1.170	.224	0.909	8.949	29 23.8
	32	0.906	1.034	.128	0.520	8.560	39 23.8
	42	0.995	1.077	.082	0.333	8.373	49 23.8
	52	1.070	1.122	.052	0.211	8.251	59 23.8
	62	1.056	1.068	.012	0.049	8.089	14 9 23.8
	72	1.105	1.097	− .008	−0.032	8.008	19 23.8
	84	1.132	1.083	− .049	−0.199	7.841	21 23.8
	93	1.144	1.090	− .054	−0.219	7.721	30 23.8
	101	1.164	1.118	− .046	−0.187	7.853	48 23.8
	112	1.125	1.105	−0.020	−0.081	7.959	59 23.8

A set of three other plates was selected, Nos. C 9188, 9189, 9190, covering a portion of the decline, minimum, and rise of U Cephei under the date of June 13, 1896. The quality of the plates was better than that of the previous pair, and it was possible to measure images at more frequent intervals. Two good comparison stars, B.D. 81°30 and 81°29, were used throughout.

The results of the measures are given in Table IV. Columns 1 and 2 give the number of the image on the plate, and the corresponding Greenwich mean time of the exposure. Columns 3 and 4 give the resulting magnitude of the variable as determined from the two comparison stars *f* and *h* respectively. The fifth column gives the direct difference of column 3 minus column 4 and forms the basis for suspected variability between *f* and *h*. The differences of magnitude have all been referred to B.D. 81°30 as before. The magnitudes of B.D. 81°30 and B.D. 81°27 were adopted as for Plate R 71, and the magnitude of B.D. 81°29 was determined from the measures of the plates.

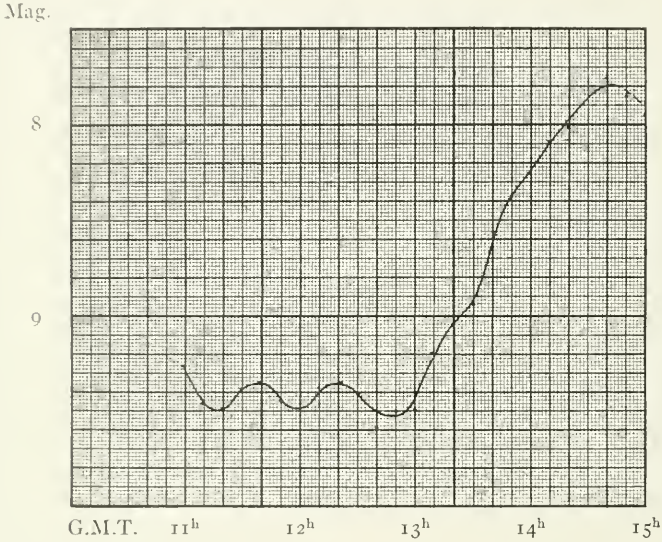

FIG. 15.—U Cephei, minimum of December 29, 1898. Plates C 11686 and C 11687.

The curve platted from these measures and represented in Fig. 16 was anything but what was expected, and indicated sources of difficulty not anticipated. Determinations of probable error pointed to variations of hundredths, whereas the curve advertised discrepancies in the tenths' place. Remeasures of offending points gave almost identical results with but slight variations in the last place, indicating that the source of trouble was not in the apparatus

TABLE IV

U CEPHEI, MINIMUM FOR JUNE 13, 1896. J.D. 2413693

Comparison Stars Used: *f*, B.D. 81°30; *g*, B.D. 81°29; *h*, B.D. 81°27

Adopted Magnitudes: *f*, 8.04; *g*, 8.73; *h*, 8.79

SUMMARY PLATE 9188

Image	G.M.T.	Mag. *f*,	Mag. *h*.	Diff. Mag. *f*– Mag. *h*	Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*.– Mag. *h*.
2.....	$14^h26^m29^s$	7.666	7.658	0.008	60.....	$15^h24^m29^s$	8.150	8.216	−0.066
4.....	28 29	7.704	7.678	.026	62.....	26 29	8.112	8.048	.064
8.....	32 29	7.670	7.764	.096	65.....	29 29	8.170	8.332	− .162
10.....	34 29	7.718	7.768	− .050	68.....	32 29	8.284	8.166	.118
12.....	36 29	7.766	7.708	.058	70.....	34 29	8.310	8.344	− .034
15.....	39 29	7.790	7.802	− .012	72.....	36 29	8.272	8.258	.014
18.....	42 29	7.826	7.752	.074	75.....	39 29	8.286	8 418	− .132
20.....	44 29	7.852	7.862	− .010	78.....	42 29	8.430	8.488	− .058
22.....	46 29	7.896	7.932	− .036	80.....	44 29	8.468	8.530	− .062
25.....	49 29	7.874	7.850	.024	82.....	46 29	8.348	8 518	− .170
28.....	52 29	7.862	7.894	− .032	85.....	49 29	8.474	8.762	− .288
30.....	54 29	7.800	7.824	− .024	88.....	52 29	8.392	8.356	.036
32.....	56 29	7.956	7.924	.032	90.....	54 29	8.474	8.374	.100
35.....	59 29	7.898	8.004	.094	92.....	56 29	8.678	8.680	− .002
38.....	15 02 29	7.970	7.902	.068	95.....	59 29	8.494	8.540	− .046
40.....	04 29	8.138	7.802	.336	98.....	16 02 29	8.688	8.800	− .112
42.....	06 29	8.060	8.032	.028	100.....	04 29	8.560	8.634	− .074
45.....	09 29	8.130	7.988	.142	102.....	06 29	8.788	8.884	− .096
48.....	12 29	8.064	8.036	.028	105.....	09 29	8.824	8.868	− .044
50.....	14 29	8.086	7.970	.116	107.....	11 29	8.968	9.014	− .046
52.....	16 29	8.064	7.918	.146	109.....	13 29	8.818	9.052	−0.234
55.....	19 29	8.064	8.040	0.024					

TABLE IV—*Continued*

SUMMARY PLATE 9189

Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*– Mag. *h*	Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*.– Mag. *h*.
5.....	$16^h32^m29^s$	9.374	9.482	−0.108	58.....	$17^h25^m29^s$	9.916	9.876	.040
7.....	34 29	9.408	9.464	− .056	61.....	28 29	9.658	9.662	− .004
10.....	37 29	9.296	9.416	− .120	65.....	32 29	9.580	9.506	.074
12.....	39 29	9.282	9.464	− .182	68.....	35 29	9.772	9.766	.006
15.....	42 29	....	....	....	71.....	38 29	9.764	9.768	−0.004
17.....	44 29	....	....	....	75.....	42 29	9.484	9 418	0.066
19.....	46 29	9.312	9.406	− .094	78.....	45 29	9.538	9.448	.090
23.....	50 29	9.388	9.400	− .012	81.....	48 29	9.618	9.508	.110
25.....	52 29	9.406	9.342	.064	85.....	52 29	9.488	9.348	.140
28.....	55 29	9.420	9.476	− .056	88.....	55 29	9.466	9.348	.128
31.....	58 29	....	....	....	91.....	58 29	9 710	9.530	.180
35.....	17 02 29	9.402	9.538	− .136	95.....	18 02 29	9.630	9.428	.202
41.....	08 29	9.532	9.476	.056	98.....	05 20	9.550	9.458	.092
45.....	12 29	9.344	9.346	− .002	100.....	07 29	......	......	....
48.....	15 29	9.264	9.262	.002	105.....	12 29	9.460	9 388	.072
51.....	18 29	9.320	9.418	− .098	107.....	14 29	9.554	9.438	0.126
55.....	22 29	9.588	9.748	− .160					

TABLE IV—*Concluded*

SUMMARY PLATE 9190

Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*– Mag. *h*.	Image	G.M.T.	Mag. *f*.	Mag. *h*.	Diff. Mag. *f*.– Mag. *h*.
4.....	$18^h28^m29^s$	9.776	9.826	−0.050	58.....	$19^h22^m29^s$	8.582	8.704	−0.122
6.....	30 29	9.482	9.426	+ .056	61.....	25 29	8.416	8.446	−0.030
8.....	32 29	9.506	9.522	− .016	63.....	27 29	8.438	8.428	+ .010
11.....	35 29	9.338	9.440	− .102	66.....	29 29	8.432	8.400	+ .032
13.....	37 29	9.388	9.444	− .056	68.....	32 29	8.538	8.536	+ .002
15.....	39 29	9.404	9.356	+ .048	71.....	35 29	8.350	8.370	− .020
18.....	42 29	9.254	9.380	− .126	74.....	38 29	....	....	....
21.....	45 29	9.290	9.338	− .048	77.....	41 29	8.328	8.228	+ .100
23.....	47 29	9.528	9.556	− .028	81.....	45 29	8.302	8.142	+ .160
25.....	49 29	9.330	9.274	+ .056	83.....	47 29	8.264	8.216	+ .048
28.....	52 29	9.222	9.322	− .100	85.....	49 29	8.174	8.036	+ .138
31.....	55 29	9.122	9.160	− .038	88.....	52 29	8.140	8.058	+ .082
33.....	57 29	9.182	9.318	− .136	91.....	55 29	8.138	8.052	+ .086
35.....	59 29	8.778	8.868	− .090	93.....	57 29	8.196	8.084	+ .112
38.....	19 02 29	8.918	9.010	− .092	95.....	59 29	8.094	8.022	+ .072
41.....	05 29	8.742	8.770	− .028	98.....	20 02 29	8.052	7.988	+ .064
43.....	07 29	9.070	9.136	− .066	101.....	05 29	8.090	8.038	+ .052
45.....	09 29	....	8.848	....	103.....	07 29	7.984	8.040	− .056
48.....	12 29	8.712	8.856	− .144	105.....	09 29	8.038	7.828	+ .210
51.....	15 29	8.608	8.690	− .082	108.....	12 29	7.932	7.884	+ .048
53.....	17 29	8.764	8.724	+ .040	110.....	14 29	7.920	7.824	+0.096
55.....	19 29	8.876	8.748	+ .128					

nor in the method of measurement. The discrepancies were therefore photographic, to be interpreted as irregularities in the exposure due to rapid changes of atmospheric transparency, as an uneven condition of sensitiveness in the plate film, or, as seemed hardly likely, due to actual changes of light among the stars in question.

The singular feature is that for the most part the general character of the light-curve is the same for either comparison star, indicating that the chief cause of the discrepancy is in the images of U Cephei. If the large irregularities are due to atmospheric changes, it is difficult to see why the variable alone should be affected, while the two comparison stars fail to show a corresponding disturbance. The most outstanding difficulty is the behavior of the curve during minimum, around 17^h20^m, where, within the short space of half an hour, both comparison stars indicate the variable to experience a change of nearly half a magnitude in light. It has seemed wiser in this preliminary graph to leave the points connected with straight lines rather than to bias judgment by any attempt at drawing a mean curve until more complete data are at hand. The combination of several such curves at minimum would doubtless "smooth out" many irregularities, but the argument perhaps needs renewed emphasis that the combination of phenomena at separate minima only tends to mask any singularities not a definite function of the 2.49 days.

Another fact at once apparent from Fig. 16 is that the curves drawn from the data furnished by the two comparison stars cross at stated intervals. The curve belonging to B.D. 81°30, for example, clearly lies above that drawn for B.D. 81°29, from 16^h to 17^h, whereas from 17^h30^m to 18^h30^m the reverse is true. A third crossing of the two curves occurs at about 19^h30^m. This behavior suggests a variability in one or the other of the comparison stars, and becomes more apparent when we examine the two comparison stars platted against each other as in Fig. 17. Here, in spite of the large residuals from a mean curve, the rise and fall of the curve at an approximate interval of an hour and three-quarters is at once evident. Sufficient measures of a third star, B.D. 81°26, were made to indicate that the eighth-magnitude star B.D. 81°30 was the cause of the discrepancy. We must therefore either try to account for the rise and fall in Fig. 17 on the ground of a progressive undulating change in atmospheric transparency continuing

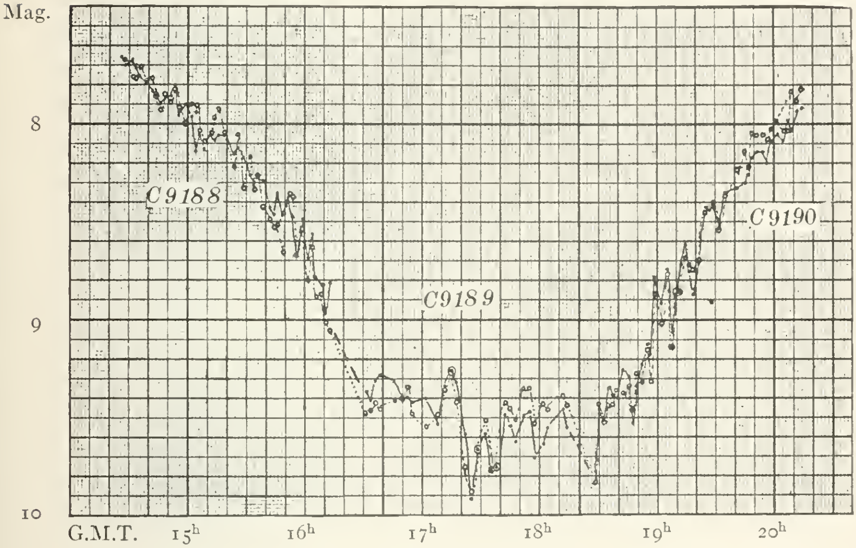

Fig. 16.—U Cephei, minimum of June 13, 1896
● = comparison star *f*; ○ = comparison star *h*

throughout the entire exposure, but not appreciably affecting the adjacent star, or else suspect the comparison star B.D. 81°30 of variability. If this should be the case, it might well be a contributing factor in the cause of the various forms of the light-curve of U Cephei previously published, when this star has been the principal comparison star used.[1]

Whatever may be gleaned from these preliminary results, it is hard to see how a straight flat-bottom curve, as demanded on the grounds of an eclipse theory alone, can be reconciled with the observations. If the binary system be held to account for the main period of the star,[2] it may seem necessary to suppose other vital factors operative in the light-curve of the star. In any event, enough has been exhibited already to indicate that with every determination complexities arise which forbid an easy verification of anything more than a mean light-curve of general characteristics. With the continuance of the method of investigation here outlined it is hoped that the completion of further measures in all parts of the light-curve may at length afford sufficient data for the formation of more conclusive ideas regarding the behavior of the light-curve of this most interesting object.

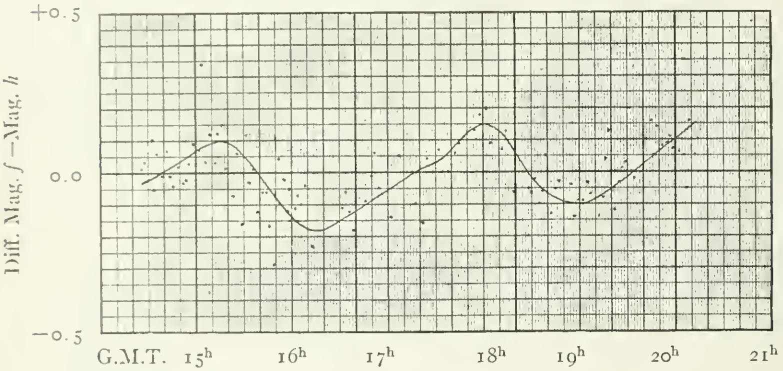

FIG. 17—Curve showing suspected variation in B.D. 81°30

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[1] Compare with the results of Capodimonte, to which reference has already been made (*Astronomische Nachrichten*, **199**, 217, 1914).

[2] For a discussion of irregularities in the main period, see Yendell's article (*loc. cit.*). Further investigation of this point is being undertaken by Shapley.

In order to determine the order of precision which could be expected in the measurements of the Harvard plates, the probable error was computed from a number of measures of the same star-image, of U Cephei near minimum, and found to be $\pm 0^{\mathrm{M}}.009$ for twenty settings or corresponding to about $\pm 0^{\mathrm{M}}.025$ for a set of readings as ordinarily taken for each star. Results from measuring diameters with a micrometer microscope and reducing to magnitudes by means of the square-root formula yielded a probable error nearly five times as great. This large value resulted from the fact that, the plate being unguided, the images were somewhat enlarged, and the variable was so faint at the minimum that the images then taken were still gray and showed no well-defined periphery. The use of the thermopile promises well for the measurement of star-images not suitable for reduction by the diameter method. With longer exposures on the variable at minimum and special care in the development of the plate it should be possible to reduce the probable error to a still lower figure. Experiments with the ferrous-oxalate developer show a great advantage to be gained by its use for photometric purposes, especially for plates to be measured thermo-electrically, inasmuch as good contrast results without the slightest possibility of chemically fogging the unexposed portion of the film.

Sources of error in the film itself are without doubt a cause of many discrepancies, and further refinement must be looked for in adopting a *modus operandi* which will reduce the effect of such residual errors to a minimum.

CONCLUSION

In the development of the present method of investigation, the first consideration has been to adapt a piece of apparatus to the measurement of stellar magnitudes from photographic plates which should eliminate so far as possible some of the difficulties incident to other methods. In so doing care has been taken to consider the errors involved and to reduce them to a minimum. The degree of precision attainable by means of the thermo-electric measurements has been tested under widely different circumstances and enough numerical results tabulated to indicate what additional contributions are to be expected from the use of such an instrument in stellar photometry.

The several lines of special investigation suggested by the results of these preliminary determinations indicate a fruitful field for extended study in many directions. It has seemed wise, therefore, in the present treatment to outline in considerable detail the essential steps in the development of the instrument and methods of reduction, with a preliminary application to several different problems, rather than to devote an early investigation in a new field to a more exhaustive treatment of a single topic.

The precise determination of magnitudes by means of the thermo-electric method of measurement has already been discussed. The difficulties mentioned are not new to the present method; rather, they are those which have long occupied the attention of investigators in stellar photometry. It is to be hoped that the measurement of plates in the manner herein described, by reducing another element of uncertainty, may facilitate more rapid progress in the field of photometry.

As has been shown, the thermopile may be used for the direct measurement of plate opacities, covering the same field as that of the Hartmann microphotometer with about the same degree of accuracy. The points of difference between the two instruments when so used may again be mentioned. The reductions of the Hartmann readings depend upon the calibration of the photographic wedge used in comparison, whereas with the thermopile arrangement the deflections of the galvanometer may be taken at once as proportional to opacities on the plate. The thermopile further has the advantages of restricting a much smaller area for examination than the microphotometer, and of making the comparison entirely independent of physiological effects or personal equation.

In measuring opacities of extended areas such as are found on solar, lunar, or nebular photographs, these advantages may be in part offset by the uncertainty in the assumption of constant absorption of glass and gelatine throughout the plate. Such a source of error will not prove troublesome whenever adjacent settings on the comparatively uniform unexposed portion of the film can be made.

The ease with which the apparatus may be adapted to the service of spectrophotometry in the measurement of objective-prism plates affords opportunity for an important study of the relation of color-index to spectral type, and may afford further means for a more precise definition of "magnitude."

In a similar manner the application of such spectral measures to the determination of the radial velocities of faint stars photographed with the objective prism is a promising field for investigation. In such a case it should be possible to locate with considerable accuracy the position of the absorption band of neodymium used as a comparison spectrum. The method becomes equally applicable to line-of-sight measures for stars having broad-line spectra.

A line of work most promising for the thermo-electric photometer is that of the photographic study of short-period variables. In the case of fields where close comparison stars are available and a long series of exposures are made on the same plate, the large and uncertain errors of atmospheric transparency, plate constants, and reduction to a standard scale are reduced to a minimum and the accuracy of the measures appears to be limited only by the quality and uniformity of the plate. Combining the advantages of photography with refinement of measurement, the thermopile so used may yet be able to accomplish for the fainter stars what the photo-electric cell and the thermopile in a more direct way are promising for the stars of brighter magnitudes.[1]

In conclusion, the writer wishes to express his appreciation to both astronomers and physicists who in any way have aided in the progress of this investigation. He especially wishes to acknowledge his indebtedness to Professor Fox, of the Dearborn Observatory, Northwestern University, for the provision and loan of important pieces of apparatus; to Professors Michelson and Millikan, of the University of Chicago, for so kindly placing at his disposal the resources of the Ryerson Physical Laboratory; to Professor E. C. Pickering, for the loan of valuable plates from the Harvard College Observatory, and for many important suggestions; to Professor Frost, for the use of the facilities of the Yerkes

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[1] See W. W. Coblentz, "Radiometric Measurements of 110 Stars with the Crossley Reflector," *Lick Observatory Bulletin*, **8**, 104, 1914.

Observatory, and for his kindly interest in the progress of the work; and to Professor Parkhurst, for the use of certain plates taken by him, and for that constant appreciation and material encouragement which have done much to further the application of laboratory methods to the problems of astronomical photometry.

SUMMARY

The results of the foregoing paper may be briefly summarized as follows:

1. An instrument has been devised, using a delicate thermopile as a receiver, whereby the magnitudes of stars may be determined from either focal or extrafocal plates by purely physical means.

2. A reduction formula has been adopted which shows a fourth-root relation between galvanometer deflections and stellar magnitudes, and this relation has been checked by the measurements of magnitudes from plates taken under various conditions, and with four different telescopes ranging from six to forty inches in aperture.

3. Provision has been made for the adaptation of the apparatus to the measurement of spectral intensities, and the method illustrated in the measurement of objective-prism spectra of B-type stars for purposes of spectral photometry.

4. An extensive study of the eclipsing variable U Cephei has been begun, the preliminary measures here presented indicating the presence of light-changes not explained on the eclipse theory alone, and revealing a suspected variability in the much-used comparison star B.D. 81°30.

5. From the measurements of the variable-star plates a somewhat higher degree of accuracy is to be expected from thermoelectric measures than from measures of diameters, the thermopile promising distinct advantage in the case of images too ill-defined to be suitable for the micrometer microscope. The principal sources of error appear to be the changing atmospheric conditions during the exposure at the telescope and irregularities in the photographic plate.

YERKES OBSERVATORY
November 1915

THE PRODUCTION IN THE ELECTRIC FURNACE OF THE BANDED SPECTRA ASCRIBED TO TITANIUM OXIDE, MAGNESIUM HYDRIDE, AND CALCIUM HYDRIDE[1]

By ARTHUR S. KING

The purpose of this investigation was to produce the banded spectra of titanium oxide, magnesium hydride, and calcium hydride in the tube-resistance furnace, to compare the conditions required for their appearance with those observed in the arc and spark, and to note any phenomena resulting from the special control of conditions possible with the furnace.

All of these banded spectra have been identified in the spectra of sunspots, and the bands attributed to titanium oxide also occur in the spectra of third-type, or Antarian, stars, which fact has led Fowler to designate them as the "Antarian bands." The presence of such banded spectra is taken as evidence of a relatively low temperature, while the appearance of the titanium oxide bands furnishes direct evidence of the presence of oxygen in third-type stars and in the sun.

THE TITANIUM OXIDE BANDS

An extended investigation by Fowler[2] furnished concordant evidence that the Antarian bands in the arc and spark result from the presence of titanium oxide. The flutings occur in a series of groups, all degraded toward the red, throughout most of the visible spectrum. Fowler gives the following wave-lengths for the first head in each group:

4353.68
4462.34
4584.62
4761.08
4954.78
5167.00
5448.48
5597.92
5760.15
5954.66
6158.86
6357.9
6651.5
7054.5

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[1] *Contributions from the Mt. Wilson Solar Observatory*, No. 114.

[2] *Proceedings of the Royal Society*, A, **79**, 509, 1907.

It was noted by the writer,[1] during an examination of the line spectrum of titanium in the furnace at different temperatures, that these bands did not appear with the furnace *in vacuo*. Experiments were recently undertaken to see whether the use of oxygen would bring them out. The photographs were made with a 15-foot concave grating spectrograph, this being set for the first observations to cover the region from λ 5500 to λ 7200 in the first order, with a dispersion of 1 mm = 3.7 A. The furnace was usually operated at about 2300° C., which gives a strong titanium spectrum without excessive vaporization of carbon.

Admitting air to one-half atmospheric pressure gave no bands. They again failed to appear when the furnace was full of air, the outlet valve being left open. It seemed probable that a strong current of oxygen was needed, since, there being no circulation through the horizontal tube, the incandescent carbon probably used up the small supply of oxygen before the titanium was fully vaporized. The windows in the ends of the furnace chamber were therefore removed, a silica tube inserted in one end of the furnace tube, and a strong blast of air forced through by means of a bellows. The whole series of bands then came out with great brilliancy. Temperatures of 2300° and 2600° C. were used with no perceptible difference except in exposure time. A noteworthy effect was the almost complete suppression of the line spectrum of titanium when a strong current of air was used, though the lines of several impurities appeared as usual. Evidently a sufficient supply of oxygen resulted in turning all of the titanium vapor into the oxide, which seems to give only the band spectrum.

Pure oxygen in the furnace was next used, this being led directly into the tube from a tank of compressed gas. The spectrum could be changed at will from a mixture of lines and bands to a pure band spectrum by increasing the current of oxygen. When the bands were at their strongest, the image on the slit was a greenish yellow, similar to the envelope of a long carbon arc containing titanium, which is one of the best of the usual sources for obtaining the band spectrum.

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[1] *Mt. Wilson Contr.*, No. 76; *Astrophysical Journal*, **39**, 139, 1914.

There was no evidence of a material change in temperature caused by the introduction of oxygen. The same exposure time with and without oxygen gave the same general intensity of spectrum, this being composed of bands in the one case and lines in the other. Pyrometer measurements of the interior of the tube were practically the same in the two cases. Allowing for differences in clearness of the vapor, the change of temperature could scarcely have been greater than 50°.

Comparing this source with the usual flames employed in producing spectra, we have here the metallic vapor heated by the graphite tube instead of by a union of flame gases, and then the process of oxidation taking place. The results are probably essentially the same as in a flame of the same temperature, but the furnace with oxygen gives a method of obtaining a series of flame temperatures for which different combinations of gases are usually required. There is also the possibility, by the use of very high furnace temperatures combined with a stream of oxygen, of exceeding any of the flame temperatures regularly attainable.

It may be noted that a distinct difference in gradation of intensity with the wave-length appeared for the titanium band spectrum in the furnace as compared with the arc, the bands at the red end being relatively stronger in the furnace.

A supplementary set of photographs was made for the region as far as λ 4200 with the furnace in vacuum, in air at atmospheric pressure, and with varying amounts of oxygen. The results obtained for the yellow and red region were repeated. By the use of a strong current of oxygen, so that a flame extended several centimeters from the end of the furnace tube, a strong band spectrum appeared, while the titanium line spectrum was eliminated except for the faint appearance of a few of the lines strongest in the regular furnace. It was noteworthy, however, that the more prominent lines of iron and chromium, as well as λ 4227 of calcium and λ 4607 of strontium, appeared as strongly as could be expected from the slight impurities present. The evidence thus seems fairly conclusive that the bands of titanium oxide have been correctly ascribed to an actual compound, since otherwise, without a large reduction of temperature, there would seem to be no reason for the disappearance of the line spectrum.

Reproductions are given in Plate VI of the titanium spectrum from λ 4427 to λ 5065 in which the four strips represent respectively the arc in air; the furnace with a moderate stream of oxygen, both lines and bands appearing; the furnace with a strong stream of oxygen, the spectrum consisting almost wholly of bands; and the furnace in air without extra supply of oxygen, in which case the bands are absent. The wave-lengths are those given by Fowler for the principal heads.

THE BANDS OF MAGNESIUM IN HYDROGEN

In a paper giving detailed measurements of the fluted spectrum obtained when magnesium is burned in an atmosphere of hydrogen, Fowler[1] summarizes previous investigations as to the dependence of the spectrum on hydrogen and as to the probability of its corresponding to a chemical compound. The bands consist of three main groups, beginning at λλ 4844.9, 5211.1, 5621.6 and shading toward the violet. They appear best with the arc or spark in an atmosphere of hydrogen, but may often be seen when only a small residue of hydrogen is present. The furnace results are in close agreement with those for the arc, which is a fact of some significance, since the arc discharge is greatly modified by a hydrogen atmosphere, and this might account for the appearance of another spectrum.

The strongest of the bands, with its head at λ 5211, was situated most favorably for observation in the furnace photographs, having a portion of its length undisturbed by the carbon spectrum. The first experiments were made by simply admitting hydrogen to the furnace chamber. Photographs at 1900°, 2200°, and 2500° C. were made for each of three pressure conditions, viz., with a pressure below 1 cm of mercury, the residual gas being air, with hydrogen at a pressure of 10 cm, and with hydrogen at atmospheric pressure. The results were somewhat variable, the main effect being that a high temperature was unfavorable for the bands, which, if they appeared at 2500°, were usually in absorption. The band at

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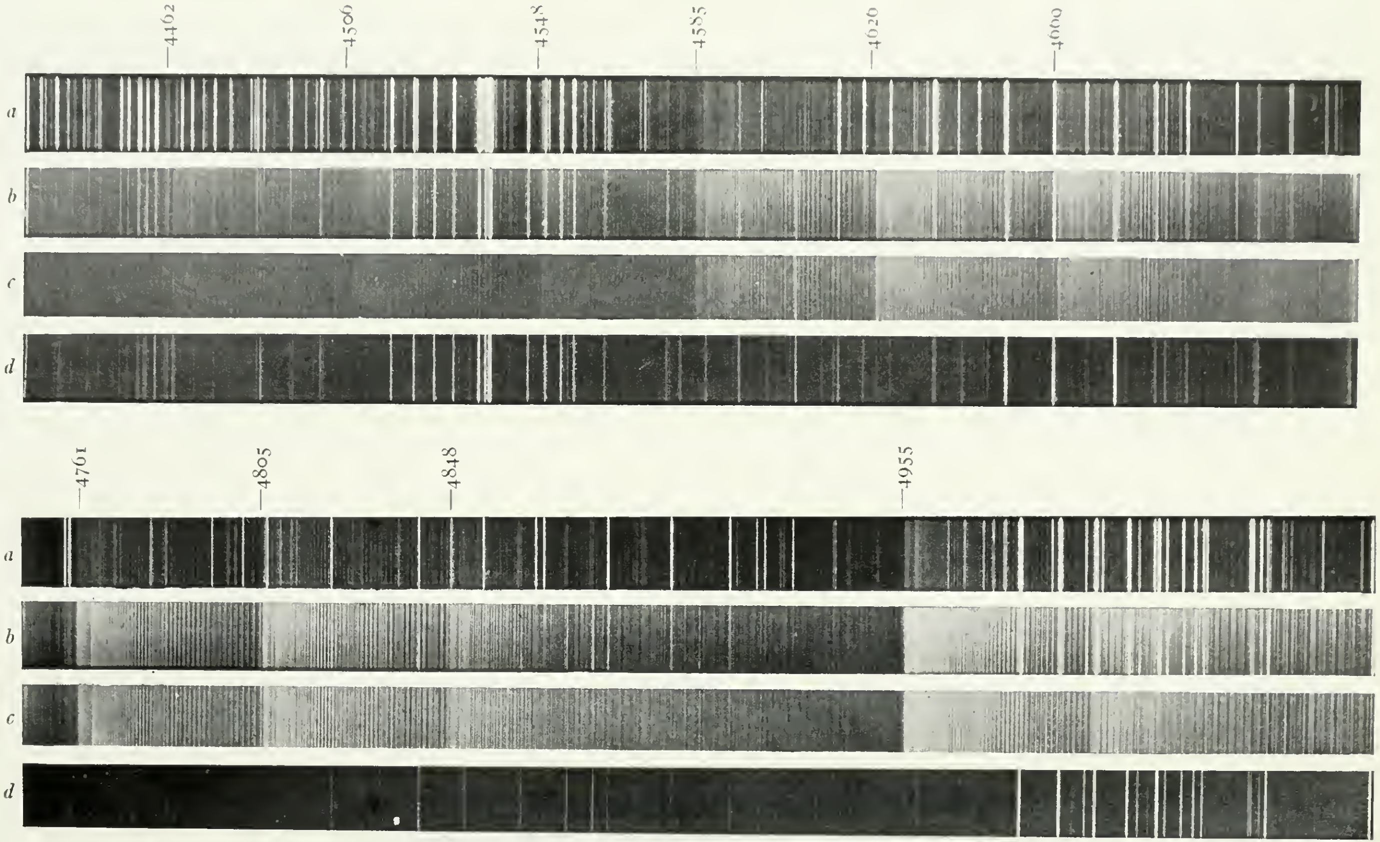
[1] *Philosophical Transactions of the Royal Society*. A, **209**, 447, 1909.

PLATE VI



Banded Spectra of Titanium Oxide

a. Spectrum of the titanium arc in air.
b. Spectrum of titanium in the electric furnace with a moderate current of oxygen through the tube.
c. Same with a strong current of oxygen.

λ 5211 occasionally appeared distinctly with the furnace pumped out. The experiments with the tube-arc have shown, however, that considerable hydrogen is occluded by the graphite tube and probably by the metallic magnesium, since hydrogen lines appear strongly in the tube-arc under these conditions.

It appeared possible that the effect of hydrogen had not been fairly tested, since a certain degree of combination with the carbon would occur, probably with the formation of acetylene. Arrangements were therefore made for leading a stream of hydrogen directly into the furnace tube by means of a silica tube passing through a rubber stopper in one end of the furnace chamber. The hydrogen was generated chemically from zinc and sulphuric acid and dried by concentrated sulphuric acid and sodium hydroxide. Passing the gas through the tube in this way, the pressure being held at 10–20 cm, proved very effectual in producing the bands. They were obtained in emission at 2500°, but much better at 2200°, when the band at λ 5211 was stronger than the carbon head at λ 5165. The line spectrum of magnesium appeared with the bands, though the *b* group and λ 4571 are the only lines given strongly by the furnace in this region.

The experiments seemed to indicate a clear dependence of the bands on the presence of hydrogen, without any apparent change in the action of the source.

THE BANDS OF CALCIUM IN HYDROGEN

The calcium arc burning in hydrogen was observed by Olmsted[1] to show a banded spectrum in the red with well-marked heads at λλ 6382.2 and 6389.3 and a fainter group in the B region, in both cases shaded toward the violet. The furnace was supplied with a stream of hydrogen through the tube, as in the experiments with magnesium, and the calcium bands were found to be very similar in behavior to those of magnesium. They frequently appear, as was noted by the writer,[2] with the furnace pumped out, apparently from occluded hydrogen. A current of hydrogen through the tube brings them out strongly at temperatures from 1900° to 2200°.

[1] *Mt. Wilson Contr.*, No. 21; *Astrophysical Journal*, **27**, 66, 1908.

[2] *Mt. Wilson Contr.*, No. 35; *Astrophysical Journal*, **29**, 190, 1909.

Higher temperatures are not so favorable, though the bands have been obtained at 2500°.

The component lines of the band to the red of λ 6700 can be measured to advantage when furnace photographs are made with higher dispersion. The denser portion shows distinct maxima at approximately λ 6902 and λ 6919, while a more scattered structure extends farther to the red. The calcium line spectrum appeared regularly with the bands.

SUMMARY

1. The bands ascribed to titanium oxide show a clear dependence on the presence of oxygen in the furnace, the spectrum consisting of bands alone when a sufficient amount of oxygen is supplied.
2. The bands attributed to magnesium hydride and to calcium hydride require in each case the presence of hydrogen in the furnace to give the bands strongly, although they frequently appear with moderate strength when only a small quantity of hydrogen is present.
3. While the bands appear through a considerable range of furnace temperatures, about 2300° C. seems to be the upper limit for their greatest strength.

Mt. Wilson Solar Observatory
April 5, 1916

RESEARCHES ON SOLAR VORTICES[1]

BY CARL STÖRMER[2]

I. INTRODUCTION

Professor Hale, as is well known, has done fundamental work on sun-spots in discovering the Zeeman effect in their spectra, thus proving the existence of very strong magnetic fields in the spots and in the surrounding whirl revealed by the spectroheliograms. In an attempt to employ mathematical methods in the investigation of these phenomena, I spent some months of the summer of 1912 at the Mount Wilson Solar Observatory.

The detailed study of the collection of spectroheliograms made at the Observatory suggested to me an application of the classical researches on terrestrial cyclones made in 1876 by my countrymen, Guldberg and Mohn.[3] The hydrogen flocculi seemed to be arranged around a single spot in curved paths that were very similar to logarithmic spirals; and just the same curves were found by Guldberg and Mohn as trajectories of the air particles in the outer part of cyclones. It therefore seemed advisable to start with the hypothesis that the motion of charged electric gas molecules takes place along such spirals around the sun-spot center, and compute the resulting magnetic field.

This investigation which I made in Pasadena showed that the lines of magnetic force due to a plane whirl of the kind mentioned above are space-curves whose projection on the plane of the whirl are also logarithmic spirals intersecting the first at right angles.

This result led me to the idea first advanced by Brester[4] in 1909, that the hydrogen flocculi resemble terrestrial auroras, thus implying that the visible whirls are not real current-lines, but lines of magnetic force due to a magnetic field at a lower level—a view expressed by Deslandres[1] in 1910. Professor Hale had previously stated in his papers[2] that the Zeeman effect was probably due to a low-level vortex. In accordance with the mathematical result mentioned above, the current-lines of this hypothetical whirl at a low level would be logarithmic spirals tending more and more to become circles according as the structure of the hydrogen whirl tended to be radial. This would much better explain the strength of the Zeeman effect of sun-spots of radial structure.

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[1] *Contributions from the Mount Wilson Solar Observatory*, No. 109.

[2] Research associate of the Carnegie Institution of Washington, Mount Wilson Solar Observatory.

[3] "Etudes sur les mouvements de l'atmosphère, Première partie"; *Programme de l'Université pour le 2re semestre*, 1876, Christiania, 1876; and *Meteorologische Zeitschrift*, XII. Band, 1877, No. 14.

[4] "The Solar Vortices of Hale," *Proceedings of the Amsterdam Academy of Sciences*, January 30, 1909.

In the following pages is given a detailed account of my researches on the mathematical theory of solar vortices.

II. DEFINITION OF THE WHIRL

Let us first consider a logarithmic spiral in the XY-plane of a system of rectangular co-ordinates in space. If we use polar co-ordinates, R and ϕ, in that plane (see Fig. 1), the equation of the spiral is

$$R=Ae^{\phi \tan \omega}$$

where A and ω are constants. A only fixes the position of the spiral; for if we introduce

$$\phi=-\cot \omega \; ln \, A$$

we get

$$A=e^{-\phi_0 \tan \omega}$$

and consequently

$$R=e^{(\phi-\phi_0) \tan \omega},$$

which is the spiral

$$R=e^{\phi \tan \omega},$$

turned through the angle ϕ_0 around the Z-axis.

The other constant, ω, determines the shape of the spiral. In order to get all possible values of tan ω between $-\infty$ and $+\infty$, we can suppose ω to be within the interval $-\pi/2$ to $+\pi/2$.

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[1] *Comptes Rendus*, January 10, 1910. See also G. Hale, "Notes on Solar Magnetic Fields and Related Phenomena," *Publications of the Astronomical Society of the Pacific*, **22**, 71, 1910.

[2] *Annual Report of the Solar Observatory of the Carnegie Institution of Washington*, 1909, p. 166 (24).

On the other hand, ω is the constant angle between the tangent MT and the normal MN on the radius vector, both reckoned in the direction of the increasing angle ϕ. ω is negative if MT lies between MN and the origin, positive if MT lies on the other side. If ω is negative, R will decrease with increasing ϕ, and we approach nearer and nearer to the origin in following the spiral. If ω is positive, R will increase with increasing ϕ, so that we recede from the origin in following the spiral in the direction of increasing ϕ.

In Fig. 3 are shown various types of spirals corresponding to various values of ω. Each separate figure contains a series of congruent spirals seen from the positive side of the Z-axis, with the origin in the center; the corresponding value of ω is written above the figure. $\omega = 0$ corresponds to circles concentric around the origin. If ω approaches $-\pi/2$ or $+\pi/2$ the spirals tend to become straight lines through the origin.

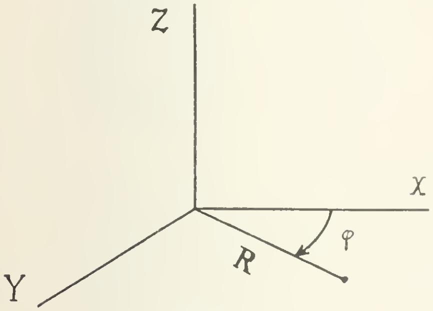


FIG. 1

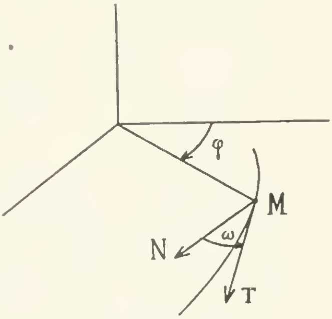


FIG. 2

After this review of the well-known properties of logarithmic spirals, we will consider an ideal vortex defined as follows:

The whirl shall be bounded by two cylinders of revolution with the Z-axis as axis and with radii equal to ρ_1 and ρ_2, by the XY-plane, and by a parallel plane at the distance h above it. We consider h small as compared with ρ_2 in order to have a flat whirl. The motion of the gas in the space mentioned is supposed to be permanent, and the trajectories of the gas particles are supposed to be logarithmic spirals parallel to the XY-plane, whose projections on that plane are

$$R = Ae^{\phi \tan \omega}$$

where ω is the same for all particles, and where A is determined by the position of the spiral. Let us suppose further that the velocity and electric charge in unit volume are functions of the distance from the Z-axis only.

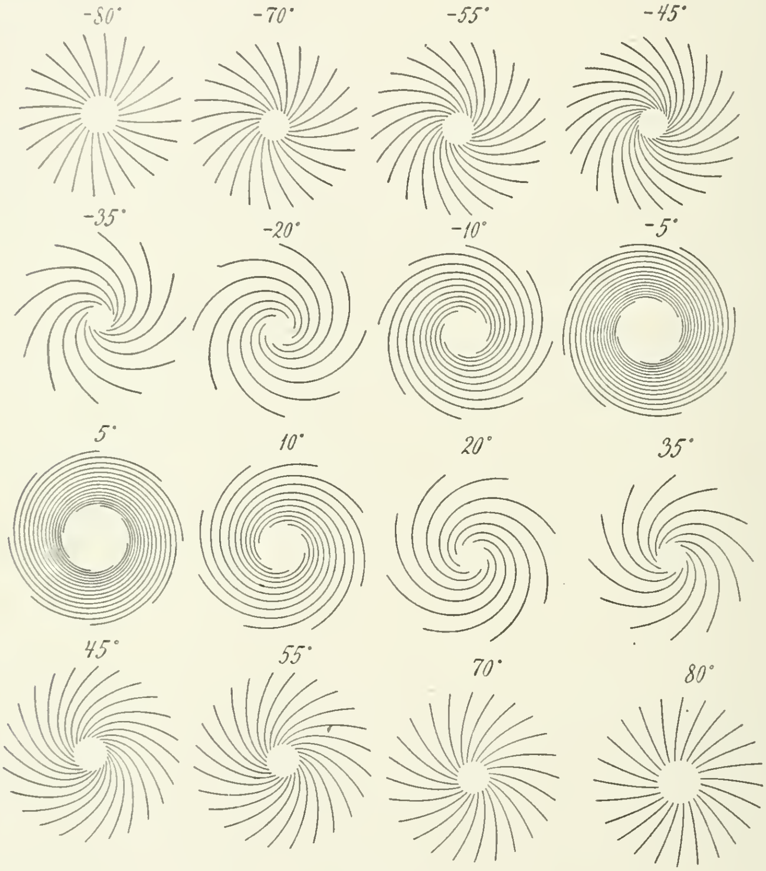


Fig. 3

It is clear that such a whirl does not exist in reality; there must be a source and a sink for the coming and going masses of gas in the whirl, but in the first approximation we will neglect the electric action of masses outside this hypothetical whirl; their action will be calculated later, when we have discussed the formulae for this first simple case.

To fix the motion without ambiguity, we will call the velocity v positive if the motion corresponds to increasing ϕ, negative if it corresponds to decreasing ϕ. Thus the motion seen from the positive side of the Z-axis will be clockwise for positive v, counter-clockwise for negative v; and the gas will approach the Z-axis if the product ωv is negative, and recede from the Z-axis if it is positive, as is seen in Fig. 4.

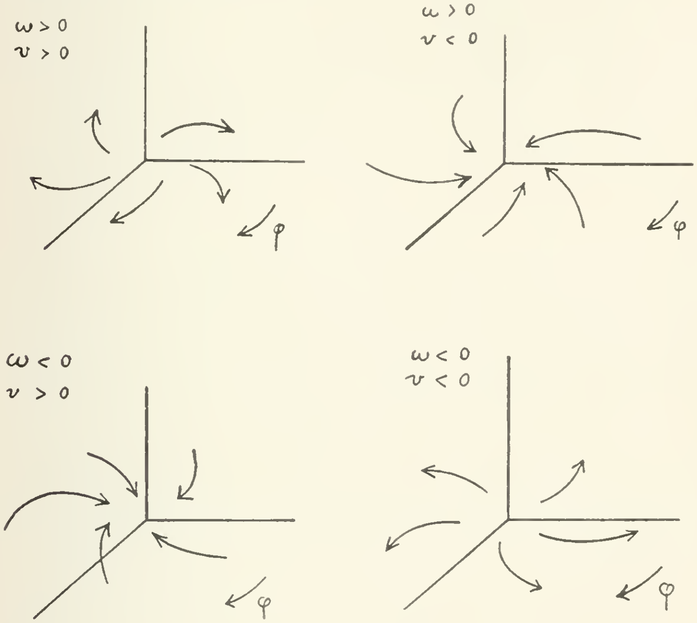


FIG. 4

Let us now calculate the magnetic action of this hypothetical whirl on a unit of north magnetism placed at a point outside the whirl. For the sake of brevity we first suppose ω, v, and the electric charge to be positive. In order to have a solenoid element bounded by current-lines, let us consider an element of the whirl, bounded by the following surfaces:

The two planes

$$z=\zeta , \qquad z=\zeta+\Delta\zeta ;$$

the two cylinders

$$R=\rho , \qquad R=\rho+\Delta\rho ;$$

and the two cylinders

$$R=Ae^{\phi \tan \omega}, \qquad R=Ae^{(\phi+\Delta\theta) \tan \omega},$$

where $\Delta\zeta$, $\Delta\rho$, and $\Delta\theta$ are positive infinitesimal quantities, and $\Delta\theta$ and $\Delta\zeta$ infinitesimal compared with $\Delta\rho$.

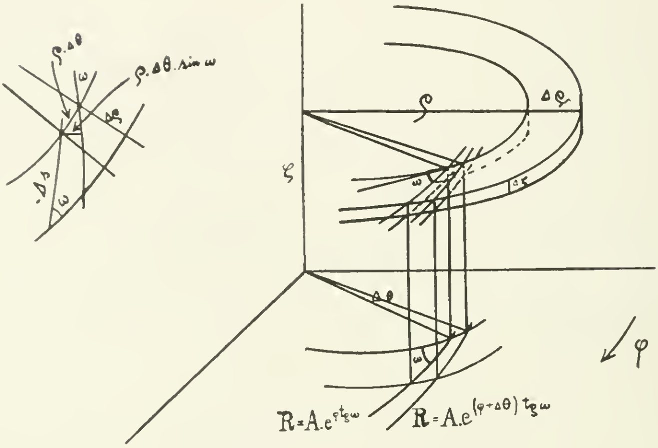


FIG. 5

Let us make use of the c.g.s. system of units, and the *electrostatic* system for the electric charge. The cross-section of our element will be

$$\rho\Delta\theta \sin \omega \, \Delta\zeta .$$

If ε is the electric charge in a unit volume of the gas, there will pass as a convection current through this section in one second

$$\varepsilon v\rho \sin \omega \, \Delta\theta\Delta\zeta \text{ electrostatic units,}$$

and consequently

$$\frac{1}{3\times 10^{10}} \varepsilon v\rho \sin \omega \, \Delta\theta\Delta\zeta \text{ electromagnetic units.}$$

If by i we denote the current in amperes passing through the element, we have

$$i=\frac{1}{3\times 10^{9}} \varepsilon v\rho \sin \omega \, \Delta\theta\Delta\zeta .$$

On the other hand, the length of the element is

$$\Delta s = \frac{\Delta \rho}{\sin \omega} \, .$$

Now, as is well known, the magnetic action of this current-element on one unit of north magnetism situated at the point M with co-ordinates x, y, and z will have the following components parallel with X-, Y-, and Z-axes:

$$p_x = \frac{1}{10} i \frac{\Delta s}{r_1^2} (\cos \beta_1 \cos \gamma_2 - \cos \gamma_1 \cos \beta_2) \, ,$$

$$p_y = \frac{1}{10} i \frac{\Delta s}{r_1^2} (\cos \gamma_1 \cos \alpha_2 - \cos \alpha_1 \cos \gamma_2) \, ,$$

$$p_z = \frac{1}{10} i \frac{\Delta s}{r_1^2} (\cos \alpha_1 \cos \beta_2 - \cos \beta_1 \cos \alpha_2) \, ,$$

where r_1 is the distance from the element to the point M; α_1, β_1, γ_1, the angles between the current-element and the positive directions of the X-, Y-, and Z-axes; and α_2, β_2, γ_2, the angles between the direction from (M) to the current-element and the same three axes.

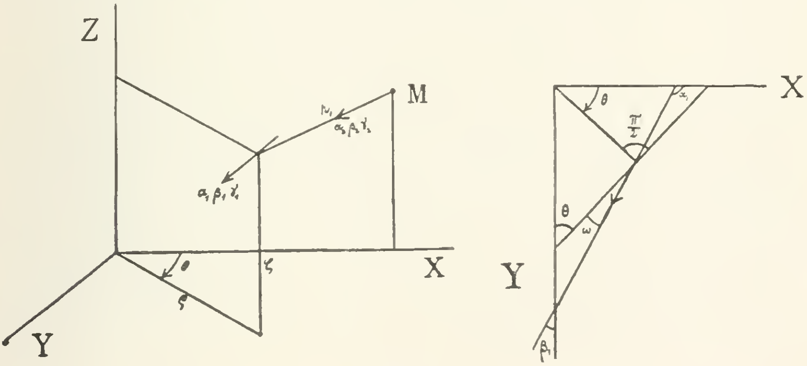


FIG. 6

Now choose the point M in the XZ-plane (Fig. 6) so that:

$$y = 0 \, .$$

The co-ordinates of the current-element are

$$\rho \cos \theta, \quad \rho \sin \theta, \quad \zeta \, ,$$

and consequently

$$r_1^2=(x-\rho\cos\theta)^2+(\rho\sin\theta)^2+(z-\zeta)^2$$

and

$$\cos\alpha_2=\frac{\rho\cos\theta-x}{r_1}\,,\qquad \cos\beta_2=\frac{\rho\sin\theta}{r_1}\,,\qquad \cos\gamma_2=\frac{\zeta-z}{r_1}\,.$$

On the other hand, we have

$$\alpha_1=\frac{\pi}{2}-\omega+\theta\,,\qquad \beta_1=\theta-\omega\,,\qquad \gamma_1=\frac{\pi}{2}\,,$$

which give

$$\cos\alpha_1=\sin(\omega-\theta)\,,\qquad \cos\beta_1=\cos(\omega-\theta)\,,\qquad \cos\gamma_1=0\,.$$

If we substitute the values of the angles α and β, we get

$$p_x=-\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\cos(\omega-\theta)$$

$$p_y=\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\sin(\omega-\theta)$$

$$p_z=\frac{1}{10}\,i\,\frac{\Delta s}{r_1^3}\,\{x\cos(\omega-\theta)-\rho\cos\omega\}\,.$$

These formulae are valid for ε, v, and ω positive. But if we change the sign of ε or of v, the magnetic action will change direction, that is, p_x, p_y, and p_z will change their signs. But this will also be the case with i and consequently the formulae are valid for both signs of the quantities v and ε. In an analogous manner we see that they are valid for all values of the angle ω.

The components of action of an element situated symmetrically with respect to the XZ-plane are obtained by changing θ into $-\theta$; we find for these components

$$p_x'=-\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\cos(\omega+\theta)\,,$$

$$p_y'=\frac{1}{10}\,i\,\frac{\Delta s\,(z-\zeta)}{r_1^3}\sin(\omega+\theta)\,,$$

$$p_z'=\frac{1}{10}\,i\,\frac{\Delta s}{r_1^3}\,\{x\cos(\omega+\theta)-\rho\cos\omega\}\,.$$

If we introduce the values of i and of Δs and add, we get

$$p_x+p'_x=-\frac{2\cos\omega}{3\times10^{10}}\cdot(z-\zeta)\,\varepsilon\rho v\cos\theta\,\frac{\Delta\rho\Delta\zeta\Delta\theta}{r_1^3}\,,$$

$$p_y+p'_y=\frac{2\sin\omega}{3\times10^{10}}\cdot(z-\zeta)\,\varepsilon\rho v\cos\theta\,\frac{\Delta\rho\Delta\zeta\Delta\theta}{r_1^3}\,,$$

$$p_z+p'_z=\frac{2\cos\omega}{3\times10^{10}}\cdot(x\cos\theta-\rho)\,\varepsilon\rho v\,\frac{\Delta\rho\Delta\zeta\Delta\theta}{r_1^3}\,.$$

From these the action of the part of the whirl bounded by the two planes

$$z=\zeta,\qquad z=\zeta+\Delta\zeta\,,$$

and the two circular cylinders

$$R=\rho,\qquad R=\rho+\Delta\rho$$

is obtained by an integration with respect to θ, which gives the components

$$-\frac{2\cos\omega}{3\times10^{10}}(z-\zeta)\,\varepsilon\rho v\,\Delta\rho\Delta\zeta\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\,,$$

$$\frac{2\sin\omega}{3\times10^{10}}(z-\zeta)\,\varepsilon\rho v\,\Delta\rho\Delta\zeta\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\,,$$

$$\frac{2\cos\omega}{3\times10^{10}}\,\varepsilon\rho v\,\Delta\rho\Delta\zeta\int_0^\pi\frac{x\cos\theta-\rho}{r_1^3}\,d\theta\,.$$

Here ε and v are independent of the angle θ according to the supposition mentioned above that they are functions of ρ alone.

The action of a ring bounded by the same two planes and the two cylinders $R=\rho_1$ and $R=\rho_2$ is obtained by an integration with respect to ρ, which gives the components

$$-\frac{2\cos\omega}{3\times10^{10}}(z-\zeta)\,\Delta\zeta\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\right)d\rho\,,$$

$$\frac{2\sin\omega}{3\times10^{10}}(z-\zeta)\,\Delta\zeta\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi\frac{\cos\theta\,d\theta}{r_1^3}\right)d\rho\,,$$

$$\frac{2\cos\omega}{3\times10^{10}}\,\Delta\zeta\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi\frac{x\cos\theta-\rho}{r_1^3}\,d\theta\right)d\rho\,.$$

Here ε and v are functions of ρ only.

A final integration with respect to ζ gives the components of the action of the whirl itself:

$$-\frac{2\cos\omega}{3\times 10^{10}}\int_0^h (z-\zeta)\left[\int_{\rho_1}^{\rho_2}\varepsilon\rho v\left(\int_0^\pi \frac{\cos\theta\, d\theta}{r_1^3}\right)d\rho\right]d\zeta$$

with analogous expressions for the other components.

But here we suppose the product $\varepsilon\rho v$ to be a continuous function of ρ; and the point $(x,0,z)$ being outside the whirl, r_1 is positive over the entire field of integration. This field being also finite, the succession of the integrations is indifferent, so that we can write more simply

$$-\frac{2\cos\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi (z-\zeta)\,\varepsilon\rho v\,\frac{\cos\theta}{r_1^3}\,d\theta d\rho d\zeta.$$

On the other hand, as everything is symmetrical around the Z-axis, we get the components in an arbitrary point (x,y,z) by substituting $R=\sqrt{x^2+y^2}$ for x; hence

$$\left.\begin{aligned}
H_R &= -\frac{2\cos\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(z-\zeta)\,\varepsilon\rho v\cos\theta}{D^3}\,d\theta d\rho d\zeta\\
H_\phi &= \frac{2\sin\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(z-\zeta)\,\varepsilon\rho v\cos\theta}{D^3}\,d\theta d\rho d\zeta\\
H_z &= \frac{2\cos\omega}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(R\cos\theta-\rho)\,\varepsilon\rho v}{D^3}\,d\theta d\rho d\zeta
\end{aligned}\right\}\quad (1)$$

where

$$D^2=R^2-2R\rho\cos\theta+\rho^2+(z-\zeta)^2.$$

H_R is the component normal to the Z-axis, H_z the component parallel to that axis, and H_ϕ the component normal to H_R and H_ϕ. As regards the signs, H_R is positive in the direction from the

Z-axis, H_z is positive in the direction of increasing z, and H_ϕ positive in the direction of increasing ϕ (see Fig. 7).

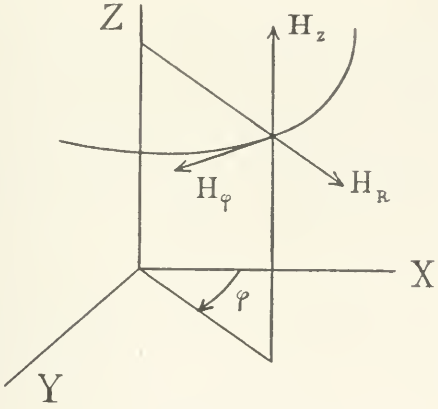


FIG. 7

III. PROPERTIES OF THE COMPONENTS OF THE MAGNETIC ACTION EQUATIONS OF THE LINES OF MAGNETIC FORCE IN SPACE

Let us, in the formula for H_R, H_ϕ, and H_z, introduce the function

$$\Phi = \frac{2R}{3\times 10^{10}} \int_0^h \int_{\rho_1}^{\rho_2} \int_0^\pi \frac{\varepsilon\rho v \cos\theta}{D}\, d\theta d\rho d\zeta \qquad (2)$$

As the point (x,y,z) is outside the field of integration, and consequently $D>0$, we obtain the derivatives $\frac{\delta\Phi}{\delta R}$ and $\frac{\delta\Phi}{\delta z}$ as follows:

$$\frac{\delta\Phi}{\delta R} = \frac{2}{3\times 10^{10}} \int_0^h \int_{\rho_1}^{\rho_2} \varepsilon\rho v \left[\frac{\delta}{\delta R}\int_0^\pi \frac{R\cos\theta}{D}\, d\theta\right] d\rho d\zeta\,,$$

$$\frac{\delta\Phi}{\delta z} = \frac{2}{3\times 10^{10}} \int_0^h \int_{\rho_1}^{\rho_2} \varepsilon\rho v \left[\frac{\delta}{\delta z}\int_0^\pi \frac{R\cos\theta}{D}\, d\theta\right] d\rho d\zeta\,.$$

But

$$\frac{\delta}{\delta R}\int_0^\pi \frac{R\cos\theta}{D} d\theta = \int_0^\pi \frac{\cos\theta\, d\theta}{D} + R\int_0^\pi \frac{(\rho - R\cos\theta)\, d\theta}{D^3} - \int_0^\pi \frac{R\rho\, \sin^2\theta\, d\theta}{D^3}\,.$$

Integrating by parts, the last integral in the right member becomes equal to the first and we get

$$\frac{\delta}{\delta R}\int_0^\pi \frac{R\cos\theta}{D}\,d\theta = R\int_0^\pi \frac{\rho - R\cos\theta}{D^3}\,d\theta\,.$$

Further

$$\frac{\delta}{\delta z}\int_0^\pi \frac{R\cos\theta}{D}\,d\theta = -R\int_0^\pi \frac{(z-\zeta)\cos\theta\,d\theta}{D^3}\,.$$

This gives us

$$\frac{\delta\Phi}{\delta R} = \frac{2R}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(\rho - R\cos\theta)\,\varepsilon\rho v}{D^3}\,d\theta d\rho d\zeta\,,$$

$$\frac{\delta\Phi}{\delta z} = -\frac{2R}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(z-\zeta)\,\varepsilon\rho v\cos\theta}{D^3}\,d\theta d\rho d\zeta\,,$$

and we can therefore write

$$\left.\begin{aligned} H_R &= \frac{\cos\omega}{R}\,\frac{\delta\Phi}{\delta z} \\ H_\phi &= -\frac{\sin\omega}{R}\,\frac{\delta\Phi}{\delta z} \\ H_z &= -\frac{\cos\omega}{R}\,\frac{\delta\Phi}{\delta R} \end{aligned}\right\} \qquad (3)$$

From this it is easy to find the lines of magnetic force in space outside the whirl. These are the integral curves of the system

$$\frac{dx}{H_x} = \frac{dy}{H_y} = \frac{dz}{H_z}\,,$$

where H_x, H_y, and H_z are the components of the magnetic force, parallel to the X-, Y-, and Z-axes respectively.

But we have

$$H_R = \frac{x}{R}\,H_x + \frac{y}{R}\,H_y\,,$$

$$H_\phi = \frac{x}{R}\,H_y - \frac{y}{R}\,H_x\,,$$

and if we take R and ϕ as variables instead of x and y, we have

$$xdx+ydy=RdR\,,$$
$$xdy-ydx=R^2d\phi\,.$$

Hence

$$\frac{dx}{H_x}=\frac{dy}{H_y}=\frac{xdx+ydy}{xH_x+yH_y}=\frac{xdy-ydx}{xH_y-yH_x}=\frac{dz}{H_z}$$

and from the last three equations we get

$$\frac{dR}{H_R}=\frac{Rd\phi}{H_\phi}=\frac{dz}{H_z},$$

which are the differential equations of the lines of force in the variables R, ϕ, and z.

Now if we substitute in the equation

$$\frac{dR}{H_R}=\frac{Rd\phi}{H_\phi}$$

the above-developed expressions for H_R and H_ϕ, we get

$$\frac{dR}{R}=-\cot\omega\;d\phi\,,$$

which, integrated, gives us

$$R=Be^{-\phi\cot\omega}\,, \tag{4}$$

B being a constant of integration.

On the other hand, if we substitute in the equations

$$\frac{dR}{H_R}=\frac{dz}{H_z}$$

the values of H_R and H_z, we get

$$\frac{\delta\Phi}{\delta R}\,dR+\frac{\delta\Phi}{\delta z}\,dz=0$$

and Φ being a function of R and z only, we can integrate and obtain

$$\Phi=C \tag{5}$$

where C is a constant.

From equations (4) and (5) we have the following fundamental theorem:

The lines of magnetic force due to the spiral whirl are curves in space lying on the surfaces of revolution $\Phi=C$. *Their projections on the plane of the whirl are logarithmic spirals cutting the current-lines of the whirl everywhere in right angles.*

IV. FIRST APPLICATION TO THE THEORY OF SOLAR VORTICES

Let us first consider the various cases that can occur for different signs of ω, v, and ε. In order to find the signs of the integrals, let us consider

$$\int_0^\pi \frac{\cos\theta\, d\theta}{D^3}\,,$$

where

$$D^2=R^2-2R\rho\cos\theta+\rho^2+(z-\zeta)^2\,.$$

This integral may be written

$$\int_0^\pi \frac{\cos\theta}{D^3}\,d\theta=\int_0^{\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{D^3}+\int_{\frac{\pi}{2}}^{\pi} \frac{\cos\theta\, d\theta}{D^3}\,.$$

By taking $\theta_1=\pi-\theta$ as variable in the last integral, we get

$$\int_0^\pi \frac{\cos\theta}{D^3}\,d\theta=\int_0^{\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{D^3}-\int_0^{\frac{\pi}{2}} \frac{\cos\theta_1\, d\theta_1}{D_1^3}\,,$$

where

$$D_1^2=R^2+2R\rho\cos\theta_1+\rho^2+(z-\zeta)^2\,.$$

But for $\theta=\theta_1$, and different from $\pi/2$, we have $D_1>D$, and therefore the first integral will be greater than the second, that is,

$$\int_0^\pi \frac{\cos\theta\, d\theta}{D^3}>0\,.$$

On the other hand, ε and v have the same sign throughout the whirl, and if we are on the positive side of the XY-plane and above the whirl, $z-\zeta$ will be positive. Farther along the Z-axis we have

$$H_z=H_{z0}=-\frac{2\cos\omega}{3\times10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi\frac{\varepsilon\rho^2v}{D^3}\,d\theta d\rho d\zeta\,.$$

Hence, from this equation and from equation (1) we conclude that:

H_R and H_{z0} will have the opposite sign to εv, and H_ϕ the same sign as $\varepsilon\omega v$.

We will apply this to solar vortices around sun-spots, assuming that the hydrogen flocculi represent lines of magnetic force due to a whirl in a lower level of the sun's atmosphere. We will choose the Z-axis along a solar radius, with its positive direction pointing away from the sun's center and toward the observer. For different signs of ε, ω, and v, we have the 8 combinations shown in Table I, with corresponding signs for the components H_R, H_ρ, and H_{z0}.

TABLE I

	ϵ	ω	v	H_R	H_ϕ	H_{z0}
1	+	+	+	−	+	−
2	+	+	−	+	−	+
3	+	−	+	−	−	−
4	+	−	−	+	+	+
5	−	+	+	+	−	+
6	−	+	−	−	+	−
7	−	−	+	+	+	+
8	−	−	−	−	−	−

This gives the diagrams shown in Fig. 8, where the whirl is seen from a distant point outside the sun. The current lines are plain curves with arrows in the direction of motion, and the projection of the lines of force situated between the whirl and the observer are dotted lines with arrows in the direction of the magnetic force. When these last arrows point to the center, H_{z0} is negative and the magnetic force along the Z-axis is directed away from the observer; when they point out, that force is directed toward the observer.

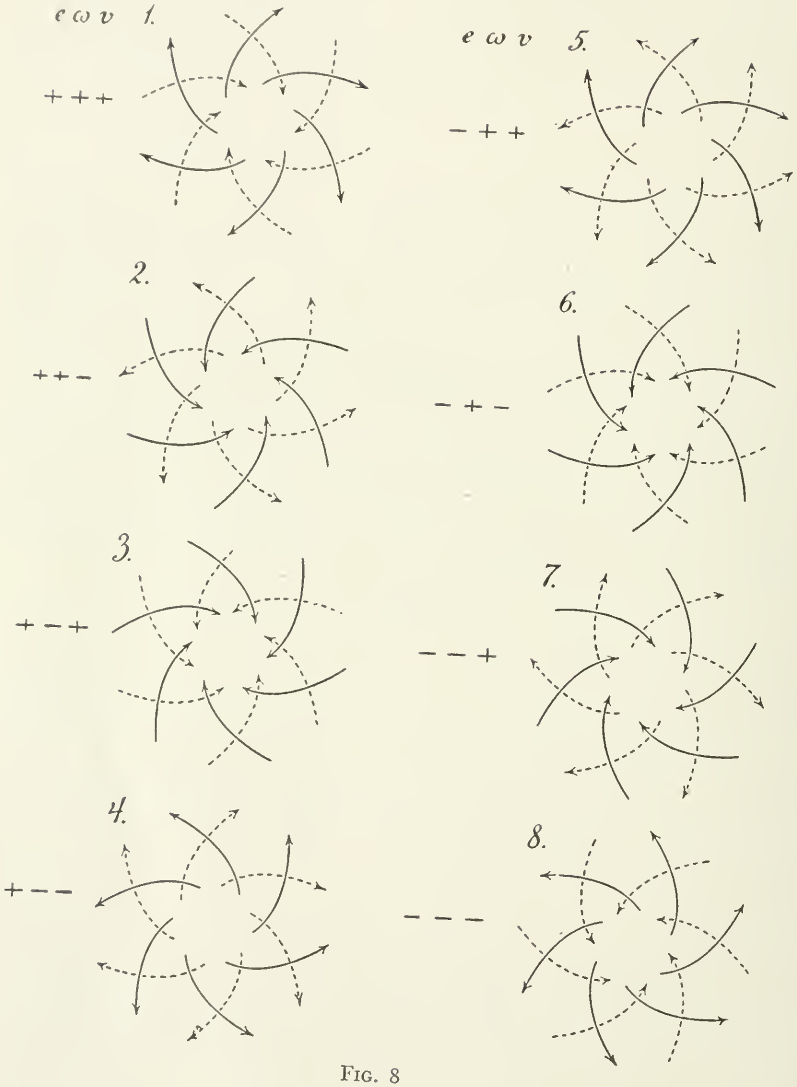
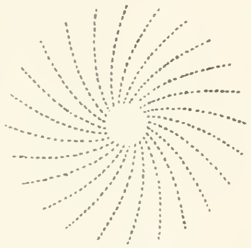


Fig. 8

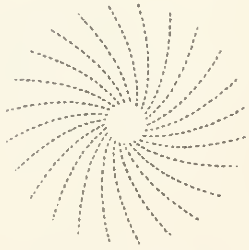
We now have to find which of these 8 cases corresponds with the observed facts. This can be done by combining Hale's classical researches on the Zeeman effect with the recent investigations on radial motion in sun-spots by St. John.[1]

In his paper "The Probable Existence of a Magnetic Field in Sun-Spots," Hale states the following facts: "The magnetic force in the center of the whirl is directed toward the observer when the direction of motion indicated by the hydrogen flocculi whirl is clockwise, and directed away from the observer when the direction of motion is counter-clockwise."[2]

The meaning of the definition of clockwise and counter-clockwise motion will be seen[3] in Fig. 9. There is here no question about direction of motion along the current-lines; the definition is purely geometrical. According to this, cases Nos. 1, 4, 6, and 7 must be rejected. On the other hand, through the recent researches of St. John,[4] the existence of a whirl at a lower level giving the Zeeman effect and causing the observed arrangement of the hydrogen flocculi at the higher levels is rendered extremely probable. In this low-level whirl, probably situated at or below the levels oo and ooo in the figure in St. John's paper, St. John finds a motion outward from the axis of the whirl. This excludes cases Nos. 2 and 3, and thus only cases Nos. 5 and 8 remain as corresponding to observed facts. In both cases ε is negative, that is to say, we come to the same conclusion as Professor Hale: *The Zeeman effect is due to a whirl of negatively charged electric particles.*



Clockwise



Counter-clockwise

FIG. 9

This whirl, however, is distinct from the observed hydrogen whirl, and is situated in a much lower level of the sun's atmosphere.

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[1] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 322, 1913.

[2] *Mt. Wilson Contr.*, No. 30; *Astrophysical Journal*, **28**, 315, 1908.

[3] *Op. cit.*

[4] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, **37**, 322, 1913.

The sign of electricity in the whirl being determined, we have to choose the probable value of the angle ω, which determines the form of the current-lines. This can be done by comparing Fig. 10 with a series of photographs of hydrogen flocculi whirls taken at

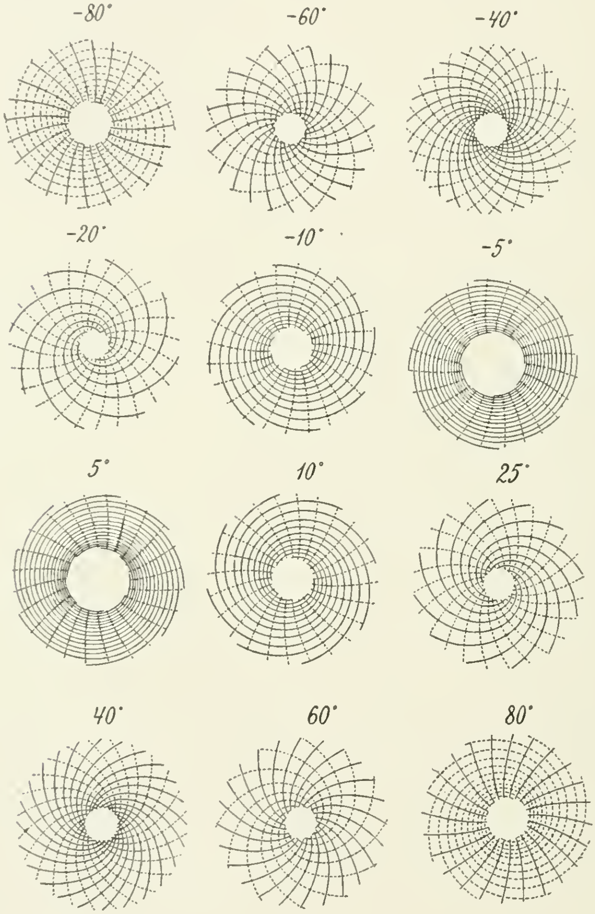

Fig. 10

Mount Wilson. It will thus be seen that values of ω lying between $15°$ and $25°$, or between $-15°$ and $-25°$, agree well with the reality; the dotted lines must then be identified with the hydrogen flocculi. The current-lines and projection of lines of magnetic force (hydrogen flocculi) for the values $\omega = 20^\circ$ and $\omega = -20^\circ$ are shown on a larger scale in Fig. 11. This agrees well with the strong magnetic effect shown by solar vortices where the structure of the hydrogen flocculi is nearly radial; for the more nearly radial this structure is, the more nearly circular are the current-lines of the whirl.

In order to get some idea regarding the amount of electricity in motion in the whirl, more quantitative calculations are necessary.

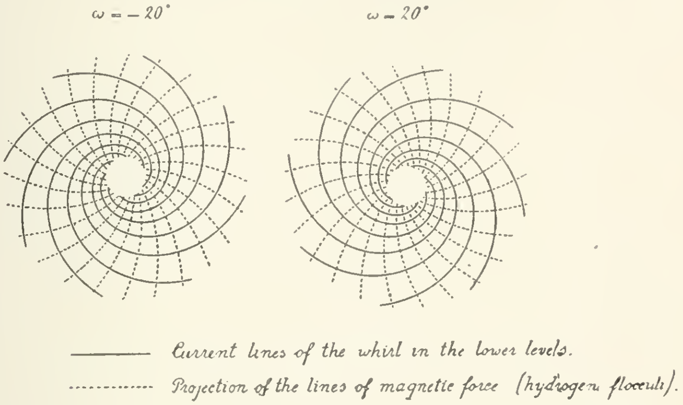
ω = −20°
ω = 20°
Current lines of the whirl in the lower levels.
Projection of the lines of magnetic force (hydrogen flocculi).

FIG. 11

For the sake of simplicity we will put forward another hypothesis concerning ε and v as functions of ρ, as will be seen in the next paragraph.

V. HYPOTHESIS REGARDING THE PRODUCT $\varepsilon\rho v$ AND THE CORRESPONDING FORMULAE FOR NUMERICAL CALCULATIONS

We will now, for the sake of simplicity, make the following assumptions:

1. The density of the gas in the whirl shall be constant throughout the whirl, and the charge ε proportional to the density; that is, ε is supposed to be a constant.

2. The velocity v shall be *inversely proportional* to the distance ρ from the axis. This hypothesis, as already known, was made by Guldberg and Mohn for the *outer* part of terrestrial cyclones. Thus $\varepsilon\rho v$ will be constant throughout the whirl. Let us put

$$\frac{\varepsilon\rho v}{3\times10^{10}}=c\,.$$

Hence

$$\left.\begin{aligned} H_R &= -2c\cos\omega \int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{(z-\zeta)\cos\theta}{D^3}\,d\theta d\rho d\zeta \\ H_\phi &= 2c\sin\omega \int_0^h\int_1^{\rho_2}\int_0^\pi \frac{(z-\zeta)\cos\theta}{D^3}\,d\theta d\rho d\zeta \\ H_z &= 2c\cos\omega \int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{R\cos\theta-\rho}{D^3}\,d\theta d\rho d\zeta \end{aligned}\right\} \tag{6}$$

and

$$\Phi = 2cR\int_0^h\int_1^{\rho_2}\int_0^\pi \frac{\cos\theta}{D}\,d\theta d\rho d\zeta \tag{7}$$

where

$$D^2 = R^2 - 2R\rho\cos\theta + \rho^2 + (z-\zeta)^2 .$$

We now have

$$\int \frac{z-\zeta}{D^3}\,d\zeta = \frac{1}{D} + \text{constant} ,$$

$$\int \frac{R\cos\theta-\rho}{D^3}\,d\rho = \frac{1}{D} + \text{constant} .$$

Hence we may write

$$\begin{aligned} H_R &= -2c\cos\omega\,[f(h)-f(0)] \\ H_\phi &= 2c\sin\omega\,[f(h)-f(0)] \\ H_z &= 2c\cos\omega\,[g(\rho_2)-g(\rho_1)] \end{aligned}$$

where

$$f(\zeta) = \int_{\rho_1}^{\rho_2}\int_0^\pi \frac{\cos\theta}{D}\,d\theta d\rho , \quad g(\rho) = \int_0^h\int_0^\pi \frac{d\theta d\zeta}{D} .$$

But we have further, ln being the natural logarithm, with base $e = 2.71828 \ldots\ldots$:

$$\int \frac{d\rho}{D} = \quad ln\,(\rho - R\cos\theta + D) + \text{constant}\,,$$

$$\int \frac{d\zeta}{D} = -ln\,(z - \zeta + D) + \text{constant}\,,$$

which gives

$$f(\zeta) = f(\rho_2, \zeta) - f(\rho_1, \zeta)\,,$$

$$g(\rho) = g(\rho, h) - g(\rho, 0)\,,$$

where

$$\left.\begin{aligned} f(\rho,\zeta) &= \int_0^\pi ln\,(\rho - R\cos\theta + D)\cos\theta\, d\theta \\ g(\rho,\zeta) &= -\int_0^\pi ln\,(z - \zeta + D)\, d\theta \end{aligned}\right\} \quad (8)$$

Substituting into the expressions for the components, we finally get

$$\left.\begin{aligned} H_R &= -2c\cos\omega\,[f(\rho_2, h) - f(\rho_1, h) - f(\rho_2, 0) + f(\rho_1, 0)] \\ H_\phi &= \quad 2c\sin\omega\,[f(\rho_2, h) - f(\rho_1, h) - f(\rho_2, 0) + f(\rho_1, 0)] \\ H_z &= \quad 2c\cos\omega\,[g(\rho_2, h) - g(\rho_1, h) - g(\rho_2, 0) + g(\rho_1, 0)] \end{aligned}\right\} \quad (9)$$

For each system of values, R, z, ρ, ζ, the integrals $f(\rho,\zeta)$ and $g(\rho,\zeta)$ can now be easily calculated numerically. Integraphs and planimeters may be employed, or the approximate formula of Simpson, which is well known.

We thus have the means of calculating in every case the magnetic field in space outside the whirl.

If the point (x,y,z) be on the axis of the whirl, R is zero and

$$D^2 = \rho^2 + (z - \zeta)^2\,,$$

which gives

$$f(\rho,\zeta)=ln\ (\rho+D)\int_0^{\pi}\cos\theta\, d\theta=0\ ,$$

$$g(\rho,\zeta)=-\pi\ ln\ (z-\zeta+D)\ .$$

Hence $H_R=0$, $H_\phi=0$, and $H_z=H_{z0}$, where

$$H_{z0}=-2\pi c\cos\omega\ ln\ \frac{(z-h+\sqrt{(z-h)^2+\rho_2^2})(z+\sqrt{z^2+\rho_1^2})}{(z-h+\sqrt{(z-h)^2+\rho_1^2})(z+\sqrt{z^2+\rho_2^2})} \qquad (10)$$

It will be interesting to obtain a verification. Let h and k be infinitesimal, and

$$\rho_2=\rho+k\ ,\quad \rho_1=\rho\ ,\quad \omega=0\ .$$

If we put

$$\sqrt{\rho^2+z^2}=u$$

we have

$$z-h+\sqrt{(z-h)^2+(\rho+k)^2}=$$

$$(z+u)\left[1-\frac{1}{u}h+\frac{\rho}{(z+u)u}k+\frac{\rho^2}{2(z+u)u^3}h^2+\frac{\rho z}{(z+u)u^3}hk\right.$$

$$\left.+\frac{z^2}{2(z+u)u^3}k^2+\ \dots\right].$$

Hence

$$ln\ \frac{z-h+\sqrt{(z-h)^2+(\rho+k)^2}}{z-h+\sqrt{(z-h)^2+\rho^2}}=\frac{\rho}{(z+u)u}k+\frac{\rho}{u^3}hk-\frac{u^2+uz+z^2}{(z+u)u^3}k^2+\ \dots$$

This gives

$$ln\ \frac{z+\sqrt{z^2+\rho^2}}{z+\sqrt{z^2+(\rho+k)^2}}=-\frac{\rho}{(z+u)u}k-\frac{u^2+uz+z^2}{(z+u)u^3}k^2+\ \dots$$

and finally

$$H_z=-2\pi c\frac{\rho}{u^3}hk+\ \dots$$

where

$$c = \frac{\varepsilon\rho v}{3\times 10^{10}} .$$

If we suppose ε and v to be positive, εhkv electrostatic units will pass through the cross-section of the current in one second, which corresponds to a current of

$$i = \frac{\varepsilon hkv}{3\times 10^{9}}$$

amperes. If we introduce this value,

$$H_z = -\frac{1}{10}\, i\, \frac{2\pi\rho^2}{u^3} + \ldots .$$

and this first term represents exactly the magnetic action in the point (0,0,z) of the circular current whose section is the infinitesimal product hk, and where the direction of the current is that of increasing ϕ (see Fig. 12).

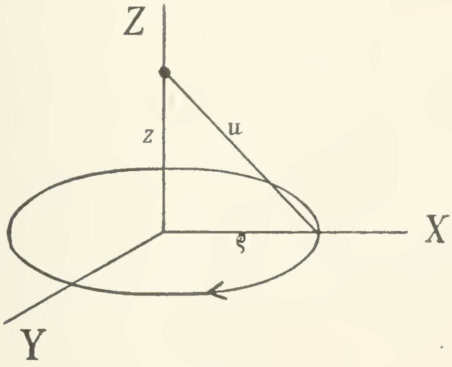

FIG. 12

VI. SPECIAL CASE IN WHICH THE THICKNESS OF THE WHIRL IS VERY SMALL COMPARED WITH ITS DIAMETER

We will now consider the special case in which the thickness of the whirl is very small compared with its diameter. Then for points (x,y,z) not very near the whirl, we can put

$$f(\rho,h) - f(\rho,0) = hf'(\rho,0) ,$$
$$g(\rho,h) - g(\rho,0) = hg'(\rho,0) ,$$

where $f'(\rho,0)$ and $g'(\rho,0)$ are the partial derivatives $\frac{\delta f}{\delta \zeta}$ and $\frac{\delta g}{\delta \zeta}$ for the value $\zeta=0$.

Now

$$f'(\rho,\zeta)=-\int_0^\pi \frac{z-\zeta}{\rho-R\cos\theta+D}\cdot\frac{\cos\theta\, d\theta}{D}\,,\quad g'(\rho,\zeta)=\int_0^\pi \frac{d\theta}{D}$$

and

$$f'(\rho,0)=-z\int_0^\pi \frac{\cos\theta\, d\theta}{(\rho-R\cos\theta+D_0)D_0}=z\int_0^\pi \frac{(\rho-R\cos\theta)\cos\theta\, d\theta}{(R^2\sin^2\theta+z^2)D_0}$$

$$g'(\rho,0)=\int_0^\pi \frac{d\theta}{D_0}\,,$$

where

$$D_0^2=R^2-2R\rho\cos\theta+\rho^2+z^2\,.$$

This must be substituted in the formulae

$$\left.\begin{aligned} H_R&=-2ch\cos\omega\,[f'(\rho_2,0)-f'(\rho_1,0)]\\ H_\phi&=2ch\sin\omega\,[f'(\rho_2,0)-f'(\rho_1,0)]\\ H_z&=2ch\cos\omega\,[g'(\rho_2,0)-g'(\rho_1,0)]\end{aligned}\right\}\qquad(11)$$

The same result can also be obtained by starting from formula (6), and considering h as infinitesimal. Then

$$H_R=-2ch\cos\omega\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{z\cos\theta}{D^3}\,d\theta d\rho$$

and an integration with respect to ρ gives the foregoing result.

The function Φ can easily be found in this case. We have

$$\Phi=2ch\,R\int_{\rho_1}^{\rho_2}\int_0^\pi \frac{\cos\theta}{D_0}\,d\theta d\rho\,,$$

and as

$$\int\frac{d\rho}{D_0}=ln\,(\rho-R\cos\theta+D_0)+\text{constant},$$

we get

$$\Phi = 2chR\left[f(\rho_2,\,0)-f(\rho_1,\,0)\right] \tag{12}$$

where the function $f(\rho,\zeta)$ is given by equation (8), when $D=D_0$.

The integrals occurring in the formulae for the components can be expressed by elliptic integrals of the first and second kind; but only the reduction of the integral for $g'(\rho,0)$ is sufficiently simple to be of any importance in the present problem. The integral in this case simplifies by the well-known substitution[1]

$$\frac{\pi}{2}-\frac{\theta}{2}=\psi$$

as follows:

$$\int_0^{\pi}\frac{d\theta}{\sqrt{R^2-2R\rho\cos\theta+\rho^2+z^2}}=\frac{2}{\lambda}\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}} \tag{13}$$

where

$$k^2=\frac{4R\rho}{(R+\rho)^2+z^2}\,,\qquad \lambda=\sqrt{(R+\rho)^2+z^2}\,.$$

As this integral is well tabulated in Legendre's *Traité des Fonctions Elliptiques* (Paris, 1826), **2**, 223, the calculation is rather easy.

The other integrals are most conveniently computed by Simpson's formula, or by machines.

VII. NUMERICAL APPLICATION TO A SOLAR VORTEX

We will now apply the preceding formula to an ideal solar vortex, in order to see how many of the observed facts can be accounted for.

According to the researches mentioned above by St. John, the vortex giving the Zeeman effect is probably situated at a level of the sun's atmosphere equal to or lower than the level Fe, 00, where the pressure is probably about 10 atmospheres. As the pressure increases very rapidly in descending into the sun's atmosphere, it seems probable that the thickness of the outflowing whirl in question can be only a very small fraction of the diameter indicated by the hydrogen flocculi whirl in the upper levels. This is also analogous to terrestrial cyclones, whose thickness is very small compared with their diameter. In other words, h must be very small compared with ρ_2. As to the size of ρ_1 compared with h and ρ_2, we can make a probable assumption by starting from the fact observed by Hale, namely, that the magnetic effect of the vortex decreases very rapidly upward along the axis of the vortex, so that there is only a very small fraction of its original value when we come to the level of 5000 miles above the photosphere.

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[1] See, for instance, Appell, *Traité de Mécanique Rationnelle*, **3**, chap. xxxv.

In order to do this, we will start from the formula

$$H_{z0} = -2\pi \cos\omega \, ln \, \frac{(z-h+\sqrt{(z-h)^2+\rho_2^2})(z+\sqrt{z^2+\rho_1^2})}{(z-h+\sqrt{(z-h)^2+\rho_1^2})(z+\sqrt{z^2+\rho_2^2})} \, .$$

This gives

$$\frac{dH_{z0}}{dz} = 2\pi c \cos\omega \left\{ \frac{1}{\sqrt{z^2+\rho_2^2}} - \frac{1}{\sqrt{(z-h)^2+\rho_2^2}} - \frac{1}{\sqrt{z^2+\rho_1^2}} + \frac{1}{\sqrt{(z-h)^2+\rho_1^2}} \right\}$$

The geometrical signification of the radicals will be seen in Fig. 13.

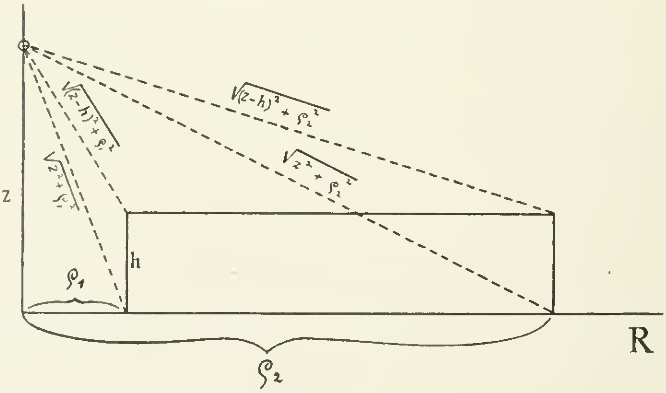


FIG. 13

On studying this figure, it will be seen that if we choose h small compared with the elevation of the high levels of the sun's atmosphere, and, on the other hand, suppose ρ_1 to be of the same size as, or smaller than, h, the magnetic force at these high levels will be only a very small fraction of what it is at the level $z=h$.

We will choose as a numerical example

$$h=\rho_1=500\text{ km}=5\times10^7\text{ cm}\,,\quad \rho_2=50{,}000\text{ km}=5\times10^9\text{ cm}\,.$$

The last value is chosen to correspond with the size of the observed hydrogen whirls around sun-spots.

Further, we take $\omega=20°$. With regard to the constant c, we will start from the value of about 3000 gausses observed in sun-spots by the Zeeman effect, and suppose that this is the value of H_{z0} for $z=h$. We then get $c=-583.1$. Starting from these values we find the following values of H_{z0} along the Z-axis:

z in km	H_{z0} in Gausses	z in km	H_{z0} in Gausses
500	3000	10,000	142.6
1000	1901	20,000	55·33
2000	916.7	30,000	28.46
3000	583.2	40,000	16.48
5000	318.4	50,000	10.62

A probable system of values for h, ρ_1, ρ_2, ω, and c being found, it is of the greatest interest to compute the magnetic field above the spot. We have done this for each point encircled in Fig. 14.

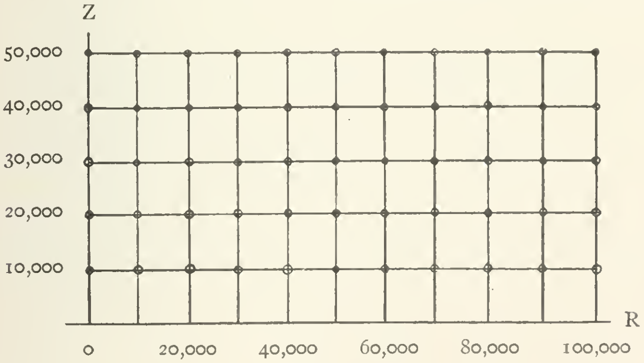


FIG. 14

We have used the formula given in Section VI for the case in which h is small compared with ρ_2. In the following paragraphs we will give these computations in detail.

VIII. COMPUTATION OF THE COMPONENTS H_z AND H_R

We have

$$H_z=4hc\cos\omega\left[\frac{T_2}{\lambda_2}-\frac{T_1}{\lambda_1}\right]$$

where the subscripts 1 and 2 denote the values for $\rho=\rho_1$ and $\rho=\rho_2$ of the functions

$$\lambda=\sqrt{(R+\rho)^2+z^2}\,, \quad T=\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}\,,$$

where

$$k=\sqrt{\frac{4R\rho}{(R+\rho)^2+z^2}}$$

and further

$$h=5\times10^7 \qquad \omega=20^\circ$$
$$\rho_1=5\times10^7 \qquad c=-583.1.$$
$$\rho_2=5\times10^9$$

By means of Legendre's well-known tables, we find the results given in Table II.

TABLE II

VALUES OF H_z

z in Km	R in Km									
	10.000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000....	9.47	7.89	5.84	3.84	+2.22	+1.09	+0.39	+0.01	−0.17	−0.25
40,000....	14.98	11.74	8.01	4.64	+2.18	+0.68	−0.08	−0.39	−0.48	−0.38
30,000....	24.86	18.09	10.94	5.22	+1.51	−0.31	−0.92	−0.99	−0.88	−0.73
20,000....	44.84	28.27	14.49	4.85	−0.65	−2.34	−2.21	−1.73	−1.31	−0.99
10,000....	87.65	42.00	17.72	2.33	−6.58	−5.95	−3.76	−2.43	−1.66	−1.18

The calculation of the other component, H_R, has been much more arduous. We have used Simpson's formula for the approximate computation of the definite integrals:

$$\int_a^b f(x)\,dx=\frac{h}{3}\,[y_0+4y_1+2y_2+4y_3+2y_4+\;\ldots\;+2y_{n-2}+4y_{n-1}+y_n]\,,$$

where the interval of integration $b-a$ is divided into n parts, each of them equal to h, and where $y_0, y_1, y_2 \ldots\ldots y_{n-1}, y_n$ are the values of $f(x)$ for the points of division. Now

$$H_R=-2chz\cos\omega\,[Y_2-Y_1]$$

where Y_2 and Y_1 are the values for $\rho=\rho_2$ and $\rho=\rho_1$ of the integral

$$Y=\int_0^\pi \frac{(\rho-R\cos\theta)\cos\theta}{(R^2\sin^2\theta+z^2)\,D}\,d\theta\,,$$

and

$$D^2=R^2-2R\rho\cos\theta+\rho^2+z^2\,.$$

Here the interval from 0 to π has been divided into 12 parts, which has involved the calculation of the function under the sign of integration for each combination of the values mentioned above of R, z, and ρ, with the values

$$\theta=0^\circ,\ 15^\circ,\ 30^\circ,\ 45^\circ,\ \ldots\ldots\ 150^\circ,\ 165^\circ,\ 180^\circ\,.$$

As the value of H_R is not affected if we divide each of the quantities R, Z, ρ_1, ρ_2, and h by the same number, that is, if we choose another unit of length, we have chosen h as this unit. The values of the quantities will then be

$$h=1\,,\quad \rho_1=1\,,\quad \rho_2=100$$

$$R=20,\ 40,\ 60,\ 80,\ 100,\ 120,\ 140,\ 160,\ 180,\ 200$$

$$Z=20,\ 40,\ 60,\ 80,\ 100.$$

The calculation has been made with the "Brunswiga" machine. The results for the function

$$\frac{(\rho--R\cos\theta)\cos\theta}{(R^2\sin^2\theta+z^2)\,D}$$

were integrated by Simpson's formula. Multiplication by the constant $-2chz\cos\omega$ gave finally the results shown in Table III.

TABLE III

VALUES OF H_R

z in Km	R in Km									
	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000....	2.12	3.69	4.44	4.44	3.95	3.20	2.47	1.86	1.37	0.73
40,000....	3.82	6.37	7.24	6.79	5.61	4.20	2.99	2.08	1.45	1.02
30,000....	7.50	11.65	12.25	10.57	7.93	5.30	3.36	2.15	1.40	0.91
20,000....	17.04	22.71	21.17	16.58	10.92	6.12	3.33	1.90	1.15	0.73
10,000....	49.68	45.86	35.56	25.48	13.83	5.30	2.28	1.16	0.66	0.42

By the formula

$$H_\phi = -H_R \tan \omega ,$$

where

$$\tan \omega = \tan 20^\circ = 0.36397,$$

we find the tabulated values of H_ϕ (Table IV).

TABLE IV

VALUES OF $H\phi$

z in Km	R in Km									
	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000 .	− 0.77	− 1.34	− 1.62	−1.62	−1.44	−1.17	−0.90	−0.68	−0.50	−0.27
40,000 .	− 1.39	− 2.32	− 2.64	−2.47	−2.04	−1.53	−1.09	−1.76	−0.53	−0.37
30,000 .	− 2.73	− 4.24	− 4.46	−3.85	−2.89	−1.93	−1.22	−0.78	−0.51	−0.33
20,000 .	− 6.20	− 8.27	− 7.70	−6.03	−3.97	−2.23	−1.21	−0.69	−0.42	−0.27
10,000 .	−18.08	−16.69	−12.94	−9.27	−5.03	−1.93	−0.83	−0.42	−0.24	−0.15

By means of these values we can calculate the magnetic force

$$H = \sqrt{H_R^2 + H_z^2 + H_\phi^2} .$$

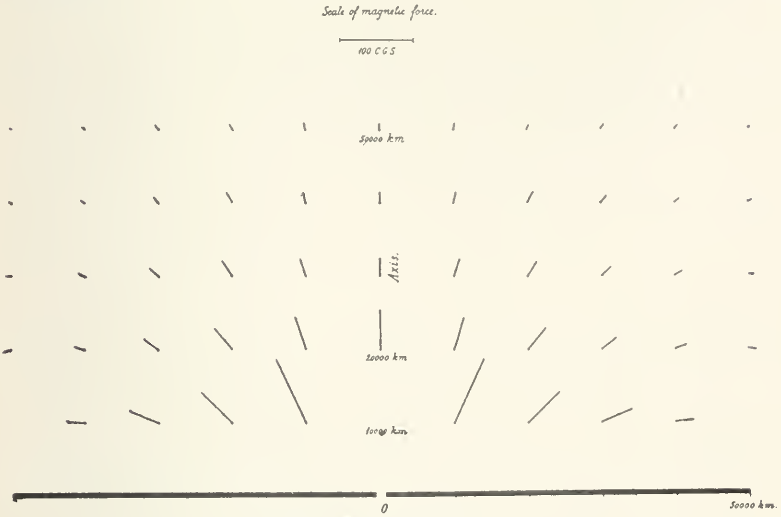
TABLE V

VALUES OF H

z in Km	R in Km									
	10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000	100,000
50,000....	9.74	8.80	7.51	6.09	4.75	3.58	2.66	1.98	1.47	0.81
40,000....	15.52	13.56	11.11	8.59	6.36	4.52	3.18	2.25	1.61	1.15
30,000....	26.10	21.93	17.02	12.40	8.57	5.65	3.69	2.49	1.73	1.21
20,000....	48.37	37.20	26.79	18.30	11.64	6.92	4.18	2.66	1.79	1.26
10,000....	102.4	64.39	41.78	27.21	16.13	8.20	4.48	2.73	1.81	1.26

The component in the meridian plane through the axis of the whirl has the components H_R and H_z and is everywhere tangent to the curves Φ = constant, where Φ is the function given by equation (12).

The component in the meridian plane for some of the points (R,z) in the tables mentioned above is shown in Fig. 15. It will be seen that the magnetic force decreases rapidly upward from the whirl.



Magnetic field of a solar vortex.

FIG. 15

IX. CALCULATION AND CONSTRUCTION OF THE LINES OF FORCE

We have seen that the lines of force in space are the curves of intersection between cylinders through the logarithmic spirals

$$R=Be^{-\phi \cot \omega}$$

and the surfaces of revolution, whose meridian curves are

$$\Phi=C\ .$$

In the case in question we have

$$\Phi=2\ chR\ [f(\rho_2,\ 0)-f(\rho_1,\ 0)]$$

where

$$f(\rho,0)=\int_0^\pi ln\ (\rho-R\cos\theta+D)\cos\theta\ d\theta,$$

$$D^2=R^2-2R\rho\cos\theta+\rho^2+z^2\ .$$

In order to draw the lines $\Phi=$constant, the following method has been used: For the computation of the integral by Simpson's formula, we have first computed the function

$$\log(\rho-R\cos\theta+D)\cos\theta,$$

where the logarithm is to the base 10 and where ρ, R, z have the values given before, with h as unit of length.

If we denote by

$$y_0,\ y_1,\ y_2,\ \ldots\ldots\ y_{12}$$

the values of this function for

$$\theta=0,\ 15^\circ,\ 30^\circ,\ 45^\circ,\ \ldots\ldots\ 165^\circ \text{ and } 180^\circ$$

and S_1 and S_2 the values of the expression

$$y_0+4y_1+2y_2+4y_3+\ \ldots\ldots\ +4y_{11}+y_{12}$$

for $\rho=\rho_1$ and $\rho=\rho_2$, we then have approximately

$$\frac{\pi}{36}S_1=\int_0^\pi \log(\rho_1-R\cos\theta+D_1)\cos\theta\,d\theta$$

$$\frac{\pi}{36}S_2=\int_0^\pi \log(\rho_1-R\cos\theta+D_2)\cos\theta\,d\theta$$

where D_1 and D_2 are the values of D for $\rho=\rho_1$ and $\rho=\rho_2$. As

$$\log A=\log e\cdot \ln A=0.43429\,\ln A$$

where e is the base of the hyperbolic logarithms, we get approximately

$$\Phi=\frac{\pi ch}{18\log e}\cdot R\,(S_2-S_1)\,.$$

Thus the function $R(S_2-S_1)$ is a function of R and of z, which differs from the function Φ only by a constant factor and this even if we express R, ε, ρ_1, ρ_2, and h with h as unit of length.

Thus the curves $\Phi=$constant are identical with the curves $\Phi_1=$constant, where $\Phi_1=R(S_2-S_1)$, if Simpson's formula is assumed to be accurate enough for our purpose. From this we have then drawn the curves with R as abscissa and Φ_1 ordinate, and where z has a constant value for each curve.

The points (R_{01}, z_0) and (R_{02}, z_0) where Φ has a given value, Φ_0, can then be found graphically by the intersection of the curve and the line $\Phi=\Phi_0$ (Fig. 16). Similar curves have been drawn with z as abscissa and Φ as ordinate, for controlling the results. This has finally given enough points for the construction of the curves $\Phi_1=$ constant.

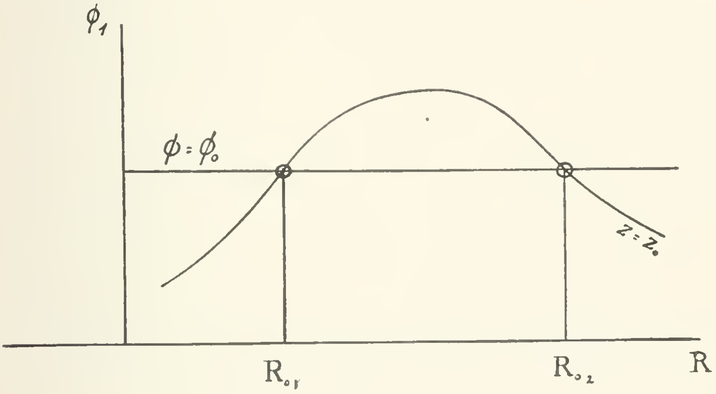


Fig. 16

In order to control the results, the function Φ_1 has also been calculated for $z=0$, in which case the integrals can be reduced to elliptic integrals of the first and second kind by a partial integration.

The curves $\Phi_1=$ constant are seen in Fig. 17, and the directions of the components of magnetic force in the meridian plane are also given. It will be seen that the agreement is fairly good, in spite of the approximate method of computation and construction.

With the aid of the curves $\Phi_1=$ constant and the logarithmic spirals

$$R=Be^{-\phi \cot \omega},$$

where B is a constant and $\omega=\pm 20°$, I have constructed wire models of the lines of magnetic force in space over the whirl. The dimensions are the same as for the case already mentioned.

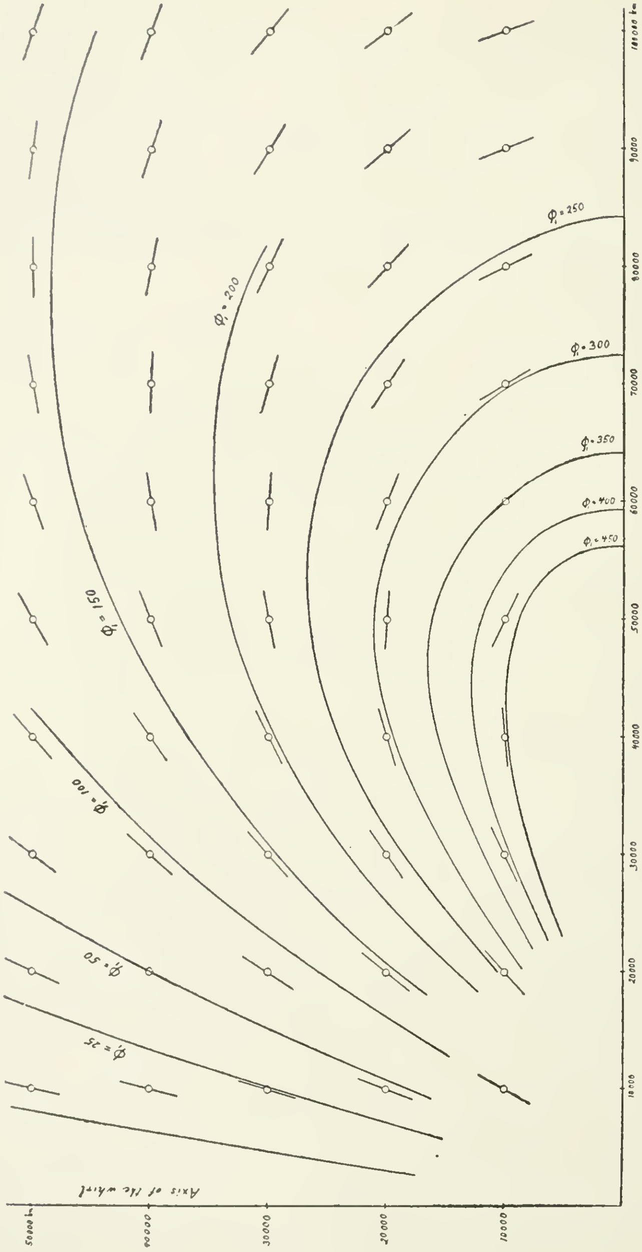
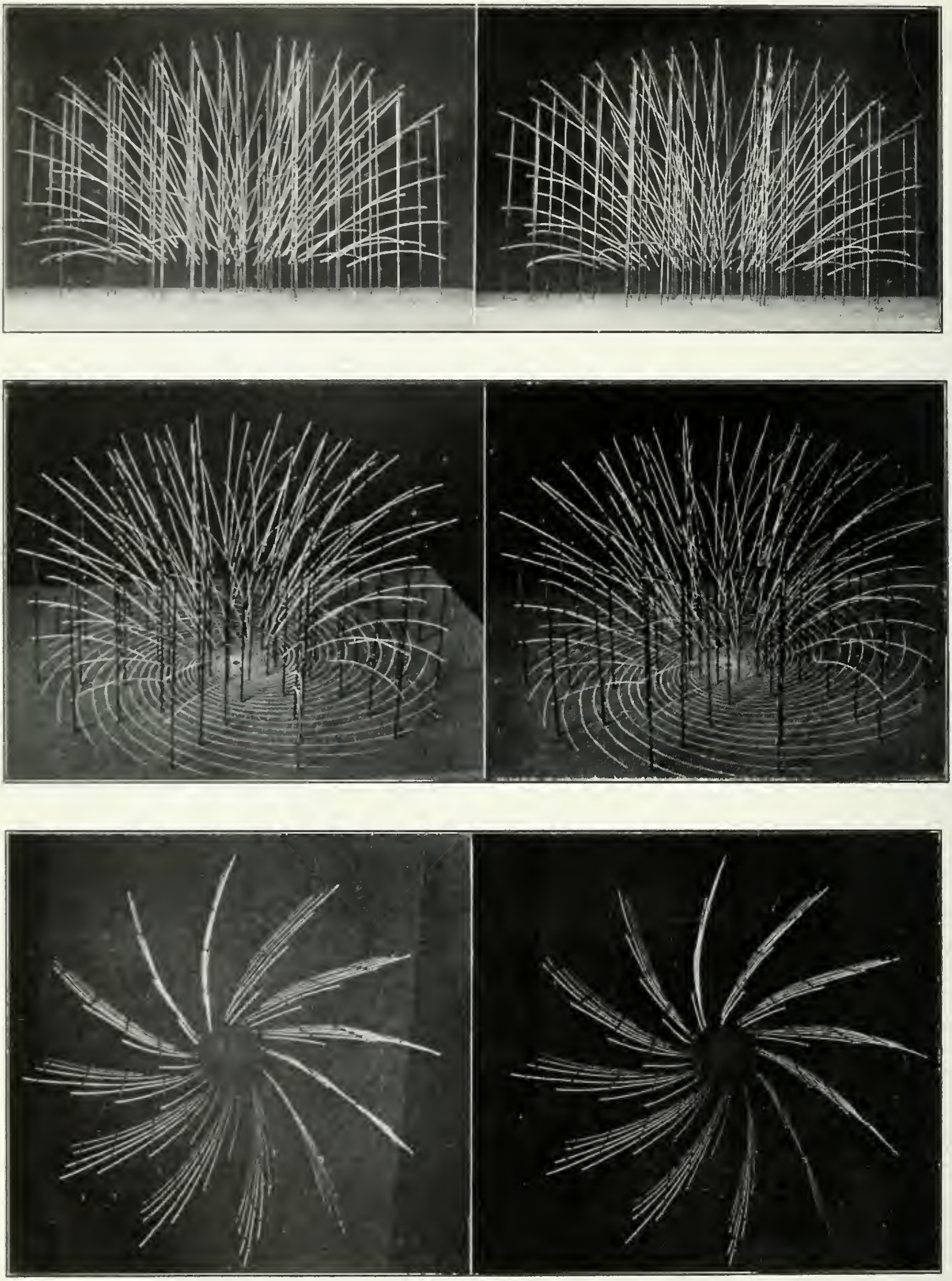

Φ₁ = 25
Φ₁ = 50
Φ₁ = 100
Φ₁ = 150
Φ₁ = 200
Φ₁ = 250
Φ₁ = 300
Φ₁ = 350
Φ₁ = 400
Φ₁ = 450
Axis of the whirl
10000
20000
30000
40000
50000 km
10000
20000
30000
40000
50000
60000
70000
80000
90000
100000 km

FIG. 17

PLATE VII



Stereoscopic Views of Model Showing Distribution of Lines of Magnetic Force for $\omega = -20^\circ$
Current Lines in Lower Levels of Whirl Are Traced on the Base of the Model

In Plate VII are seen some stereoscopic representations of these models for the case $\omega = -20°$, viewed from various points. On the base are drawn the current-lines, and the lines of force in space are supported by dark pins, standing perpendicular to the plane of the whirl.

If we compare the curves Φ_1=constant with the well-known spectroheliograph pictures of calcium prominences around a sun-spot at the sun's limb, shown by Professor Slocum in his paper[1] "Attractions of Sun-Spots for Prominences," a very striking resemblance appears. It seems as if these prominences are arranged along lines of magnetic force from a whirl round the sun-spot.

X. THE MAGNETIC FIELD ROUND A BIPOLAR SPOT-GROUP

Let us now consider two whirls and their combined magnetic field of force. For simplicity we will suppose that each of the whirls is of the kind just mentioned, and that they are lying in the same plane with parallel axes perpendicular to the plane. The magnetic resultant force in each point of space over the whirls can then be found by the parallelogram law, because we know the force due to each of the whirls considered separately.

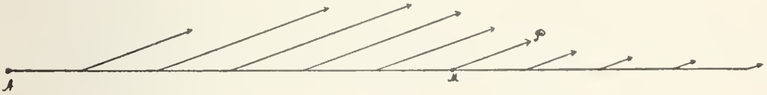


FIG. 18

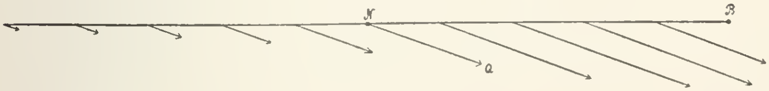


FIG. 19

The graphical method used for finding the resultant components at a given level above the plane of the whirls is as follows: on a sheet of tracing paper are marked the components of the magnetic force due to the whirl A (Fig. 18), and on another sheet the components due to the whirl B (Fig. 19), both components for points along a radius vector in the selected level $z=$ constant. The two sheets are then placed with the points A and B coinciding with the centers of the whirls (Fig. 20), and turned to produce coincidence of the points M and N on the sheets. On a third underlying sheet of tracing paper the points M and R are marked by pricking with a pin, and the resultant force MR is then constructed.

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[1] *Astrophysical Journal*, 36, 265, 1912.

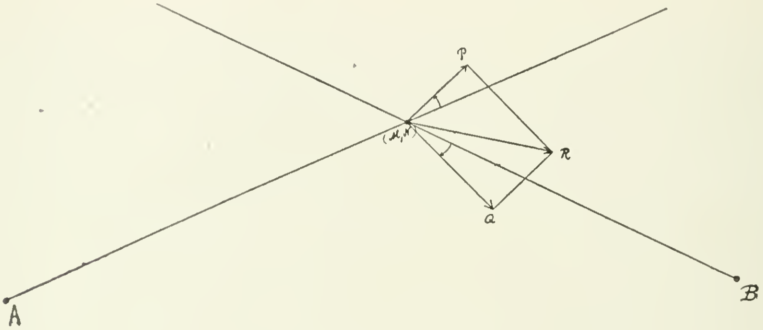

FIG. 20

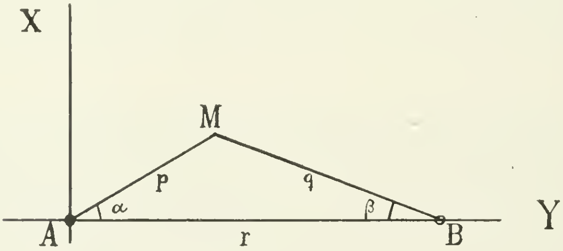

FIG. 21

With regard to the calculation of the resultant force the necessary formula may be developed as follows: Let us consider a given plane parallel to the plane of the whirls, and let A and B be the points where the axes of the whirls intersect this plane. Let r be their mutual distance. Let M be a point in the plane and H_R, H_ϕ, and H_z the components at M due to the whirl with axis A, and K_R, K_ϕ, and K_z the components due to the whirl B. Let us finally denote the distances AM and BM by p and g and the angles MAB and MBA by α and β. Then

$$\cos\alpha=\frac{p^2+r^2-g^2}{2pr}\,,$$

$$\cos\beta=\frac{g^2+r^2-p^2}{2gr}\,.$$

If we choose a system of rectangular co-ordinates with the origin in the center of the whirl A, and the X-, Y-, and Z-axes parallel to AX (perpendicular to AB, see Fig. 21), to AY, and to the axis of the whirl, and if we denote by P_x, P_y, and P_z the components of the resulting magnetic force at the point M, parallel with the X-, Y-, and Z-axes, we find

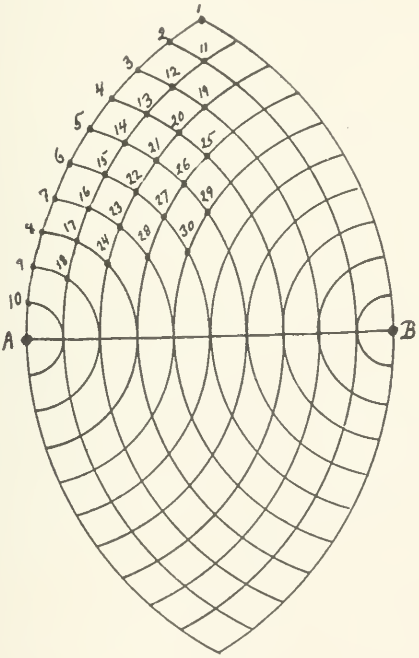

Fig. 22

$$P_x=H_R\sin\alpha-H_\phi\cos\alpha+K_R\sin\beta+K_\phi\cos\beta\,,$$
$$P_y=H_R\cos\alpha+H_\phi\sin\alpha-K_R\cos\beta+K_\phi\sin\beta\,,$$
$$P_z=H_z+K_z\,.$$

For the practical computation of P_x, P_y, and P_z, sin α, cos α, sin β, and cos β can first be calculated. If, for instance, $r = 10{,}000$ km, the distances p and g have the values:

10,000, 20,000, ... 90,000 and 100,000 km,

and we obtain the values in the following tables corresponding to the numbered points in Fig. 22. The values of these functions will be useful for the calculation of P_x, P_y, and P_z in special cases.

TABLE VI

$g = 100{,}000$

	p									
	100,000	90,000	80,000	70,000	60,000	50,000	40,000	30,000	20,000	10,000
sin α	0.8660	0.8930	0.9165	0.9367	0.9539	0.9682	0.9798	0.9887	0.9950	0.9987
cos α......	0.5000	0.4500	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500
sin β......	0.8660	0.8037	0.7332	0.6557	0.5723	0.4841	0.3919	0.2965	0.1990	0.0998
cos β......	0.5000	0.5950	0.6800	0.7550	0.8200	0.8750	0.9200	0.9550	0.9600	0.9950
Point no...	1	2	3	4	5	6	7	8	9	10

TABLE VII

$g = 90{,}000$

	p							
	90,000	80,000	70,000	60,000	50,000	40,000	30,000	20,000
sin α..........	0.8315	0.8549	0.8741	0.8888	0.8980	0.8992	0.8843	0.8181
cos α..........	0.5556	0.5187	0.4857	0.4583	0.4400	0.4375	0.4668	0.5750
sin β..........	0.8315	0.7599	0.6798	0.5924	0.4988	0.3995	0.2947	0.1814
cos β..........	0.5556	0.6501	0.7334	0.8056	0.8667	0.9167	0.9556	0.9834
Point no........	11	12	13	14	15	16	17	18

TABLE VIII

$g = 80{,}000$

	p					
	80,000	70,000	60,000	50,000	40,000	30,000
sin α....................	0.7806	0.7945	0.8000	0.7924	0.7599	0.6613
cos α....................	0.6250	0.6072	0.6000	0.6100	0.6500	0.7501
sin β....................	0.7806	0.6953	0.6000	0.4953	0.3799	0.2480
cos β....................	0.6250	0.7187	0.8000	0.8687	0.9250	0.9687
Point no................	19	20	21	22	23	24

TABLE IX

$g = 70{,}000$

	p			
	70,000	60,000	50,000	40,000
sin α	0.6998	0.6887	0.6500	0.5464
cos α	0.7143	0.7250	0.7600	0.8375
sin β	0.6998	0.5903	0.4642	0.3122
cos β	0.7143	0.8072	0.8857	0.9500
Point no.	25	26	27	28

TABLE X

$g = 60{,}000$

	p	
	60,000	50,000
sin α	0.5528	0.4560
cos α	0.8333	0.8900
sin β	0.5528	0.3799
cos β	0.8333	0.9250
Point no.	29	30

XI. SOME SIMPLE CASES OF BIPOLAR SPOT-GROUPS

There are two interesting cases which we will study in greater detail, namely, the case of two identical whirls, and the case of two equal and opposite whirls. In both cases there is a certain symmetry in the arrangement of the lines of force, and their projection on the plane of the whirls.

First case: two identical whirls.—Let us consider the components P_x and P_y and their resultant P_h in a plane parallel to the plane of the whirls. Let C (Fig. 23) be the middle point of the line AB joining points A and B where the axes of the whirls cut the plane. Let M and M' be two points in the plane situated symmetrically with regard to C, and let us denote by accents the letters corresponding to the point M'. We then have

$$\alpha' = -\beta \quad , \qquad \beta' = -\alpha$$
$$H'_R = \quad K_R \, , \qquad H_\phi = K_\phi$$
$$K'_R = \quad H_R \, , \qquad K'_\phi = H_\phi$$

which gives

$$P'_x = -H_R \sin\alpha + H_\phi \cos\alpha - K_R \sin\beta - K_\phi \cos\beta = -P_x$$
$$P'_y = -H_R \cos\alpha - H_\phi \sin\alpha + K_R \cos\beta - K_\phi \sin\beta = -P_y\,,$$

i.e., the *component* P'_h *in the point* M' *is obtained by turning the component* P_h *in the point* M *through an angle of 180° around C as center.*

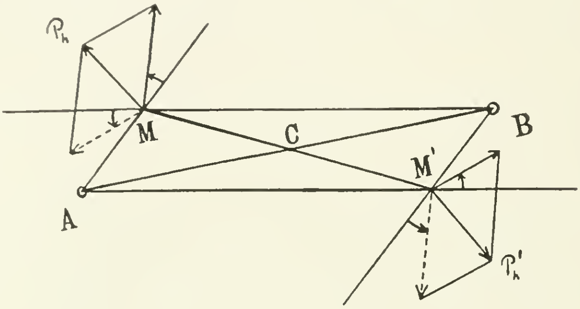

FIG. 23

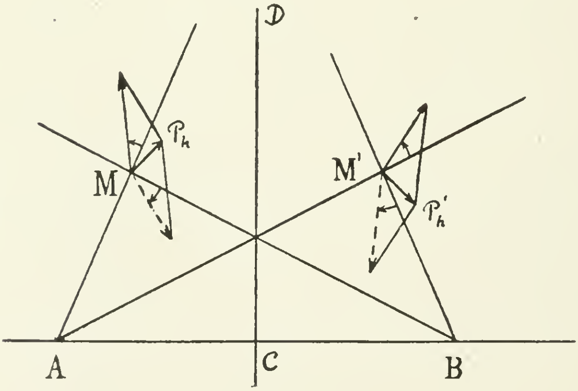

FIG. 24

In Fig. 25 are seen the directions of the components P_h in a plane, corresponding to the level 20,000 km over the two whirls in the sun's atmosphere. For the two identical whirls we have chosen case No. 5 of Fig. 8, with

$$h = 500 \text{ km}\,, \qquad \omega = 20°$$
$$\rho_1 = 500 \text{ km}\,, \qquad c = -581.3$$
$$\rho_2 = 50{,}000 \text{ km}\,, \qquad r = 100{,}000 \text{ km}.$$

Second case: two whirls corresponding to cases Nos. 5 and 8 in Fig. 8.—For the whirl A we have ε negative and ω and v positive, and for the whirl B,

$$\varepsilon' = \varepsilon\,, \qquad \omega' = -\omega\,, \qquad v' = -v\,,$$

consequently

$$c' = \rho\varepsilon' v' = -c\,.$$

Further, for both whirls h, ρ_1, and ρ_2 are supposed to be the same.

Let us consider (Fig. 24) as before a given level above the plane of the two whirls, and let CD be the perpendicular through the central point C of the line AB. Let M and M' be two points situated symmetrically with respect to this line, and let us denote by accents the letters corresponding to the point M'.

We have

$$\alpha' = \beta\,, \qquad \beta' = \alpha\,.$$

Further by formula (11), H_R positive, H_ϕ negative and

$$\begin{aligned} H'_R &= -K_R\,, & K'_R &= -H_R \\ H'_\phi &= K_\phi\,, & K'_\phi &= H_\phi \end{aligned}$$

we find

$$\begin{aligned} P' &= +H'_R \sin \alpha' - H'_\phi \cos \alpha' + K'_R \sin \beta' + K'_\phi \cos \beta' \\ &= -H_R \sin \alpha + H_\phi \cos \alpha - K_R \sin \beta - K_\phi \cos \beta = -P_x \end{aligned}$$

and

$$\begin{aligned} P'_y &= H'_R \cos \alpha' + H'_\phi \sin \alpha' - K'_R \cos \beta' + K'_\phi \sin \beta' \\ &= H_R \cos \alpha + H_\phi \sin \alpha - K_R \cos \beta + K_\phi \sin \beta = P_y\,. \end{aligned}$$

Hence the straight lines passing through M and M' in the direction of the force lie symmetrically with respect to the line CD.

If M is on the line CD, we have

$$H_R = -K_R\,, \qquad H_\phi = K_\phi\,, \qquad \alpha = \beta$$

and

$$P_x = 0\,, \quad P_y = 2R \cos \alpha + 2H_\phi \sin \alpha\,,$$

i.e., the component P_h is parallel to the line AB.

In Fig. 26 are seen the directions of the component P_h in a plane corresponding to a level of 20,000 km, in the sun's

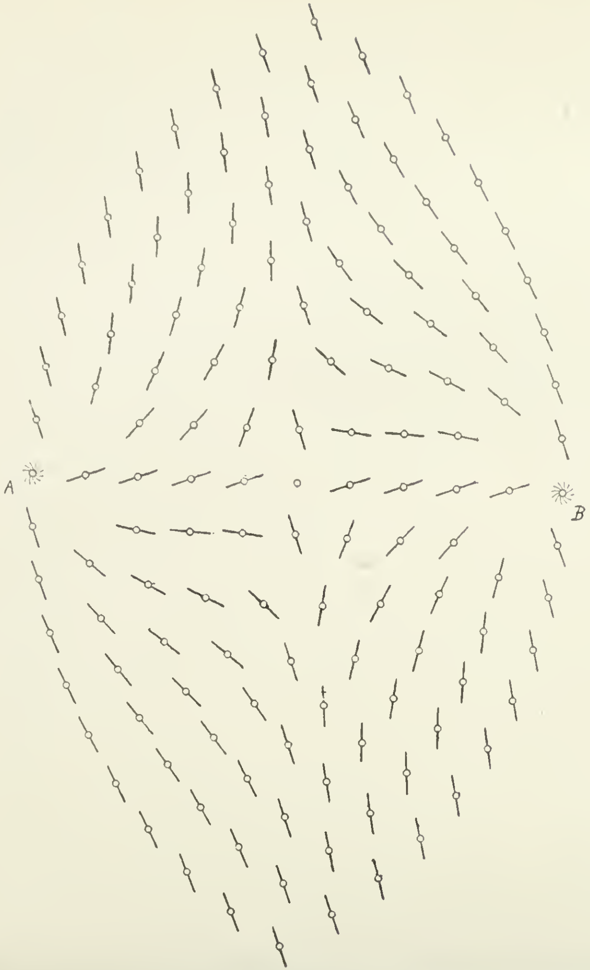

FIG. 25

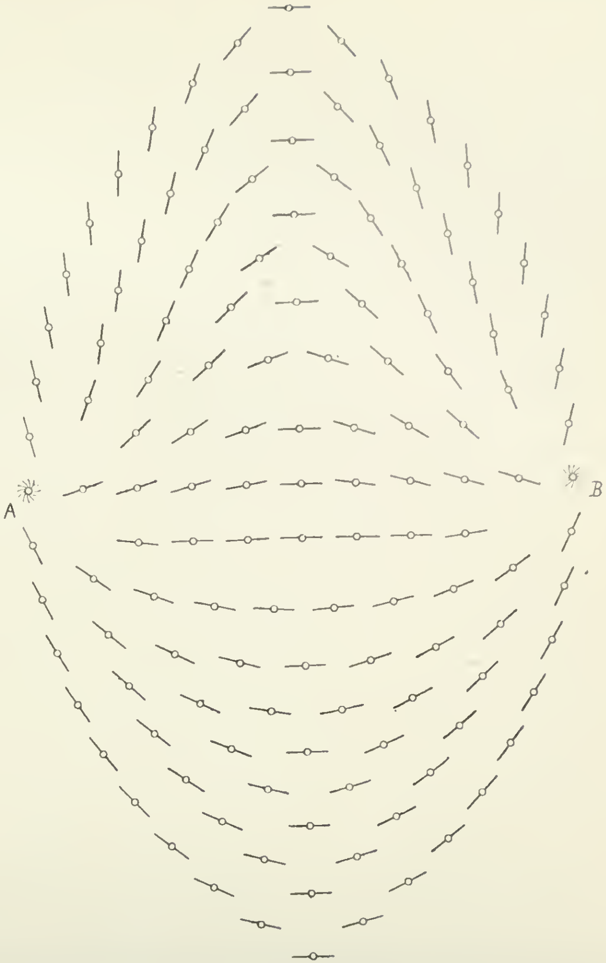

FIG. 26

atmosphere, where the hydrogen flocculi are probably to be found. The dimensions and the distance of the whirls are chosen as in the first case.

It is naturally of considerable interest to compare this with spectroheliograms of bipolar sun-spot groups. This will be done later, when more material has been obtained.

XII. THE ELECTRIC CHARGE OF THE SOLAR VORTEX AND THE ELECTROSTATIC FIELD OVER IT. DIFFICULTIES IN EXPLAINING THE ABSENCE OF THE STARK EFFECT. HYPOTHESIS OF GALVANIC CURRENTS

We have seen that the charge of the low-level whirl probably causing the Zeeman effect is negative. The value of the constant c will give us the means of determining this charge and of calculating the electrostatic field in space over the whirl. We had, by definition (see Section V),

$$c=\frac{\rho\varepsilon v}{3\times 10^{10}},$$

where ε is the charge of electricity in 1 cubic cm of the whirl, measured in electrostatic units, and ρ and v two corresponding values of the distance from the axis and the velocity of a gas-particle of the whirl, measured in the c.g.s. system of units. ε was supposed constant throughout the whirl.

If we measure directly the velocity at a given distance from the axis of the whirl, we can compute ρv, and c being known from the Zeeman effect, we can find ε.

Let us make an estimate of the probable value of the charge ε. As stated in his paper "Radial Motions in Sun-Spots,"[1] St. John has found, at the level Fe, 00 and at a distance of 11,250 km from the axis, an *outward* motion of about 1 km per second. If we assume the current-line to be a logarithmic spiral corresponding to $\omega=20°$, the velocity will then be

$$v=\frac{10^5}{\sin 20°}\text{ cm},$$

---

[1] *Mt. Wilson Contr.*, No. 69; *Astrophysical Journal*, 37, 322, 1913.

i.e., about 3×10^5 cm. If we choose the value of c determined in Section VII, namely, $c = -583.1$ we find $\varepsilon = -0.052$. From this we can compute the number N of corpuscles to a cubic cm of the gas, as we know the elementary charge of a negative corpuscle, which is[1] -4.65×10^{-10}. We then find $N = 1.1 \times 10^8$.

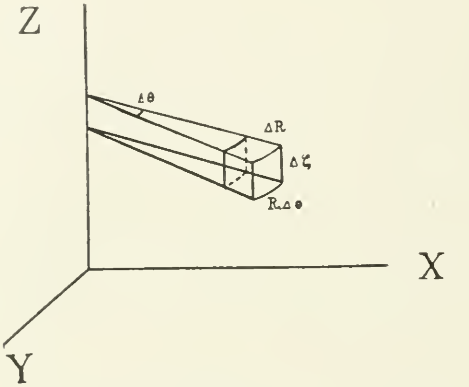

FIG. 27

Let us now compute the electrostatic field above the whirl. Consider an element of the whirl bounded by the planes (Fig. 27)

$$z = \zeta, \qquad z = \zeta + \Delta\zeta,$$

the planes

$$\phi = \theta, \qquad \phi = \theta + \Delta\theta,$$

and the cylinders

$$R = \rho, \qquad R = \rho + \Delta\rho.$$

The volume of this element is equal to $\rho\Delta\rho\Delta\theta\Delta\zeta$ and its charge is $\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta$.

Let δE_x, δE_y, and δE_z be the components parallel to the axes OX, OY, and OZ of the electrostatic action of this element on one electrostatic unit of positive electricity in the point $(x,0,z)$ of the XZ-plane. Then

---

[1] See Rutherford, *Radioactive Substances and Their Radiations*, § 20, Cambridge, 1913.

$$\delta E_x = \frac{x-\rho\cos\theta}{r_1^3}\,\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta\,,$$

$$\delta E_y = \frac{-\rho\sin\theta}{r_1^3}\,\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta\,,$$

$$\delta E_z = \frac{z-\zeta}{r_1^3}\,\varepsilon\rho\Delta\rho\Delta\theta\Delta\zeta\,.$$

The action of an element lying symmetrically with respect to the XZ-plane is obtained by changing the sign of θ. Here r_1 is the distance from the point $(x,0,z)$ to the element.

The components of the resultant action of the two elements will be

$$2\delta E_x\,,\qquad 0\,,\qquad 2\delta E_z\,.$$

If we suppose the whirl to be of the same shape as in Section II, the components of its action on an arbitrary point (x,y,z) will be

$$\left.\begin{aligned} E_R &= 2\varepsilon\int_{\rho_1}^{\rho_2}\int_0^h\int_0^\pi \frac{(R-\rho\cos\theta)\rho d\rho d\zeta d\theta}{D^3} \\ E_z &= 2\varepsilon\int_{\rho_1}^{\rho_2}\int_0^h\int_0^\pi \frac{(z-\zeta)\rho d\rho d\zeta d\theta}{D^3} \end{aligned}\right\}\qquad(14)$$

Here E_R and E_z are the components normal to and parallel to the Z-axis, lying in the meridian plane and with the same orientation as the components H_R and H_z derived in Section II; further.

$$D^2 = R^2+\rho^2-2R\rho\cos\theta+(z-\zeta)^2\,.$$

If we introduce the potential

$$W = 2\varepsilon\int_{\rho_1}^{\rho_2}\int_0^h\int_0^\pi \frac{\rho d\rho d\zeta d\theta}{D}\qquad(15)$$

we can write

$$E_R = -\frac{\delta W}{\delta R}\,,\qquad E_z = -\frac{\delta W}{\delta z}\,.$$

The components of action of the whirl on an electric charge of ε' electrostatic units in the point (x,y,z) are consequently $\varepsilon' E_R$ and $\varepsilon' E_z$.

If the thickness h of the whirl is very small compared with its radius ρ_2, we can use the formula corresponding to h infinitesimal, namely,

$$W = 2\varepsilon h \int_{\rho_1}^{\rho_2} \int_0^{\pi} \frac{\rho d\rho d\theta}{D_0}$$

with

$$E_R = -\frac{\delta W}{\delta R}, \qquad E_z = -\frac{\delta W}{\delta z},$$

where

$$D_0^2 = R^2 + \rho^2 - 2R\rho \cos\theta + z^2.$$

The expression for W can easily be transformed into another form that is better adapted to numerical computation. We have

$$\int \frac{\rho d\rho}{D_0} = D_0 + R\cos\theta \, ln(\rho - R\cos\theta + D_0) + \text{constant},$$

which gives

$$\left.\begin{aligned} & W = W(\rho_2) - W(\rho_1), \\ & \text{where} \\ & W(\rho) = 2\varepsilon h \int_0^{\pi} D_0 d\theta + 2\varepsilon h R \int_0^{\pi} \cos\theta \, ln(\rho - R\cos\theta + D_0) d\theta \end{aligned}\right\} \quad (16)$$

Here the second integral is the same as in formula (12) for the function Φ, and has been denoted by $f(\rho,0)$. The first integral can be expressed by elliptic functions by the substitution quoted in Section VI, which gives

$$\int_0^{\pi} D_0 d\theta = 2\lambda \int_0^{\frac{\pi}{2}} \sqrt{1 - k^2 \sin^2\psi} \, d\psi$$

λ and k having the same definition as before. This integral can then be calculated by Legendre's tables.

Let us now consider the components E_R and E_z. We must first find the partial derivatives of $W(\rho)$ with respect to R and to z.

We have

$$\frac{\delta W(\rho)}{\delta R}=2\varepsilon h\int_0^\pi\frac{R-\rho\cos\theta}{D_0}\,d\theta+2\varepsilon h\,\frac{\delta}{\delta R}\left[R\int_0^\pi\cos\theta\; ln(\rho-R\cos\theta+D_0)d\theta\right].$$

But the partial derivative in the right member becomes

$$\int_0^\pi\cos\theta\; ln(\rho-R\cos\theta+D_0)d\theta+R\int_0^\pi\frac{R\cos\theta-(\rho+D_0)\cos^2\theta}{D_0(\rho-R\cos\theta+D_0)}\,d\theta\,.$$

Integrated by parts, the first term of this becomes

$$-\int_0^\pi\frac{(R\rho+RD_0)\sin^2\theta}{D_0(\rho-R\cos\theta+D_0)}\,d\theta$$

which, combined with the second term, gives for the partial derivative

$$-R\int_0^\pi\frac{d\theta}{D_0}.$$

Consequently,

$$\frac{\delta W(\rho)}{\delta R}=-2\varepsilon h\rho\int_0^\pi\frac{\cos\theta\,d\theta}{D_0}\,.$$

If we use the transformation of Section VI, we further obtain this integral expressed in terms of elliptic functions in the form

$$\left.\begin{aligned}&E_R=2\varepsilon h[T(\rho_2)-T(\rho_1)]\,,\\ &\text{where}\\ &T(\rho)=\frac{2\rho}{\lambda k^2}\left[(2-k^2)\int_0^{\frac{\pi}{2}}\frac{d\psi}{\sqrt{1-k^2\sin^2\psi}}-2\int_0^{\frac{\pi}{2}}\sqrt{1-k^2\sin^2\psi}\;d\psi\right]\end{aligned}\right\}\quad(17)$$

Finally we have to determine E_z. By partial differentiation

$$\frac{\delta W(\rho)}{\delta z}=2\varepsilon hz\int_0^\pi\frac{d\theta}{D_0}+2\varepsilon hRz\int^\pi\frac{\cos\theta\,d\theta}{(\rho-R\cos\theta+D_0)D_0}\,.$$

Here we have the integrals which in Section VI we have denoted by $f'(\rho,0)$ and $g'(\rho,0)$. We find

$$E_z = -2\varepsilon h z[g'(\rho_2, 0) - g'(\rho_1, 0)] + 2\varepsilon h R[f'(\rho_2, 0) - f'(\rho_1, 0)] \tag{18}$$

By formulae (16), (17), and (18), the functions W, E_R, and E_z can easily be computed.

Along the axis of the whirl,

$$\left.\begin{aligned} W &= 2\pi\varepsilon h[\sqrt{\rho_2^2+z^2} - \sqrt{\rho_1^2+z^2}] \\ E_R &= 0 \\ E_z &= 2\pi\varepsilon h\left[\frac{z}{\sqrt{z^2+\rho_1^2}} - \frac{z}{\sqrt{z^2+\rho_2^2}}\right] \end{aligned}\right\} \tag{19}$$

Let us apply these to the solar vortex, studied in Section VII. We have

$$h = 5\times 10^7 \text{ cm}, \qquad \rho_1 = 5\times 10^7 \text{ cm}, \qquad \rho_2 = 5\times 10^9 \text{ cm},$$
$$\varepsilon = -0.052 \text{ electrostatic units.}$$

Let us choose $z = 10^9$, 2×10^9, 3×10^9, 4×10^9, and 5×10^9 cm. We then find:

$z = 10{,}000$ km	20,000	30,000	40,000	50,000
$E_z = -1.3\times 10^7$	-10^7	-0.8×10^7	-0.6×10^7	-0.5×10^7

Here, as we have seen, E_z is the force acting on one electrostatic unit of positive electricity.

The immense values of this force compared with the absence of any appreciable Stark effect over sun-spots[1] seem quite improbable. Even with a much smaller convection whirl the observed magnitude of the Zeeman effect leads us, as Salet has already remarked,[2] to electric forces corresponding to several million volts per centimeter. We have tried to explain this paradox by assuming with Deslandres that the sun's atmosphere is composed of different levels of opposite charges. If one of these levels is in rapid motion about a sun-spot and the corresponding layer relatively quiet, we shall have large magnetic action and a very small resultant electric field over the layers; we must then assume their location to be below the layers accessible to spectroscopic investigations. In this manner we succeed in bringing the electric field to a very small fraction of the value given above, but yet not small enough to explain the absence of the Stark effect.

---

[1] According to a letter from Professor Hale dated June 30, 1914.

[2] *Bulletin de la Société Astronomique de France*, 28, 220, 1914.

It seems as if the only possible explanation will be the hypothesis that the Zeeman effect is due to galvanic currents instead of convection currents. This hypothesis will be developed in another paper.

XIII. THE MOTION OF ELECTRIC CORPUSCLES IN THE SUPERPOSED ELECTRIC AND MAGNETIC FIELDS OVER A SOLAR VORTEX

There is considerable interest in the study of the motion of electric corpuscles over sun-spots because of various applications to the theory of prominences and to the theory of the aurora, especially the latter, if we suppose with Birkeland and others that the aurora is caused by such corpuscles coming from the neighborhood of sun-spots.

If we limit our studies to the highest stratum of the sun's atmosphere, which may be considered as a vacuum, this investigation may be made by the methods which I have applied to the theory of the aurora.[1] In the following we neglect the action of the general magnetic field of the sun.

Let us first consider the motion along the axis of the whirl. This axis being a line of magnetic force, the magnetism has no influence on the motion; we have only to take into account the electrostatic field. In order to consider the most general case, we will take into account the variation of mass with velocity, a hypothesis well founded when the velocity is so great that it is comparable with the velocity of light.

We will follow the equations of Schott.[2] Let ε' be the electric charge of the corpuscle in electrostatic units; then the force acting on it will, with the same notation as before, be $\varepsilon' E_z$ and we obtain

---

[1] See my memoirs in the *Archives des Sciences Physiques et Naturelles*, Geneva, 1907 and 1911–1912.

[2] See *Electromagnetic Radiation and the Mechanical Reactions Arising from It*, Cambridge University Press, 1912, Appendix F.

$$\frac{d}{dt}\left(\frac{cmv}{\sqrt{c^2-v^2}}\right)=\varepsilon' E_z=-\varepsilon'\frac{\delta W}{\delta z}\,,$$

where m is the mass of the corpuscle for zero velocity; c is here the velocity of light, and v the velocity of the corpuscle.

From this equation we find, as Schott has shown,

$$\frac{c^3m}{\sqrt{c^2-v^2}}=-\varepsilon' W+C'\,,$$

C' being a constant of integration, which can be determined if we suppose the velocity given for a given value of z. Let us for instance suppose $v=0$ for $z=z_0$, and let W_0 be the value of W for $z=z_0$. We get

$$c^2m=-\varepsilon' W_0+C'\,,$$

that is,

$$\frac{c^3m}{\sqrt{c^2-v^2}}=c^2m+\varepsilon'(W-W_0)\,.$$

Here

$$W=2\pi\varepsilon h[\sqrt{\rho_2^2+z^2}-\sqrt{\rho_1^2+z^2}]$$

$$W_0=2\pi\varepsilon h[\sqrt{\rho_2^2+z_0^2}-\sqrt{\rho_1^2+z_0^2}]$$

If we consider the motion over the whirl in general, the mathematical treatment is especially interesting for the case in which the current-lines are circles, as they are when $\omega=0$. If we do not then take into account the general magnetic field of the sun but consider only the solar vortex, we have a field that is symmetrical around an axis, which affords very important simplifications in the mathematical treatment.

Let us first suppose the velocity of the corpuscle negligible as compared with the velocity of light. The corpuscle being in motion *in vacuo* and under the action of a superposed electric and magnetic field, we can apply the researches which I have published in the *Comptes Rendus* for March 11, 1911.

The equations of motion are

$$m\frac{d^2x}{dt^2}=\frac{\varepsilon'}{c}\left[H_y\frac{dz}{dt}-H_z\frac{dy}{dt}\right]+\varepsilon' E_x\,,$$

$$m\frac{d^2y}{dt^2}=\frac{\varepsilon'}{c}\left[H_z\frac{dx}{dt}-H_x\frac{dz}{dt}\right]+\varepsilon' E_y\,,$$

$$m\frac{d^2z}{dt^2}=\frac{\varepsilon'}{c}\left[H_x\frac{dy}{dt}-H_y\frac{dx}{dt}\right]+\varepsilon' E_z\,.$$

There the z-axis is the axis of the whirl, H_x, H_y, and H_z are the components of the magnetic force parallel to the X-, Y-, and Z-axes, respectively, and E_x, E_y, and E_z are the components of the electric force. m and ε' are the mass and charge of the corpuscle and $c=3\times 10^{10}$ the velocity of light. ε' is measured in electrostatic units.

Now in our case the components H_R, H_ϕ, and H_z, previously introduced, have the values

$$H_R=\frac{1}{R}\frac{\delta\Phi}{\delta z}\,,\qquad H_\phi=0\,,\qquad H_z=-\frac{1}{R}\frac{\delta\Phi}{\delta R}\,,$$

and

$$H_x=\frac{x}{R}H_R\,,\qquad H_y=\frac{y}{R}H_R\,,$$

where

$$\Phi=\frac{2R}{3\times 10^{10}}\int_0^h\int_{\rho_1}^{\rho_2}\int_0^\pi\frac{\varepsilon\rho v\cos\theta}{D}\,d\theta d\rho d\zeta$$

is the previously introduced function of R and z.

In the same manner

$$E_x=\frac{x}{R}E_R\,,\qquad E_y=\frac{y}{R}E_R$$

and

$$E_R=-\frac{\delta W}{\delta R}\,,\qquad E_z=-\frac{\delta W}{\delta z}\,,$$

where W is the electric potential, also a function of R and z only.

By introducing semipolar co-ordinates, R and ϕ, we get, as in my paper,

$$\frac{1}{2}mv^2 = C - \varepsilon' W$$

$$mR^2 \frac{d\phi}{dt} = C_1 - \frac{\varepsilon'}{c}\Phi$$

which give further

$$\frac{d^2R}{dt^2} = \frac{1}{2}\frac{\delta Q}{\delta R}, \qquad \frac{d^2z}{dt^2} = \frac{1}{2}\frac{\delta Q}{\delta z},$$

$$\left(\frac{dR}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = Q,$$

where

$$Q = \frac{2C}{m} - \left(\frac{C_1 - \frac{\varepsilon'}{c}\Phi}{mR}\right)^2 - 2\frac{\varepsilon'}{m}W.$$

Here v is the velocity and C and C' are two constants of integration. If s denotes the arc along the orbit and θ the angle between the tangent and the plane through the point of contact and the Z-axis, we have

$$\sin\theta = R\frac{d\phi}{ds}$$

and as

$$v = \frac{ds}{dt}$$

we get, as in my paper,

$$\sin\theta = \frac{C_1 - \frac{\varepsilon'}{c}\Phi}{R\sqrt{2mC - 2m\varepsilon' W}}.$$

Hence the orbit must lie within the space defined by

$$-1 \overset{=}{<} \frac{C_1 - \frac{\varepsilon'}{c}\Phi}{R\sqrt{2mC - 2m\varepsilon' W}} \overset{=}{<} 1.$$

The detailed study of these regions is of fundamental importance in the discussion of the orbits, as in the mathematical theory of the aurora, where corresponding inequalities have been found.

As has already been said, the equations of motion take another form if we suppose the velocity of the corpuscle not to be negligible as compared with the velocity of light.

We have then to use the fundamental equations due to Schott,[1] and we get

$$\frac{d}{dt}\left(\frac{mc}{\sqrt{c^2-v^2}}\frac{dx}{dt}\right)=\frac{\varepsilon'}{c}\left[H_y\frac{dz}{dt}-H_z\frac{dy}{dt}\right]+\varepsilon' E_x\,,$$

$$\frac{d}{dt}\left(\frac{mc}{\sqrt{c^2-v^2}}\frac{dy}{dt}\right)=\frac{\varepsilon'}{c}\left[H_z\frac{dx}{dt}-H_x\frac{dz}{dt}\right]+\varepsilon' E_y\,,$$

$$\frac{d}{dt}\left(\frac{mc}{\sqrt{c^2-v^2}}\frac{dz}{dt}\right)=\frac{\varepsilon'}{c}\left[H_x\frac{dy}{dt}-H_y\frac{dx}{dt}\right]+\varepsilon' E_z\,,$$

where *m* is the mass of the corpuscle for zero velocity, and where *c* is the velocity of light.

From this, as Schott has shown, we get

$$\frac{mc^3}{\sqrt{c^2-v^2}}=C'-\varepsilon' W\,,$$

C' being a constant of integration and W the electric potential.

If now we introduce instead of the time t, a new variable τ defined by

$$\frac{dt}{d\tau}=\frac{c}{\sqrt{c^2-v^2}}\,,$$

the equations of motion can be written

$$m\frac{d^2x}{d\tau^2}=\frac{\varepsilon'}{c}\left[H_y\frac{dz}{d\tau}-H_z\frac{dy}{d\tau}\right]+\varepsilon' E'_x\,,$$

$$m\frac{d^2y}{d\tau^2}=\frac{\varepsilon'}{c}\left[H_z\frac{dx}{d\tau}-H_x\frac{dz}{d\tau}\right]+\varepsilon' E_y\,,$$

$$m\frac{d^2z}{d\tau^2}=\frac{\varepsilon'}{c}\left[H_x\frac{dy}{d\tau}-H_y\frac{dx}{d\tau}\right]+\varepsilon' E'_z\,,$$

---

[1] *Loc. cit.*

where

$$E'_x = \frac{x}{R} E'_R, \qquad E'_y = \frac{y}{R} E'_R$$

and

$$E'_R = -\frac{\delta W'}{\delta R}, \qquad E'_z = -\frac{\delta W'}{\delta z}$$

with

$$W' = -\frac{(C' - \varepsilon' W)^2}{2m\varepsilon' c^2}.$$

This remarkable transformation is due to Schott. Further, we find as before

$$mR^2 \frac{d\phi}{d\tau} = C_1 - \frac{\varepsilon'}{c}\Phi$$

and, as Schott has shown in his book (published in 1912),

$$\frac{d^2R}{d\tau^2} = \frac{1}{2}\frac{\delta P}{\delta R}, \quad \frac{d^2z}{d\tau^2} = \frac{1}{2}\frac{\delta P}{\delta z},$$

$$\left(\frac{dR}{d\tau}\right)^2 + \left(\frac{dz}{d\tau}\right)^2 = P,$$

where

$$P = -c^2 - \left(\frac{C_1 - \frac{\varepsilon'}{c}\Phi}{mR}\right)^2 + \left(\frac{C' - \varepsilon' W}{mc}\right)^2.$$

Further,

$$\frac{d\tau}{dt} = \frac{\sqrt{c^2 - v^2}}{c} = \frac{mc^2}{C' - \varepsilon' W},$$

which gives

$$R\frac{d\phi}{dt} = \frac{c^2}{C' - \varepsilon' W} \frac{C_1 - \frac{\varepsilon'}{c}\Phi}{R},$$

and since

$$v = \frac{ds}{dt} = c\frac{\sqrt{(C' - \varepsilon' W)^2 - m^2c^4}}{C' - \varepsilon' W},$$

we find

$$R\frac{d\phi}{ds} = \frac{c}{\sqrt{(C' - \varepsilon' W)^2 - m^2c^4}} \cdot \frac{C_1 - \frac{\varepsilon'}{c}\Phi}{R}.$$

As $R\frac{d\phi}{ds}=\sin\theta$, we have the conditions

$$-1 \leqq \frac{c}{\sqrt{(C'-\varepsilon' W)^2-m^2c^4}}, \qquad \frac{C_1-\frac{\varepsilon'}{c}\Phi}{R} \leqq 1,$$

which define the regions in space to which the orbits are restricted.

We have here a wide field for research which can be carried out by the same methods as those employed for the aurora.

As I have shown in the *Comptes Rendus* for March 6, 1911, November 25, 1912, and February 10 and 17, 1913, and in *Videnskapsselskapets Skrifter*, Christiania, 1913, these considerations may also be applied to other cosmical problems, e.g., to the theory of the solar corona and to cosmogony.

XIV. CONCLUDING REMARKS. IMPORTANCE OF SIMULTANEOUS INVESTIGATIONS OF THE AURORA BOREALIS AND SOLAR PHENOMENA

As we have seen in Section XII, the difficulties in explaining the absence of the Stark effect, supposing the Zeeman effect due to convection currents, lead to the hypothesis of galvanic currents around sun-spots. In the first approximation we can apply the theory of currents in plates and try to determine the current lines and the lines of magnetic force in space over the plate for comparison with the configuration of $H\alpha$ flocculi.

The conclusions based on the hypothesis of convection currents have then to be modified. This will be done in another paper, which is in preparation. We shall return to these hypotheses when more data regarding sun-spot fields have been collected.

In this connection a detailed study of the aurora, its situation, and its altitude will be of considerable interest, especially if we have auroras connected with definite sun-spot phenomena as have sometimes been observed, for it is most probable that the electric corpuscles that cause the aurora are the same as those thrown out from the sun's atmosphere; and having studied their physical properties in the aurora, we can draw corresponding conclusions for the phenomena in the sun. This connection between solar phenomena and auroras, which has been elaborated by physical methods by Professor Birkeland[1] and mathematically by myself,[2] will probably be of importance in future solar work. Furthermore, the photographic method of measuring the altitude of the aurora by means of two stations connected by telephone which I have employed on my expeditions[3] in 1910 and 1913 is capable of giving very exact information regarding the auroral phenomena, thus laying a firm foundation for the conclusions we may draw from them. During the coming period of solar activity, special attention will be paid to these points of view.

Christiania
January 1914

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[1] *The Norwegian Aurora Polaris Expedition*, 1, sections 1 and 2.

[2] *Archives des Sciences Physiques et Naturelles*, Geneva, 1907 and 1911–1912.

[3] "Bericht über eine Expedition nach Bossekop zwecks photographischer Aufnahmen und Höhenmessungen von Nordlichtern," *Videnskapsselskapets Skrifter. I. Math.-Naturw. Klasse*, 1911, No. 17, Christiania; and "On an Auroral Expedition to Bossekop in the Spring of 1913," *Astrophysical Journal*, **38**, 311, 1913.

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